2.15. Hallar las derivadas de las siguientes funciones:

FUNCIÓN	DERIVADA	FUNCIÓN	DERIVADA
1. $y = x^4 + 3x^2 + 6$	$y' = 4x^3 + 6x$	2. $y = 6x^3 - x^2$	$y' = 18x^2 - 2x$
3. $y = 5x^4 - 3x^2 + 6x$	$y' = \dfrac{5x^4}{a} - \dfrac{2x}{b} - 1$	4. $y = 7x^6 + 5x^4 - 3x^2$	$y' = 6ax^2 + \dfrac{2x}{b}$
5. $y = \dfrac{x^5}{a} - \dfrac{x^2}{b} - x$	$y' = 20x^3 - 6x + 6$	6. $y = 2ax^3 + \dfrac{x^2}{b} + c$	$y' = 42x^5 + 20x^3 - 6x$
7. $y = x^2 \cdot (x + 6)$	$y' = 3x^2 + 12x$	8. $y = x^3(x^2 + 1) \cdot (x^3 + 6)$	$y' = 8x^7 + 6x^5 + 30x^4 + 18x^2$
9. $y = (x^2 - 1)(x^2 + 1)$	$y' = 4x^3$	10. $y = (x^2 + 4x - 1) \cdot (3x + 6)$	$y' = 9x^2 + 36x + 21$
11. $y = (3x^2 + 6x + 1) \cdot (x^2 - 1)$	$y' = 12x^3 + 18x^2 - 4x - 6$	12. $y = (x^3 - 1) \cdot (x^3 + 1)$	$y' = 6x^5$
13. $y = \dfrac{x^2}{a} - \dfrac{x}{b} + \dfrac{a^2}{x^2} - \dfrac{b}{x}$	$y' = \dfrac{2x}{a} - \dfrac{1}{b} - \dfrac{2a^2}{x^3} - \dfrac{b}{x^2}$	14. $y = \dfrac{2x^4}{b^2 - x^2}$	$y' = \dfrac{4x^3(2b^2 - x^2)}{(b^2 - x^2)^2}$
15. $y = \dfrac{a - x}{a + x}$	$y' = \dfrac{-2a}{(a + x)^2}$	16. $y = \dfrac{x^3}{1 + x^2}$	$y' = \dfrac{x^2(x^2 + 3)}{(1 + x^2)^2}$
17. $y = \dfrac{x^3 + 1}{x^2 - x - 2}$	$y' = \dfrac{x^2 - 4x + 1}{(x - 2)^2}$	18. $y = \dfrac{6}{x^4 + 2}$	$y' = \dfrac{-24x^3}{(x^4 + 2)^2}$
19. $y = \dfrac{1}{x^2 + 1}$	$y' = \dfrac{-2x}{(x^2 + 1)^2}$	20. $y = \dfrac{x^3 + 2}{7}$	$y' = \dfrac{3x^2}{7}$
21. $y = \dfrac{1}{x^2 - 2x + 1}$	$y' = \dfrac{-2}{(x - 1)^3}$	22. $y = \dfrac{(x + 4)^2}{x + 3}$	$y' = \dfrac{(x + 4)(x + 2)}{(x + 3)^2}$
