

FRACCIONES ALGEBRAICAS

1.- Factoriza y simplifica al máximo las siguientes fracciones algebraicas:

$$\begin{array}{llll} \text{a)} \frac{5x^2}{2x^2 - x} & \text{b)} \frac{x^2 - 3x + 2}{x^2 - 2x + 1} & \text{c)} \frac{x^2 - 6x + 9}{2x - 6} & \text{d)} \frac{x^2 - 4x + 4}{x^2 - 4} \\ \text{e)} \frac{x^2 - 5x}{x^2 - 6x + 5} & \text{f)} \frac{x^2 - 9}{x^2 + x - 12} & \text{g)} \frac{x^2 - 2x}{2x - 4} & \text{h)} \frac{x^2 - x - 20}{x^2 + x - 12} \end{array}$$

2.- Reducir a mínimo común denominador las siguientes fracciones:

$$\begin{array}{lll} \text{a)} \frac{x+1}{x^2-4} ; \frac{x^2}{x+2} & \text{b)} \frac{x-1}{x^2-5x+4} ; \frac{5}{x^2-1} & \text{c)} \frac{1}{x+2} ; \frac{4}{x-2} ; \frac{3x}{x^2-4} \end{array}$$

3.- Realizar las siguientes operaciones con fracciones:

$$\begin{array}{lll} \text{a)} \frac{3x^2+1}{3x} - \frac{x+1}{3x} & \text{b)} \frac{-2x}{x^2} + \frac{4x+1}{2x^2} & \text{c)} \frac{3}{x-2} + \frac{1}{2+x} \\ \text{d)} \frac{2x}{x-1} - \frac{3x+1}{x^2-1} & \text{e)} x - \frac{x}{x-1} & \text{f)} \frac{3}{x-2} - \frac{x+1}{x^2-4} \end{array}$$

4.- Efectuar los siguientes productos de fracciones:

$$\begin{array}{lll} \text{a)} \frac{-a}{4x^2} \cdot \frac{x}{2a^2} & \text{b)} \frac{3mn^2}{x^2} \cdot \frac{-2x}{nm^3} & \text{c)} \frac{2x}{1-x^2} \cdot \frac{x^2+2x+1}{4x^2+4x} \\ \text{d)} \frac{x^2-16}{2x} \cdot \frac{4x^2}{x^2-5x+4} & \text{e)} \frac{2x+2}{x^2-1} \cdot \frac{x^2-x}{x+2} & \text{f)} \frac{x^2-2x}{3x+1} \cdot \frac{9x^2-1}{x^2-4} \end{array}$$

5.- Efectuar los siguientes cocientes de fracciones:

$$\begin{array}{lll} \text{a)} \frac{3x^2}{y^3} : \frac{-6x}{4y^2} & \text{b)} \frac{ax^2}{4b} : \frac{-ax}{b} & \text{c)} \frac{x+2}{x+1} : \frac{x^2+4x}{2x+2} \\ \text{d)} \frac{2x+1}{x+3} : \frac{4x^2-1}{x^2+3x} & \text{e)} \frac{x+1}{x-3} : \frac{x^2-1}{x^2-x-6} & \text{f)} \frac{x^2-5x+6}{x^2-x-2} : \frac{x^2+2x}{x^2+x} \end{array}$$

SOLUCIONES

1.- Factoriza y simplifica al máximo las siguientes fracciones algebraicas:

$$\text{a) } \frac{5x^2}{2x^2 - x} = \frac{5x^2}{x(2x - 1)} = \frac{5x}{2x - 1}$$

$$\text{b) } \frac{x^2 - 3x + 2}{x^2 - 2x + 1} = \frac{(x - 2)(x - 1)}{(x - 1)(x - 1)} = \frac{x - 2}{x - 1}$$

$$\text{c) } \frac{x^2 - 6x + 9}{2x - 6} = \frac{(x - 3)(x - 3)}{2(x - 3)} = \frac{x - 3}{2}$$

$$\text{d) } \frac{x^2 - 4x + 4}{x^2 - 4} = \frac{(x - 2)(x - 2)}{(x + 2)(x - 2)} = \frac{x - 2}{x + 2}$$

$$\text{e) } \frac{x^2 - 5x}{x^2 - 6x + 5} = \frac{x(x - 5)}{(x - 1)(x - 5)} = \frac{x}{x - 1}$$

$$\text{f) } \frac{x^2 - 9}{x^2 + x - 12} = \frac{(x - 3)(x + 3)}{(x - 3)(x + 4)} = \frac{x + 3}{x + 4}$$

$$\text{g) } \frac{x^2 - 2x}{2x - 4} = \frac{x(x - 2)}{2(x - 2)} = \frac{x}{2}$$

$$\text{h) } \frac{x^2 - x - 20}{x^2 + x - 12} = \frac{(x + 4)(x - 5)}{(x + 4)(x - 3)} = \frac{x - 5}{x - 3}$$

