EL NÚMERO REAL. RADICALES

1. Halla el menor conjunto numérico al que pertenecen los siguientes números:

$a)\ 3 \quad b)\ 4{,}23 \quad c)\ \sqrt{13} \quad d)\ -3/7 \quad e)\ 1{,}03333\ldots \quad f)\ \sqrt[3]{125} \quad g)\ 4^{1/2} \quad h)\ \dfrac{-18}{+\sqrt{9}} \quad i)\ 2\sqrt{3}$

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2. Expresa como intervalos y representa gráficamente los siguientes conjuntos:

$a)\ \{x \in R / x \geq 3\} = [3, +\infty)$ $\quad d)\ \left\{x \in R / |x| \leq \dfrac{1}{3}\right\} = \left[-\dfrac{1}{3}, \dfrac{1}{3}\right]$

$b)\ \{x \in R / x < 0\} = (-\infty, 0)$ $\quad e)\ \{x \in R / |x| > 2\} = (-\infty, -2) \cup (2, +\infty)$

$c)\ \{x \in R / -5 \leq x < 1/2\} = [-5, \tfrac{1}{2})$ $\quad f)\ \{x \in R / -3 < x \leq 1\} = (-3, 1]$

3. Expresa los siguientes intervalos en forma de desigualdades y represéntalos en la recta real:
   a) [2 , 7/2] b) [-2 , 3/5) c) (-∞, 5) d) [-2/3 , ∞).

4. Expresa los siguientes intervalos como desigualdades:
   $a)\ [-2, 7] \quad b)\ [13, +\infty) \quad c)\ (-\infty, 0) \quad d)\ (-3, 0] \quad e)\ [3/2, 6) \quad f)\ (0, +\infty)$

5. Averigua los valores de x que verifican:
   $a)\ |2x-1| = 5 \quad b)\ |x-2| > 1 \quad c)\ |x+2| \leq 3 \quad d)\ E(1; 1/2)$

6. Expresa en forma de intervalo los valores de x que verifican las siguientes desigualdades:
   $a)\ |x| < 7 \quad b)\ |x| \geq 5 \quad c)\ |2x| < 8 \quad d)\ |x-1| \leq 6 \quad e)\ |x+2| > 9 \quad f)\ |x-5| \geq 1$

7. Escribe en forma de intervalo y de desigualdad los siguientes entornos:
   a) De centro 0 y radio 4.
   b) De centro -1 y radio 2.
   c) De centro 2,5 y radio 4,25.
   d) De centro -5 y radio 2/3.

8. Expresa como un entorno simétrico los siguientes conjuntos:
   a) (-3, 1); b) (-5, 4); c) (-2, 2) d) (-1, 2) e) (1,3 ; 2,9) f) (-2,2 ; 0,2)

9. Expresa en forma de una única potencia las siguientes expresiones:

$a)\ \sqrt[3]{2^5} \quad b)\ \dfrac{1}{\sqrt[3]{4^2}} \quad c)\ a^3 \cdot \sqrt[3]{a^{-2}} \quad d)\ \sqrt[5]{2^3} \cdot \sqrt[3]{2^2} \quad e)\ \dfrac{\sqrt[3]{3^2}}{\sqrt{3}} \quad f)\ \sqrt{5\sqrt{5\sqrt{5\sqrt{5\sqrt{5}}}}}$

$g)\ \dfrac{\sqrt[4]{x^3} \cdot \left(\sqrt[3]{x}\right)^2}{\sqrt{x}} \quad h)\ \sqrt[3]{5\sqrt[3]{1/25}} \quad i)\ \sqrt[3]{2} \cdot \sqrt[4]{8}$

10. Efectúa las siguientes operaciones dejando el resultado como un único radical de la forma más reducida posible:

$a)\ \sqrt{27} \cdot \sqrt{15} \cdot \sqrt{40} \quad b)\ \dfrac{\sqrt[4]{3^3} \cdot \sqrt[6]{3}}{\sqrt{3}} \quad c)\ \dfrac{\left(\sqrt[3]{a^2}\right)^4 \cdot \left(a^2 \cdot \sqrt{a}\right)^3}{\sqrt[6]{a^5}} \quad d)\ \sqrt[3]{a^2} \cdot \sqrt[4]{a^3 \cdot \sqrt{a}}$

