

1º) Calcular "x" en las siguientes expresiones:

a) $2^x = 64$ b) $x = \log_3 5$ c) $6^{2x} = 7$ d) $3 \cdot \log 9 + 2 \cdot \log x^3 = 5$

e) $4^x = 5$ f) $\log_x \left(\frac{1}{9}\right) = -2$ g) $\log_5 x = 3$

2º) Razonar si son ciertas las igualdades siguientes, sabiendo que $a \neq 1$:

a) $\frac{\log\left(\frac{1}{a}\right) - \log \sqrt{a}}{\log a^2} = \frac{3}{4}$

b) $\frac{\log \sqrt[3]{a} + \log\left(\frac{1}{a}\right)}{\log a^2} = -\frac{2}{9}$

3º) Determina los intervalos siguientes y representalos:

a) $\{x \in \mathbb{R} : 3 < x \leq 10\}$ b) $x \geq 2$ c) $|x - 5| \leq 3$

d) $|x + 4| < 1$ e) $|x - 3| > 2$

4º) Calcular:

a) $\sqrt[3]{2 \cdot \sqrt{5}} \cdot \sqrt[4]{2^2 \cdot 5}$

b) $2 \cdot \sqrt{18} - 3 \cdot \sqrt{20} - 5 \cdot \sqrt{125} + 7 \cdot \sqrt{32}$

5º) Racionalizar:

a) $\frac{\sqrt{2} + 3}{\sqrt{5}}$

b) $\frac{4 + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$

c) $\frac{3}{\sqrt[4]{5}}$

SOLUCIONES

1º a) $2^x = 64 \Rightarrow 2^x = 2^6 \Rightarrow \boxed{x=6}$

b) $x = \log_3 5 \Rightarrow \boxed{x=1'4649}$ con calculadora.

c) $6^{2x} = 7 \Rightarrow \log 6^{2x} = \log 7 \Rightarrow 2x \cdot \log 6 = \log 7 \Rightarrow x = \frac{\log 7}{2 \cdot \log 6} = \boxed{0'543}$

d) $3 \log 9 + 2 \cdot \log x^3 = 5 \Rightarrow \log 9^3 + \log (x^3)^2 = 5 \Rightarrow \log 729 + \log x^6 = 5 \Rightarrow$
 $\Rightarrow \log (729 \cdot x^6) = 5 \Rightarrow 10^5 = 729 \cdot x^6 \Rightarrow x^6 = \frac{10^5}{729} \Rightarrow x^6 = 137'1742 \Rightarrow$
 $\Rightarrow x = \sqrt[6]{137'1742} \Rightarrow \boxed{x=2'27}$

e) $4^x = 5 \Rightarrow \log 4^x = \log 5 \Rightarrow x \cdot \log 4 = \log 5 \Rightarrow x = \frac{\log 5}{\log 4} \Rightarrow \boxed{x=1'16}$

f) $\log_x \left(\frac{1}{9}\right) = 2 \Rightarrow \bar{x}^2 = \frac{1}{9} \Rightarrow \bar{x}^2 = \frac{1}{3^2} \Rightarrow \bar{x}^2 = 3^{-2} \Rightarrow \boxed{x=3}$

g) $\log_5 x = 3 \Rightarrow x = 5^3 \Rightarrow \boxed{x=125}$

2º a) $\frac{\log(\frac{1}{a}) - \log \sqrt{a}}{\log a^2} = \frac{\log 1 - \log a - \log a^{1/2}}{2 \cdot \log a} =$

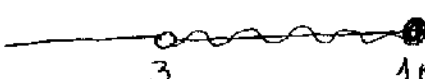
$= \frac{-\log a - \frac{1}{2} \log a}{2 \cdot \log a} = \frac{-\frac{3}{2} \log a}{2 \cdot \log a} = \frac{-3/2}{2} = -\frac{3}{4} \quad \boxed{\text{FALSO}}$

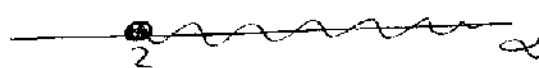
b) $\frac{\log \sqrt[3]{a} + \log(\frac{1}{a})}{\log a^3} = \frac{\log a^{1/3} + \log 1 - \log a}{3 \cdot \log a} =$