2.- Reducir a mínimo común denominador las siguientes fracciones:

$$\text{a) } \frac{2}{a^2b^3}; \frac{1}{ab^2}$$

$$\text{m.c.m. } (a^2b^3, ab^2) = a^2b^3 \Rightarrow \frac{2}{a^2b^3}; \frac{ab}{a^2b^3}$$

$$\text{b) } \frac{x - 1}{x^2 - 5x + 4}; \frac{5}{x^2 - 1}$$

$$\left. \begin{array}{l} x^2 - 5x + 4 = (x - 4)(x - 1) \\ x^2 - 1 = (x + 1)(x - 1) \end{array} \right\} \Rightarrow \text{mcm } (x^2 - 5x + 4, x^2 - 1) = (x - 4)(x + 1)(x - 1) \Rightarrow$$

$$\left\{ \begin{array}{l} \frac{x - 1}{x^2 - 5x + 4} = \frac{(x - 1)(x + 1)}{(x - 4)(x - 1)(x + 1)} = \frac{x^2 - 1}{x^3 - 4x^2 - x + 4} \\ \frac{5}{x^2 - 1} = \frac{5(x - 4)}{(x^2 - 1)(x - 4)} = \frac{5x - 20}{x^3 - 4x^2 - x + 4} \end{array} \right.$$

$$\text{c) } \frac{1}{x + 2}; \frac{4}{x - 2}; \frac{3x}{x^2 - 4}$$

$$x^2 - 4 = (x - 2)(x + 2) \Rightarrow \text{mcm } (x + 2, x - 2, x^2 - 4) = x^2 - 4$$

$$\left\{ \begin{array}{l} \frac{1}{x + 2} = \frac{(x - 2)}{(x + 2)(x - 2)} = \frac{x - 2}{x^2 - 4} \\ \frac{4}{x - 2} = \frac{4(x + 2)}{(x - 2)(x + 2)} = \frac{4x + 8}{x^2 - 4} \\ \frac{3x}{x^2 - 4} \end{array} \right.$$

3.-Realizar las siguientes operaciones con fracciones:

$$a) \frac{3x^2 + 1}{3x} - \frac{x + 1}{3x} = \frac{3x^2 + 1 - x - 1}{3x} = \frac{3x^2 - x}{3x} = \frac{x(3x - 1)}{3x} = \frac{3x - 1}{3}$$

$$b) \frac{-2x}{x^2} + \frac{4x + 1}{2x^2} = \frac{-4x}{2x^2} + \frac{4x + 1}{2x^2} = \frac{1}{2x^2}$$

$$c) \frac{3}{x - 2} + \frac{1}{2 + x} = \frac{3(2 + x)}{(2 + x)(x - 2)} + \frac{(x - 2)}{(x - 2)(2 + x)} = \frac{6 + 3x + x - 2}{(x - 2)(2 + x)} = \frac{4 + 4x}{x^2 - 4}$$

$$d) \frac{2x}{x - 1} - \frac{3x + 1}{x^2 - 1} = \frac{2x(x + 1)}{(x - 1)(x + 1)} - \frac{3x + 1}{(x - 1)(x + 1)} = \frac{2x^2 + 2x - 3x - 1}{(x - 1)(x + 1)} = \frac{2x^2 - x - 1}{(x - 1)(x + 1)} = \frac{(2x + 1)(x - 1)}{(x - 1)(x + 1)} = \frac{2x + 1}{x + 1}$$

$$e) x - \frac{x}{x - 1} = \frac{x(x - 1)}{x - 1} - \frac{x}{x - 1} = \frac{x(x - 1) - x}{x - 1} = \frac{x^2 - x - x}{x - 1} = \frac{x^2 - 2x}{x - 1}$$

$$f) \frac{3}{x - 2} - \frac{x + 1}{x^2 - 4} = \frac{3(x + 2)}{(x - 2)(x + 2)} - \frac{x + 1}{(x - 2)(x + 2)} = \frac{3x + 6 - x - 1}{(x - 2)(x + 2)} = \frac{2x + 5}{x^2 - 4}$$