11. Realiza la operación y simplifica si es posible:

$a)\ 4\sqrt{27}\cdot 5\sqrt{6} =$ $\qquad b)\ 2\sqrt{\dfrac{4}{3}}\cdot\sqrt{\dfrac{27}{8}} =$ $\qquad c)\ \sqrt{2}\cdot\sqrt{\dfrac{1}{8}} =$

$d)\ (\sqrt[3]{12})^2 =$ $\qquad e)\ (\sqrt[6]{32})^3 =$ $\qquad f)\ \sqrt[3]{24} : \sqrt[3]{3} =$

$g)\ \sqrt[3]{2}\cdot\sqrt{3} =$ $\qquad h)\ \sqrt[3]{a}\cdot\sqrt[3]{\dfrac{1}{a}}\cdot\sqrt{a} =$ $\qquad i)\ \left(\dfrac{\sqrt[6]{32}}{\sqrt{8}}\right)^3 =$

$j)\ \sqrt[3]{2\sqrt{3}} : \sqrt{\sqrt[3]{4}} =$ $\qquad k)\ \sqrt[3]{3\sqrt{3}}\cdot\sqrt[4]{9\sqrt{3}}\cdot\sqrt{3\sqrt[4]{3}} =$ $\qquad l)\ \sqrt[3]{\dfrac{a^4}{b^5}\cdot\sqrt[4]{\dfrac{b^3}{a}}\cdot\sqrt{a^2b}} =$

12. Efectúa las siguientes sumas y diferencias de radicales:

$a)\ \sqrt[6]{8}+\sqrt[4]{4}-7\sqrt{72}$ $\qquad b)\ \sqrt{75}-\dfrac{\sqrt{18}}{3}+\dfrac{3\sqrt{12}}{4}-\sqrt{\dfrac{2}{25}}$

$c)\ 3\sqrt{12}-5\sqrt{27}+\sqrt{243}-\dfrac{1}{5}\sqrt{75}$ $\qquad d)\ 2\sqrt[3]{16}+3\sqrt[3]{128}-5\sqrt[3]{54}$

$e)\ \dfrac{4}{5}\sqrt{8}-\sqrt{50}+\dfrac{7}{2}\sqrt{18}-\dfrac{3}{4}\sqrt{98}$ $\qquad f)\ 5\sqrt{4x}-3\sqrt{36x}+3\sqrt{25x}-4\sqrt{9x}$

$g)\ 3\sqrt{8x^3}-4\sqrt{72x^3}+2\sqrt{32x^3}$ $\qquad h)\ 6\sqrt[3]{x^7}+x^2\sqrt[3]{x}-5x\sqrt[3]{x^4}-3x^2\sqrt[3]{27x}$

13. Efectúa las siguientes operaciones:

$a)\ \left(2\sqrt{3}+\sqrt{5}\right)^2$ $\qquad e)\ \left(\sqrt{3}+2\sqrt{2}\right)\cdot\left(\sqrt{2}-\sqrt{3}\right)\cdot\sqrt{3}$

$b)\ 2\sqrt{6}\cdot\left(2\sqrt{5}-\sqrt{2}\right)^2$ $\qquad f)\ \left(2+\sqrt{2}\right)^2-\left(2+\sqrt{2}\right)\cdot\left(2-\sqrt{2}\right)$

$c)\ \left(\sqrt{2}+1\right)^2\cdot\sqrt{3}$ $\qquad g)\ \left(1+\sqrt{2}\right)\cdot\left(1-\sqrt{2}\right)+\left(2+\sqrt{2}\right)\cdot\left(2-\sqrt{2}\right)$

$d)\ \left[\left(\sqrt{2}-1\right)^2-1\right]\cdot\sqrt{2}$ $\qquad h)\ \left(\sqrt{72}-\sqrt{20}-\sqrt{2}\right)\cdot\left(\sqrt{2}+2\sqrt{8}-7\sqrt{2}\right)$

