03747

- CUESTIÓN B.1 Calcular la inversa de la matriz $A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix}$

$$\left(A \mid I\right) \xrightarrow[\text{elementales}]{\text{transformaciones}} \left(I \mid A^{-1}\right)$$

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix}$$

Utilizamos el método de Gauss:

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & -1 & 2 & 0 & 1 & 0 \\ 3 & 2 & -3 & 0 & 0 & 1 \end{array}\right) \xrightarrow[F_3-3F_1 \to F_3]{F_2-2F_1 \to F_2} \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -7 & 0 & -2 & 1 & 0 \\ 0 & -7 & -6 & -3 & 0 & 1 \end{array}\right) \longrightarrow$$

$$\xrightarrow{F_3 - F_2 \to F_3} \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -7 & 0 & -2 & 1 & 0 \\ 0 & 0 & -6 & -1 & -1 & 1 \end{array}\right) \longrightarrow$$

$$\xrightarrow[3F_2 \to F_2]{7F_1 \to F_1} \left(\begin{array}{ccc|ccc} 7 & 21 & 7 & 7 & 0 & 0 \\ 0 & -21 & 0 & -6 & 3 & 0 \\ 0 & 0 & -6 & -1 & -1 & 1 \end{array}\right) \longrightarrow$$

$$\xrightarrow{F_1+F_2 \to F_1} \left(\begin{array}{ccc|ccc} 7 & 0 & 7 & 1 & 3 & 0 \\ 0 & -21 & 0 & -6 & 3 & 0 \\ 0 & 0 & -6 & -1 & -1 & 1 \end{array}\right) \longrightarrow$$

$$\xrightarrow[7F \to F_3]{6F_1 \to F_1} \left(\begin{array}{ccc|ccc} 42 & 0 & 42 & 6 & 18 & 0 \\ 0 & -21 & 0 & -6 & 3 & 0 \\ 0 & 0 & -42 & -7 & -7 & 7 \end{array}\right) \longrightarrow$$

$$\xrightarrow{F_1+F_3 \to F_1} \left(\begin{array}{ccc|ccc} 42 & 0 & 0 & -1 & 11 & 7 \\ 0 & -21 & 0 & -6 & 3 & 0 \\ 0 & 0 & -42 & -7 & -7 & 7 \end{array}\right) \longrightarrow$$

$$\xrightarrow[\substack{-\frac{1}{21}F_2 \\ -\frac{1}{42}F_3}]{\frac{1}{42}F_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{42} & \frac{11}{42} & \frac{7}{42} \\ 0 & 1 & 0 & \frac{6}{21} & -\frac{3}{21} & 0 \\ 0 & 0 & 1 & \frac{7}{42} & \frac{7}{42} & -\frac{7}{42} \end{array}\right)$$

OPERACIONES APARTE

$$-2(1\ 3\ 1\ 1\ 0\ 0) \to \begin{array}{rcccccc} & \cancel{-2} & -6 & -2 & -2 & 0 & 0 \\ + & \cancel{2} & -1 & 2 & 0 & 1 & 0 \\ \hline & 0 & -7 & 0 & -2 & 1 & 0 \end{array}$$

$$-3(1\ 3\ 1\ 1\ 0\ 0) \to \begin{array}{rcccccc} & \cancel{-3} & - & -3 & -3 & 0 & 0 \\ + & \cancel{3} & 2 & -3 & 0 & 0 & 1 \\ \hline & 0 & -7 & -6 & -3 & 0 & 1 \end{array}$$

$$-(0\ -7\ 0\ -2\ 1\ 0) \quad \begin{array}{cccccc} \cancel{0} & \cancel{7} & 0 & 2 & -1 & 0 \\ \cancel{0} & \cancel{-7} & -6 & -3 & 0 & 1 \\ \hline 0 & 0 & -6 & -1 & -1 & 1 \end{array}$$

