OPERACIONES CON FRACCIONES ALGEBRÁICAS

Realiza las siguientes operaciones con fracciones simplificando al máximo:

1) $\dfrac{3-x}{x}+\dfrac{2x}{x-1}-\dfrac{x-1}{x}$

2) $\dfrac{3}{x+1}-\dfrac{2}{x-1}$

3) $\dfrac{x-1}{x+1}-\dfrac{x+1}{x-1}+\dfrac{4}{x^2-1}$

4) $\dfrac{x+3}{x-4}-\dfrac{x-3}{x+4}-\dfrac{14x}{x^2-16}$

5) $\dfrac{x}{x-1}+\dfrac{x+1}{x^2-4x+3}$

6) $\dfrac{x-6-x^3}{x^2+4x+4}-\dfrac{5}{x+2}+x$

7) $\dfrac{2}{1-x}+\dfrac{4}{1+x}-\dfrac{4}{1-x^2}$

8) $\dfrac{6}{1+x}+\dfrac{4}{x-1}-\dfrac{12x}{x^2-1}$

9) $\dfrac{2x^2-2}{x^2+2x+1}\cdot\dfrac{x^2-1}{x^2+x-2}$

10) $\dfrac{8x^2}{x^2-3x+2}\cdot\dfrac{x^2-4x+4}{4x^4-16x^2}$

11) $\dfrac{2x^2}{4x-8}:\dfrac{x^3-x^2}{x^2-3x+2}$

12) $\dfrac{x^2-5x+6}{x^2-x-2}:\dfrac{x^2+2x}{x^2+x}$

13) $\dfrac{x^2+x-2}{x+3}\cdot\dfrac{x^2+6x+9}{x^2+2x-3}$

14) $\dfrac{x^2-9}{x^2+9}:\dfrac{x^2-6x+9}{x^4-81}$

15) $\left(x-\dfrac{1}{x}\right)\cdot\left(x+\dfrac{1}{x}\right)\cdot\dfrac{x}{x^2-1}$

16) $\left(\dfrac{x^2+4}{x}+4\right):\dfrac{x+2}{2x}$

17) $\dfrac{x^2-4}{x^3}\cdot\left(\dfrac{x}{x+2}\right)^2$

18) $\left(\dfrac{1}{2x}-\dfrac{2}{x^2}\right):\left(\dfrac{x}{4}-\dfrac{4}{x}\right)$

19) $\dfrac{x^2-4x}{x^2-16}:\left(1-\dfrac{4}{x+4}\right)$

20) $\left(\dfrac{1}{2}-\dfrac{1}{x}\right):\left(\dfrac{1}{4}-\dfrac{1}{x^2}\right)$

Soluciones:

1) $\dfrac{3-x}{x}+\dfrac{2x}{x-1}-\dfrac{x-1}{x}=\dfrac{(3-x)(x-1)}{x(x-1)}+\dfrac{2x^2}{x(x-1)}-\dfrac{(x-1)^2}{x(x-1)}=\dfrac{3x-3-x^2+x+2x^2-x^2+2x-1}{x(x-1)}=\dfrac{6x-4}{x^2-x}$

2) $\dfrac{3}{x+1}-\dfrac{2}{x-1}=\dfrac{3(x-1)-2(x+1)}{(x+1)(x-1)}=\dfrac{3x-3-2x-2}{(x+1)(x-1)}=\dfrac{x-5}{x^2-1}$

3) $\dfrac{x-1}{x+1}-\dfrac{x+1}{x-1}+\dfrac{4}{x^2-1}$

$=\dfrac{(x-1)^2-(x+1)^2-4}{(x+1)(x-1)}=\dfrac{x^2-2x+1-x^2-2x-1-4}{(x+1)(x-1)}=\dfrac{-4x-4}{(x+1)(x-1)}=\dfrac{-4(x+1)}{(x+1)(x-1)}=\dfrac{-4}{x-1}$

4) $\dfrac{x+3}{x-4}-\dfrac{x-3}{x+4}-\dfrac{14x}{x^2-16}=\dfrac{(x+3)(x+4)-(x-3)(x-4)-14x}{(x-4)(x+4)}=\dfrac{x^2+3x+4x+12-x^2+3x+4x-12-14x}{(x-4)(x+4)}=0$

5) $\dfrac{x}{x-1}+\dfrac{x+1}{x^2-4x+3}=\dfrac{x(x-3)+x+1}{(x-1)(x-3)}=\dfrac{x^2-3x+x+1}{(x-1)(x-3)}=\dfrac{x^2-2x+1}{(x-1)(x-3)}=\dfrac{(x-1)^2}{(x-1)(x-3)}=\dfrac{x-1}{x-3}$

6) $\dfrac{x-6-x^3}{x^2+4x+4}-\dfrac{5}{x+2}+x=\dfrac{x-6-x^3-5(x+2)+x(x+2)^2}{(x+2)^2}=\dfrac{x-6-x^3-5x-10+x^3+4x^2+4x}{(x+2)^2}=\dfrac{4x^2-16}{(x+2)^2}=$

$=\dfrac{4(x-2)(x+2)}{(x+2)(x+2)}=\dfrac{4x-8}{x+2}$

7) $\dfrac{2}{1-x}+\dfrac{4}{1+x}-\dfrac{4}{1-x^2}=\dfrac{2(1+x)+4(1-x)-4}{(1-x)(1+x)}=\dfrac{2+2x+4-4x-4}{(1-x)(1+x)}=\dfrac{2-2x}{(1-x)(1+x)}=\dfrac{2(1-x)}{(1-x)(1+x)}=\dfrac{2}{1+x}$