14. Racionaliza y simplifica:

$a)\ \dfrac{3}{\sqrt{3}}$ $\quad b)\ \dfrac{5}{2\sqrt{5}}$ $\quad c)\ \dfrac{\sqrt[5]{2}}{\sqrt[5]{3^3}}$ $\quad d)\ \dfrac{3}{2+\sqrt{2}}$ $\quad e)\ \dfrac{\sqrt{3}}{3-2\sqrt{3}}$ $\quad f)\ \dfrac{\sqrt{5}-2}{\sqrt{5}+2}$ $\quad g)\ \dfrac{\sqrt{3}+1}{\sqrt{2}}$

$h)\ \dfrac{\sqrt{7}+1}{2\sqrt{7}+5}$ $\quad i)\ \dfrac{1}{\sqrt{\sqrt{3}+\sqrt{2}}}$ $\quad j)\ \dfrac{2}{\sqrt{2-\sqrt{3}}}$ $\quad k)\ \dfrac{2-\sqrt{2}}{2\sqrt{3}}$

15. Calcula, racionalizando previamente:

a) $\dfrac{3}{2\sqrt{5}}+\dfrac{2}{3-\sqrt{5}}=$ (Soluc: $\frac{3}{2}+\frac{4}{5}\sqrt{5}$)

b) $\dfrac{2\sqrt{3}-3}{2\sqrt{3}+3}+\dfrac{12}{\sqrt{3}}=$ (Soluc: 7)

c) $\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}-\dfrac{3}{2\sqrt{6}}=$ (Soluc: $5-\frac{9}{4}\sqrt{6}$)

d) $\dfrac{2-\sqrt{3}}{\sqrt{3}-\sqrt{2}}+\dfrac{8\sqrt{3}}{\sqrt{2}}-\dfrac{3\sqrt{2}}{\sqrt{3}}=$ (Soluc: $2\sqrt{3}+2\sqrt{2}+2\sqrt{6}-3$)

e) $\dfrac{1}{1-\sqrt{2}}-\dfrac{3+3\sqrt{2}}{\sqrt{2}-4}=$ (Soluc: $\frac{4+\sqrt{2}}{14}$)

f) $\dfrac{5\sqrt{2}}{2\sqrt{3}}+\dfrac{\sqrt{2}}{\sqrt{3}-1}=$ (Soluc: $\frac{4\sqrt{6}}{3}+\frac{\sqrt{2}}{2}$)

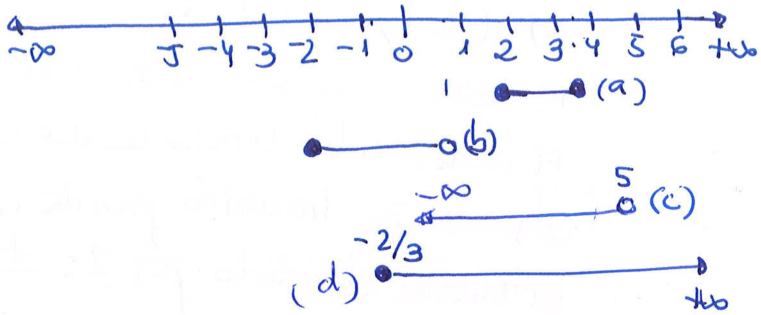
g) $\dfrac{3+2\sqrt{2}}{6+6\sqrt{2}}+\dfrac{1}{\sqrt{8}}=$ (Soluc: $1+\frac{5}{4}\sqrt{2}$)

h) $\dfrac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}=$ (Soluc: $3-2\sqrt{2}$)

i) $\dfrac{\frac{3}{4}+\frac{\sqrt{3}}{3}}{1-\frac{3}{4}\frac{\sqrt{3}}{3}}=$ (Soluc: $\frac{48+25\sqrt{3}}{39}$)

j) $\dfrac{17-9\sqrt{3}}{3\sqrt{3}-5}-\dfrac{9}{\sqrt{3}}=$ (Soluc: 2)

RESOLUCIