

ECUACIONES DE GRADO SUPERIOR A 2

Resolver las siguientes ecuaciones:

1) $x^3 + 3x^2 - x - 3 = 0$

	1	3	-1	-3
1		1	4	3
	1	4	3	0
-1		-1	-3	
	1	3	0	

$$x^3 + 3x^2 - x - 3 = (x - 1)(x + 1)(x + 3)$$

$$(x - 1)(x + 1)(x + 3) = 0 \Rightarrow \begin{cases} x - 1 = 0 \Rightarrow \boxed{x_1 = 1} \\ x + 1 = 0 \Rightarrow \boxed{x_2 = -1} \\ x + 3 = 0 \Rightarrow \boxed{x_3 = -3} \end{cases}$$

2) $x^3 + 2x^2 + 2x + 1 = 0$

	1	2	2	1
-1		-1	-1	-1
	1	1	1	0

$$x^3 + 2x^2 + 2x + 1 = (x + 1) \cdot (x^2 + x + 1)$$

$$(x + 1)(x^2 + x + 1) = 0 \Rightarrow \begin{cases} x + 1 = 0 \Rightarrow \boxed{x_1 = -1} \\ x^2 + x + 1 = 0 \end{cases}$$

Las otras dos raíces las calculamos aplicando la fórmula de la ecuación de segundo grado:

$$x^2 + x + 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1 - 4}}{2} \rightarrow \text{No tiene solución}$$

3) $x^3 + 3x^2 - 4x - 12 = 0$

	1	3	-4	-12
2		2	10	12
	1	5	6	0
-2		-2	-6	
	1	3	0	

$$x^3 + 3x^2 - 4x - 12 = (x - 2)(x + 2)(x + 3)$$

$$(x - 2)(x + 2)(x + 3) = 0 \Rightarrow \begin{cases} x - 2 = 0 \Rightarrow \boxed{x_1 = 2} \\ x + 2 = 0 \Rightarrow \boxed{x_2 = -2} \\ x + 3 = 0 \Rightarrow \boxed{x_3 = -3} \end{cases}$$

4) $x^3 - x^2 - x + 1 = 0$

	1	-1	-1	1
-1		-1	2	-1
	1	-2	1	0
1		1	-1	
	1	-1	0	

$$x^3 - x^2 - x + 1 = (x + 1)(x - 1)^2$$

$$(x + 1)(x - 1)^2 = 0 \Rightarrow \begin{cases} x - 1 = 0 \Rightarrow \boxed{x_1 = 1} \\ x + 1 = 0 \Rightarrow \boxed{x_2 = -1} \end{cases}$$

5) $x^3 - 2x^2 - 4x + 8 = 0$

	1	-2	-4	8
2		2	0	-8
	1	0	-4	0
2		2	4	
	1	2	0	

$$x^3 - 2x^2 - 4x + 8 = (x + 2)(x - 2)^2$$

$$(x + 2)(x - 2)^2 = 0 \Rightarrow \begin{cases} x - 2 = 0 \Rightarrow \boxed{x_1 = 2} \\ x + 2 = 0 \Rightarrow \boxed{x_2 = -2} \end{cases}$$

6) $6x^3 + 7x^2 - 9x + 2 = 0$

$\begin{array}{c cccc} & 6 & 7 & -9 & 2 \\ -2 & & -12 & 10 & -2 \\ \hline & 6 & -5 & 1 & 0 \end{array}$	$6x^3 + 7x^2 - 9x + 2 = (x + 2)(6x^2 - 5x + 1)$ $(x + 2)(6x^2 - 5x + 1) = 0 \Rightarrow \begin{cases} x + 2 = 0 \Rightarrow \boxed{x_1 = -2} \\ 6x^2 - 5x + 1 = 0 \end{cases}$
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$$6x^2 - 5x + 1 = 0 \Rightarrow x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 6 \cdot 1}}{6 \cdot 2} = \frac{5 \pm 1}{12} = \begin{cases} x_2 = \frac{5+1}{12} = \frac{6}{12} \Rightarrow \boxed{x_2 = \frac{1}{2}} \\ x_3 = \frac{5-1}{12} = \frac{4}{12} \Rightarrow \boxed{x_3 = \frac{1}{3}} \end{cases}$$

7) $x^4 - 1 = 0$

$$x^4 - 1 = 0 \Rightarrow x^4 = 1 \Rightarrow x = \sqrt[4]{1} = \pm 1 \Rightarrow \boxed{x = \pm 1}$$

8) $8x^3 - 14x^2 + 7x - 1 = 0$

$\begin{array}{c cccc} & 8 & -14 & 7 & -1 \\ 1 & & 8 & -6 & 1 \\ \hline & 8 & -6 & 1 & 0 \end{array}$	$8x^3 - 14x^2 + 7x - 1 = (x + 2)(8x^2 - 6x + 1)$ $(x - 1)(8x^2 - 6x + 1) = 0 \Rightarrow \begin{cases} x - 1 = 0 \Rightarrow \boxed{x_1 = 1} \\ 8x^2 - 6x + 1 = 0 \end{cases}$
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$$8x^2 - 6x + 1 = 0 \Rightarrow x = \frac{6 \pm \sqrt{6^2 - 4 \cdot 8 \cdot 1}}{8 \cdot 2} = \frac{6 \pm 2}{16} = \begin{cases} x_2 = \frac{6+2}{16} = \frac{8}{16} \Rightarrow \boxed{x_2 = \frac{1}{2}} \\ x_3 = \frac{6-2}{16} = \frac{4}{16} \Rightarrow \boxed{x_3 = \frac{1}{4}} \end{cases}$$

9) $2x^4 - 5x^3 + 5x - 2 = 0$

$\begin{array}{c ccccc} & 2 & -5 & 0 & 5 & -2 \\ 1 & & 2 & -3 & -3 & 2 \\ \hline & 2 & -3 & -3 & 2 & 0 \\ -1 & & -2 & 5 & -2 & \\ \hline & 2 & -5 & 2 & 0 & \end{array}$	$2x^4 - 5x^3 + 5x - 2 = (x + 1)(x - 1)(2x^2 - 5x + 2)$ $(x + 1)(x - 1)(2x^2 - 5x + 2) = 0 \Rightarrow \begin{cases} x - 1 = 0 \Rightarrow \boxed{x_1 = 1} \\ x + 1 = 0 \Rightarrow \boxed{x_2 = -1} \\ 2x^2 - 5x + 2 = 0 \end{cases}$
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$$2x^2 - 5x + 2 = 0 \Rightarrow x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot 2}}{2 \cdot 2} = \frac{5 \pm 3}{4} = \begin{cases} x_3 = \frac{5+3}{4} = \frac{8}{4} \Rightarrow \boxed{x_3 = 2} \\ x_4 = \frac{5-3}{4} = \frac{2}{4} \Rightarrow \boxed{x_4 = \frac{1}{2}} \end{cases}$$

10) $x^4 + 2x^2 + 3 = 0$

$$y^2 + 2y + 3 = 0 \Rightarrow y = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 3}}{2} \Rightarrow \boxed{\text{No hay solución real}}$$