8) $\dfrac{6}{1+x}+\dfrac{4}{x-1}-\dfrac{12x}{x^2-1}=\dfrac{6(x-1)+4(1+x)-12x}{(1+x)(x-1)}=\dfrac{6x-6+4+4x-12x}{(1+x)(x-1)}=\dfrac{-2-2x}{(1+x)(x-1)}=\dfrac{-2(1+x)}{(1+x)(x-1)}=\dfrac{-2}{x-1}$

9) $\dfrac{2x^2-2}{x^2+2x+1}\cdot\dfrac{x^2-1}{x^2+x-2}=\dfrac{(2x^2-2)(x^2-1)}{(x^2+2x+1)(x^2+x-2)}=\dfrac{2(x-1)^2(x+1)^2}{(x+1)^2(x+2)(x-1)}=\dfrac{2x-2}{x+2}$

10) $\dfrac{8x^2}{x^2-3x+2}\cdot\dfrac{x^2-4x+4}{4x^4-16x^2}=\dfrac{8x^2(x^2-4x+4)}{(x^2-3x+2)(4x^4-16x^2)}=\dfrac{8x^2(x-2)^2}{(x-2)(x-1)4x^2(x-2)(x+2)}=\dfrac{2}{x^2+x-2}$

11) $\dfrac{2x^2}{4x-8}:\dfrac{x^3-x^2}{x^2-3x+2}=\dfrac{2x^2(x^2-3x+2)}{(4x-8)(x^3-x^2)}=\dfrac{2x^2(x-2)(x-1)}{4(x-2)\cdot x^2(x-1)}=\dfrac{1}{2}$

12) $\dfrac{x^2-5x+6}{x^2-x-2}:\dfrac{x^2+2x}{x^2+x}=\dfrac{(x^2-5x+6)(x^2+x)}{(x^2-x-2)(x^2+2x)}=\dfrac{(x-3)(x-2)(x+1)x}{(x-2)(x+1)(x+2)x}=\dfrac{x-3}{x+2}$

13) $\dfrac{x^2+x-2}{x+3} \cdot \dfrac{x^2+6x+9}{x^2+2x-3} = \dfrac{(x^2+x-2)(x^2+6x+9)}{(x+3)(x^2+2x-3)} = \dfrac{(x+2)(x-1)(x+3)^2}{(x+3)(x+3)(x-1)} = \dfrac{(x+2)\cancel{(x-1)}\cancel{(x+3)^2}}{\cancel{(x+3)}\cancel{(x+3)}\cancel{(x-1)}} = x+2$

14) $\dfrac{x^2-9}{x^2+9} : \dfrac{x^2-6x+9}{x^4-81} = \dfrac{(x^2-9)(x^4-81)}{(x^2+9)(x^2-6x+9)} = \dfrac{\cancel{(x^2+9)}\cancel{(x-3)^2}(x+3)^2}{\cancel{(x^2+9)}\cancel{(x-3)^2}} = (x+3)^2$

15) $\left(x-\dfrac{1}{x}\right)\cdot\left(x+\dfrac{1}{x}\right)\cdot\dfrac{x}{x^2-1} = \dfrac{x^2-1}{\cancel{x}}\cdot\dfrac{x^2+1}{x}\cdot\dfrac{\cancel{x}}{x^2-1} = \dfrac{x^2+1}{x}$

16) $\left(\dfrac{x^2+4}{x}+4\right):\dfrac{x+2}{2x} = \dfrac{x^2+4+4x}{x}:\dfrac{x+2}{2x} = \dfrac{2x(x^2+4+4x)}{x(x+2)} = \dfrac{2\cancel{x}(x+2)^{\cancel{2}}}{\cancel{x}\cancel{(x+2)}} = 2x+4$

17) $\dfrac{x^2-4}{x^3}\cdot\left(\dfrac{x}{x+2}\right)^2 = \dfrac{x^2-4}{x^3}\cdot\dfrac{x^2}{(x+2)^2} = \dfrac{(x+2)(x-2)\cancel{x^2}}{x^{\cancel{3}}(x+2)^{\cancel{2}}} = \dfrac{x-2}{x^2+2x}$

18) $\left(\dfrac{1}{2x}-\dfrac{2}{x^2}\right):\left(\dfrac{x}{4}-\dfrac{4}{x}\right) = \dfrac{x-4}{2x^2}:\dfrac{x^2-16}{4x} = \dfrac{4\cancel{x}\cancel{(x-4)}}{2x^{\cancel{2}}\cancel{(x-4)}(x+4)} = \dfrac{2}{x^2+4x}$

19) $\dfrac{x^2-4x}{x^2-16}:\left(1-\dfrac{4}{x+4}\right) = \dfrac{x^2-4x}{x^2-16}:\dfrac{x+4-4}{x+4} = \dfrac{(x^2-4x)(x+4)}{x(x^2-16)} = \dfrac{x(x-4)(x+4)}{x(x^2-16)} = 1$

20) $\left(\dfrac{1}{2}-\dfrac{1}{x}\right):\left(\dfrac{1}{4}-\dfrac{1}{x^2}\right) = \dfrac{x-2}{2x}:\dfrac{x^2-4}{4x^2} = \dfrac{4x(x-2)}{2x(x^2-4)} = \dfrac{4\cancel{x}\cancel{(x-2)}}{2\cancel{x}\cancel{(x-2)}(x+2)} = \dfrac{2}{x+2}$