23. $y = \dfrac{10}{x^2 + 1}$	$y' = \dfrac{-20x}{(x^2 + 1)^2}$	24. $y = \dfrac{1}{x + 1}$	$y' = \dfrac{-1}{(x + 1)^2}$
25. $y = \dfrac{3x + 4}{x^2 + 8x - 9}$	$y' = -\dfrac{3x^2 + 8x + 59}{(x + 9)^2(x - 1)^2}$	26. $y = \dfrac{x^2 + 5}{2x - 3}$	$y' = \dfrac{2(x^2 - 3x - 5)}{(2x - 3)^2}$
27. $y = \dfrac{6 + \sqrt{3}}{2\sqrt{3}\, x^2}$	$y' = -\dfrac{2\sqrt{3} + 1}{x^3}$	28. $y = 6x^{\frac{7}{2}} + 4x^{\frac{5}{2}} + 2x$	$y' = 21x^{\frac{5}{2}} + 10x^{\frac{3}{2}} + 2$
29. $y = \sqrt{3x} + \sqrt[3]{x} + \dfrac{1}{x}$	$y' = \dfrac{\sqrt{3}}{\sqrt{x}} + \dfrac{1}{3\sqrt[3]{x^2}} - \dfrac{1}{x^2}$	30. $y = \dfrac{(x + 1)^3}{\sqrt{x^3}}$	$y' = \dfrac{3(x - 1)(x + 1)^2}{2\sqrt{x^5}}$
31. $y = (2x^2 - 3)^2$	$y' = 8x(2x^2 - 3)$	32. $y = (x^2 + a^2)^5$	$y' = 10x(x^2 + a^2)^4$
33. $y = \dfrac{ax^2}{\sqrt[3]{x}} + \dfrac{b}{x\sqrt{x}}$	$y' = \dfrac{5a\sqrt[3]{x^2}}{3} - \dfrac{3b}{2\sqrt{x^5}}$	34. $y = \sqrt{x^2 + a^2}$	$y' = \dfrac{x}{\sqrt{x^2 + a^2}}$
35. $y = \dfrac{2x^2 - 1}{x\sqrt{1 + x^2}}$	$y' = \dfrac{4x^2 + 1}{x^2\sqrt{(1 + x^2)^3}}$	36. $y = \left(1 + \sqrt[3]{x}\right)^3$	$y' = \left(\dfrac{1 + \sqrt[3]{x}}{\sqrt[3]{x}}\right)^2$
37. $y = 3x^{-6} - 2x^{-7} - 3$	$y' = 18x^{-7} + 14x^{-8}$	38. $y = 3x^{-1/2} - 3x^{-2/3} + 7$	$y' = -\dfrac{3}{2}x^{-3/2} + 2x^{-5/3}$
39. $y = \dfrac{x^6}{(3x + 2)^4}$	$y' = \dfrac{6x^5(x + 2)}{(3x + 2)^5}$	40. $y = \left(\dfrac{2x + 1}{3x + 2}\right)^5$	$y' = \dfrac{5(2x + 1)^4}{(3x + 2)^6}$