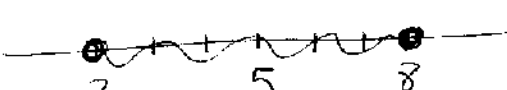
$= \frac{\frac{1}{3} \log a - \log a}{3 \log a} = \frac{-\frac{2}{3} \log a}{3 \cdot \log a} = \frac{-2/3}{3} = -\frac{2}{9}$

VERDADERO

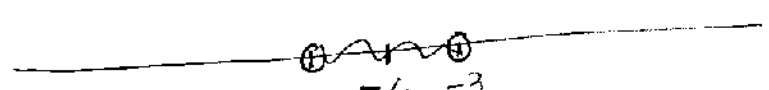
3°

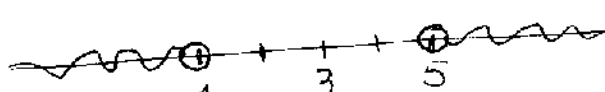
a) $\{x \in \mathbb{R} : 3 < x \leq 10\} \Rightarrow (3, 10]$ 

b) $x \geq 2 \Rightarrow [2, +\infty)$ 

c) $|x-5| \leq 3$  $[2, 8]$

d) $|x+4| < 1 \Rightarrow |x - (-4)| < 1$

 $(-5, -3)$

e) $|x-3| > 2$ 

$(-\infty, 1) \cup (5, +\infty)$

4°

a) $\sqrt[3]{2 \cdot \sqrt{5}} \cdot \sqrt[4]{2^2 \cdot 5} = \sqrt[6]{2^2 \cdot 5} \cdot \sqrt[4]{2^2 \cdot 5} =$

$= \sqrt[12]{(2^2 \cdot 5)^2} \cdot \sqrt[12]{(2^2 \cdot 5)^3} = \sqrt[12]{(2^2 \cdot 5)^5} = \sqrt[12]{2^{10} \cdot 5^5}$

b)

$$\begin{aligned}
 & 2 \cdot \sqrt{18} - 3 \cdot \sqrt{20} - 5 \cdot \sqrt{125} + 7 \cdot \sqrt{32} = \\
 & = 2 \cdot \sqrt{3^2 \cdot 2} - 3 \cdot \sqrt{2^2 \cdot 5} - 5 \cdot \sqrt{5^3} + 7 \cdot \sqrt{2^5} = \\
 & = 2 \cdot 3 \cdot \sqrt{2} - 3 \cdot 2 \cdot \sqrt{5} - 5 \cdot 5 \cdot \sqrt{5} + 7 \cdot 2^2 \cdot \sqrt{2} = \\
 & = 6\sqrt{2} - 6\sqrt{5} - 25\sqrt{5} + 28\sqrt{2} = \boxed{34\sqrt{2} - 31\sqrt{5}}
 \end{aligned}$$

5° a) $\frac{\sqrt{2}+3}{\sqrt{5}} = \frac{(\sqrt{2}+3) \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{10}+3\sqrt{5}}{\sqrt{5^2}} = \boxed{\frac{\sqrt{10}+3\sqrt{5}}{5}}$

b) $\frac{4+\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{(4+\sqrt{2}) \cdot (\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2}) \cdot (\sqrt{5}+\sqrt{2})} = \frac{4\sqrt{5}+4\sqrt{2}+\sqrt{10}+\sqrt{2}^2}{(\sqrt{5})^2 - (\sqrt{2})^2} =$

$$= \frac{4\sqrt{5}+4\sqrt{2}+\sqrt{10}+2}{5-2} = \boxed{\frac{4\sqrt{5}+4\sqrt{2}+\sqrt{10}+2}{3}}$$

c) $\frac{3}{\sqrt[4]{5}} = \frac{3 \cdot \sqrt[4]{5^3}}{\sqrt[4]{5} \cdot \sqrt[4]{5^3}} = \frac{3 \cdot \sqrt[4]{5^3}}{\sqrt[4]{5^4}} = \boxed{\frac{3 \cdot \sqrt[4]{5^3}}{5}}$