4.- Efectuar los siguientes productos de fracciones:

$$a) \frac{-a}{4x^2} \cdot \frac{x}{2a^2} = \frac{-ax}{4x^2 \cdot 2a^2} = \frac{-ax}{4x^2 \cdot 2a^2} = \frac{-1}{4x \cdot 2a} = \frac{-1}{8xa}$$

$$b) \frac{3mn^2}{x^2} \cdot \frac{-2x}{nm^3} = -\frac{6 \times m \times n^2}{x^2 \times n \times m^3} = -\frac{6n}{xm^2}$$

$$c) \frac{2x}{1 - x^2} \cdot \frac{x^2 + 2x + 1}{4x^2 + 4x} = \frac{2x \cdot (x^2 + 2x + 1)}{(1 - x^2) \cdot (4x^2 + 4x)} = \frac{2x \cdot (x + 1)^2}{(1 - x)(1 + x) \cdot 4x(x + 1)} = \frac{1}{2(1 - x)} = \frac{1}{2 - 2x}$$

$$d) \frac{x^2 - 16}{2x} \cdot \frac{4x^2}{x^2 - 5x + 4} = \frac{(x^2 - 16) \cdot 4x^2}{2x \cdot (x^2 - 5x + 4)} = \frac{(x - 4)(x + 4) \cdot 4x^2}{2x \cdot (x - 4)(x - 1)} = \frac{(x + 4) \cdot 2x}{(x - 1)} = \frac{2x^2 + 8x}{x - 1}$$

$$e) \frac{2x + 2}{x^2 - 1} \cdot \frac{x^2 - x}{x + 2} = \frac{(2x + 2) \cdot (x^2 - x)}{(x^2 - 1) \cdot (x + 2)} = \frac{2(x + 1) \cdot x(x - 1)}{(x - 1)(x + 1)(x + 2)} = \frac{2x}{x + 2}$$

$$f) \frac{x^2 - 2x}{3x + 1} \cdot \frac{9x^2 - 1}{x^2 - 4} = \frac{(x^2 - 2x) \cdot (9x^2 - 1)}{(3x + 1) \cdot (x^2 - 4)} = \frac{x(x - 2)(3x - 1)(3x + 1)}{(3x + 1)(x - 2)(x + 2)} = \frac{x(3x - 1)}{(x + 2)} = \frac{3x^2 - x}{x + 2}$$

5.- Efectuar los siguientes cocientes de fracciones:

$$a) \frac{3x^2}{y^3} : \frac{-6x}{4y^2} = \frac{3x^2 \cdot 4y^2}{y^3 \cdot (-6x)} = \frac{12x^2 \cdot y^2}{-6xy^3} = -\frac{2x}{y}$$

$$b) \frac{ax^2}{4b} : \frac{-ax}{b} = -\frac{ax^2 \cdot b}{4b \cdot ax} = -\frac{abx^2}{4abx} = -\frac{x}{4}$$

$$c) \frac{x + 2}{x + 1} : \frac{x^2 + 4x}{2x + 2} = \frac{2(x + 2)(x + 1)}{x(x + 4)(x + 1)} = \frac{2x + 4}{x^2 + 4x} =$$

$$d) \frac{2x + 1}{x + 3} : \frac{4x^2 - 1}{x^2 + 3x} = \frac{(2x + 1) \cdot x(x^2 + 3x)}{(x + 3) \cdot (4x^2 - 1)} = \frac{(2x + 1) \cdot x(x + 3)}{(x + 3)(2x + 1)(2x - 1)} = \frac{x}{2x - 1}$$

$$e) \frac{x + 1}{x - 3} : \frac{x^2 - 1}{x^2 - x - 6} = \frac{(x + 1) \cdot (x^2 - x - 6)}{(x - 3) \cdot (x^2 - 1)} = \frac{(x + 1)(x - 3)(x + 2)}{(x - 3)(x - 1)(x + 1)} = \frac{x + 2}{x - 1}$$

$$f) \frac{x^2 - 5x + 6}{x^2 - x - 2} : \frac{x^2 + 2x}{x^2 + x} = \frac{(x^2 - 5x + 6) \cdot (x^2 + x)}{(x^2 - x - 2) \cdot (x^2 + 2x)} = \frac{(x - 3)(x - 2)x(x + 1)}{(x - 2)(x + 1)x(x + 2)} = \frac{x - 3}{x - 2}$$