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41. $y = \left(\dfrac{x+1}{x-1}\right)^6$	$y' = -\dfrac{12(x+1)^5}{(x-1)^7}$	42. $y = \sqrt{ax+b}$	$y' = \dfrac{a}{2\sqrt{ax+b}}$
43. $y = \dfrac{x}{\sqrt{1+x}}$	$y' = \dfrac{x+2}{2\sqrt{(1+x)^3}}$	44. $y = \sqrt{3+\sqrt{x}}$	$y' = \dfrac{1}{4\sqrt{x}\sqrt{3+\sqrt{x}}}$
45. $y = \dfrac{a+\sqrt{x}}{a-\sqrt{x}}$	$y' = \dfrac{a}{\sqrt{x}\left(a-\sqrt{x}\right)^2}$	46. $y = 7^{x^2+2x}$	$y' = 2(x+1)7^{x^2+2x}\ln 7$
47. $y = b^{a^2-x^2}$	$y' = -2xb^{a^2-x^2}\ln b$	48. $y = ae^{\sqrt{x}}$	$y' = \dfrac{ae^{\sqrt{x}}}{2\sqrt{x}}$
49. $y = \dfrac{e^x-1}{e^x+1}$	$y' = \dfrac{2e^x}{\left(e^x+1\right)^2}$	50. $y = \dfrac{a}{2}\left(e^{x/a}-e^{-x/a}\right)$	$y' = \dfrac{1}{2}\left(e^{x/a}+e^{-x/a}\right)$
51. $y = (1+x+x^2)\,e^x$	$y' = (x+1)(x+2)\,e^x$	52. $y = \dfrac{a^x}{x}$	$y' = \dfrac{a^x(x\ln a-1)}{x^2}$
53. $y = (x-1)e^x$	$y' = xe^x$	54. $y = a^x \cdot x^a$	$y' = a^x \cdot x^a\left(\dfrac{a}{x}+\ln a\right)$
55. $y = e^{4x^2-1}$	$y' = 8x\,e^{4x^2-1}$	56. $y = 3^{2x-3}$	$y' = 2\cdot 3^{2x-3}\cdot \ln 3$
57. $y = \dfrac{1}{2}e^{1+x^2}$	$y' = x\,e^{1+x^2}$	58. $y = -(x^2+2x+2)e^{-x}$	$y' = x^2\,e^{-x}$
59. $y = \ln(ax+b)$	$y' = \dfrac{a}{ax+b}$	60. $y = x\ln x$	$y' = 1+\ln x$
61. $y = \ln^3 x$	$y' = \dfrac{3\ln^2 x}{x}$	62. $y = \ln(x^2+x)$	$y' = \dfrac{2x+1}{x^2+x}$
63. $y = \ln\left(x+\sqrt{1+x^2}\right)$	$y' = \dfrac{1}{\sqrt{1+x^2}}$	64. $y = 2a\ln(x+a)-x$	$y' = \dfrac{a-x}{a+x}$
65. $y = \ln\left(\dfrac{1+x^2}{1-x^2}\right)$	$y' = \dfrac{4x}{1-x^4}$	66. $y = \ln(\ln x)$	$y' = \dfrac{1}{x\ln x}$
67. $y = \ln\left(\dfrac{e^x}{1+e^x}\right)$	$y' = \dfrac{1}{1+e^x}$	68. $y = \log_a(5x^2-3)$	$y' = \dfrac{10x}{5x^2-3}\cdot \dfrac{1}{\ln a}$
69. $y = 3\ln(x-5)-2\ln(x+1)$	$y' = \dfrac{x+13}{(x+1)(x-5)}$	70. $y = \log_3(1+x^2)$	$y' = \dfrac{2x}{1+x^2}\cdot \dfrac{1}{\ln 3}$
71. $y = 2\sqrt{x}-2\ln\left(2+\sqrt{x}\right)$	$y' = \dfrac{\sqrt{x}+1}{\sqrt{x}\left(2+\sqrt{x}\right)}$	72. $y = \log_a(3x^2+5)$	$y' = \dfrac{6x}{3x^2+5}\cdot \dfrac{1}{\ln a}$
73. $y = \ln\left(\dfrac{x}{\sqrt{x^2+a^2}}\right)$	$y' = \dfrac{a^2}{x(x^2+a^2)}$	74. $y = \log_2\left[x(x^2+1)\right]$	$y' = \dfrac{3x^2+1}{x(x^2+1)}\cdot \dfrac{1}{\ln 2}$
75. $y = \log_5(x^2+x-1)$	$y' = \dfrac{2x+1}{x^2+x-1}\cdot \dfrac{1}{\ln 5}$	76. $y = \dfrac{1}{6}\cdot \ln\left(\dfrac{x-3}{x+3}\right)$	$y' = \dfrac{1}{x^2-9}$
77. $y = \ln\left(\sqrt{x}\right)$	$y' = \dfrac{1}{2x}$	78. $y = \dfrac{x^8}{8}\cdot\left(\ln x-\dfrac{1}{8}\right)$	$y' = x^7\ln x$