$$\begin{array}{rcccccc} & 7 & \cancel{21} & 7 & 7 & 0 & 0 \\ + & 0 & \cancel{-21} & 0 & -6 & 3 & 0 \\ \hline & 7 & 0 & 7 & 1 & 3 & 0 \end{array}$$

$$\begin{array}{cccccc} 42 & \cancel{0} & \cancel{42} & 6 & 18 & 0 \\ 0 & \cancel{0} & \cancel{-42} & -7 & -7 & 7 \\ \hline 42 & 0 & 0 & -1 & 11 & 7 \end{array}$$

luego
$$\boxed{A^{-1} = \begin{pmatrix} -\frac{1}{42} & \frac{11}{42} & \frac{1}{6} \\ \frac{2}{7} & -\frac{1}{7} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \end{pmatrix}}$$

Ahora la resolvemos utilizando determinantes con: $A^{-1}=\frac{1}{|A|}\left[Adj(A)\right]^t$

$$A=\begin{pmatrix}1 & 3 & 1\\ 2 & -1 & 2\\ 3 & 2 & -3\end{pmatrix}$$

Para calcular A^{-1}, utilizamos $A^{-1}=\frac{1}{|A|}\left[Adj(A)\right]^t$

Calculamos el determinante asociado a la matriz A:

$$|A|=\begin{vmatrix}1 & 3 & 1\\ 2 & -1 & 2\\ 3 & 2 & -3\end{vmatrix}=1\cdot(-1)(-3)+3\cdot2\cdot3+2\cdot2\cdot1-1(-1)3-3\cdot2(-3)-2\cdot2\cdot1=3+18+\cancel{4}+3+18-\cancel{4}=\boxed{42}$$

Calculamos ahora la matriz adjunta:

$$A_{11}=\begin{vmatrix}-1 & 2\\ 2 & -3\end{vmatrix}=3-4=-1 \qquad A_{12}=-\begin{vmatrix}2 & 2\\ 3 & -3\end{vmatrix}=-(-6-6)=12 \qquad A_{13}=\begin{vmatrix}2 & -1\\ 3 & 2\end{vmatrix}=4-(-3)=7$$

$$A_{21}=-\begin{vmatrix}3 & 1\\ 2 & -3\end{vmatrix}=-(-9-2)=11 \qquad A_{22}=\begin{vmatrix}1 & 1\\ 3 & -3\end{vmatrix}=-3-3=-6 \qquad A_{23}=-\begin{vmatrix}1 & 3\\ 3 & 2\end{vmatrix}=-(2-9)=7$$

$$A_{31}=\begin{vmatrix}3 & 1\\ -1 & 2\end{vmatrix}=6-(-1)=7 \qquad A_{32}=-\begin{vmatrix}1 & 1\\ 2 & 2\end{vmatrix}=-(2-2)=0 \qquad A_{33}=\begin{vmatrix}1 & 3\\ 2 & -1\end{vmatrix}=-1-6=-7$$

Luego $Adj(A)=\begin{pmatrix}-1 & 12 & 7\\ 11 & -6 & 7\\ 7 & 0 & -7\end{pmatrix}$ De ahí: $\left[Adj(A)\right]^t=\begin{pmatrix}-1 & 11 & 7\\ 12 & -6 & 0\\ 7 & 7 & -7\end{pmatrix}$

$$A^{-1}=\frac{1}{42}\begin{pmatrix}-1 & 11 & 7\\ 12 & -6 & 0\\ 7 & 7 & -7\end{pmatrix}=\begin{pmatrix}-\frac{1}{42} & \frac{11}{42} & \frac{7}{42}\\ \frac{12}{42} & -\frac{6}{42} & 0\\ \frac{7}{42} & \frac{7}{42} & -\frac{7}{42}\end{pmatrix}=\begin{pmatrix}-\frac{1}{42} & \frac{11}{42} & \frac{1}{6}\\ \frac{2}{7} & -\frac{1}{7} & 0\\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6}\end{pmatrix}$$

LA TERCERA RESPUESTA ES LA MEJOR

AUNQUE LAS OTRAS DOS SON TAMBIÉN CORRECTAS