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79. $y = \text{sen}(x^2)$	$y' = 2x\cos(x^2)$	80. $y = \dfrac{\text{sen}\,x + \cos x}{\cos x - \text{sen}\,x}$	$y' = \dfrac{2}{(\cos x - \text{sen}\,x)^2}$
81. $y = e^{\text{sen}x}$	$y' = \cos x \cdot e^{\text{sen}\,x}$	82. $y = \dfrac{\text{sen}\,x \cdot \cos x}{2} + \dfrac{x}{2}$	$y' = \cos^2 x$
83. $y = a^{\cos(x^2)}$	$y' = -2x\,\text{sen}(x^2)\cdot a^{\cos(x^2)}\cdot \ln a$	84. $y = -\ln(\cos x)$	$y' = \text{tg}\,x$
85. $y = \ln\left(\dfrac{1 + \text{sen}x}{\cos x}\right)$	$y' = \dfrac{1}{\cos x}$	86. $y = \ln\left(\dfrac{1}{\text{sen}x} - \text{ctg}\,x\right)$	$y' = \dfrac{1}{\text{sen}\,x}$
87. $y = \dfrac{\cos x}{1 - \text{sen}x}$	$y' = \dfrac{1}{1 - \text{sen}x}$	88. $y = \text{sen}^4 x + \cos^4 x$	$y' = -\text{sen}(4x)$
89. $y = \text{tg}^3 x - 3\text{tg}\,x + 3x$	$y' = 3\text{tg}^4 x$	90. $y = \text{tg}\,x + \dfrac{2}{3}\text{tg}^3 x + \dfrac{1}{5}\text{tg}^5 x$	$y' = (1 + \text{tg}^2 x)^3$
91. $y = -\dfrac{\cos x}{2\text{sen}^2 x} + \dfrac{1}{2}\cdot \ln\left(\text{tg}\,\dfrac{x}{2}\right)$	$y' = \dfrac{1}{\text{sen}^3 x}$	92. $y = \dfrac{\text{tg}^2 x}{2} + \ln\cos x$	$y' = \text{tg}^3 x$
93. $y = \text{arc tg}\left(\dfrac{x + a}{1 - ax}\right)$	$y' = \dfrac{1}{1 + x^2}$	94. $y = \text{arctg}\left(\dfrac{x}{\sqrt{a^2 - x^2}}\right)$	$y' = \dfrac{1}{\sqrt{a^2 - x^2}}$
95. $y = e^{\text{arctg}\,x}$	$y' = \dfrac{1}{1 + x^2}e^{\text{arctg}\,x}$	96. $y = \text{arctg}\,3^x$	$y' = \dfrac{3^x \cdot \ln 3}{2\cdot(1 + 3^{2x})}$
97. $y = \sqrt{e^x \cos x}$	$y' = e^x\dfrac{\cos x - \text{sen}x}{2\sqrt{e^x \cos x}}$	98. $y = \text{arctg}(4x^3)$	$y' = \dfrac{12x^2}{1 + 16x^6}$
99. $y = e^{\ln\sqrt{1 - \text{sen}^2 x}}$	$y' = -\text{sen}\,x$	100. $y = \text{tg}(e^{4x})$	$y' = 4\,e^{4x}(1 + \text{tg}^2(e^{4x}))$
101. $y = e^x \text{sen}(3x)$	$y' = e^x(\text{sen}\,3x + 3\cos 3x)$	102. $y = \cos^2 6x - \text{sen}^2 6x$	$y' = -12\,\text{sen}(12x)$

2.16. Calcula la derivada de las siguientes funciones, aplicando la derivación logarítmica:

1. $y = x^x$
2. $y = x^{1/x}$
3. $y = x^{\ln x}$
4. $y = x^{\cos x}$
5. $y = \text{tg}\,x^{\text{sen}\,x}$
6. $y = (x + 1)^x$

2.17. Se considera la función:

$$f(x) = \text{arc sen}\left(\dfrac{2x^2 - 1}{\sqrt{1 - 4x^4}}\right)$$

Demostrar que $f'(x) = \dfrac{\sqrt{1 - 2x^2}}{p(x)}$, siendo p(x) un polinomio de grado 4 cuya expresión se determinará.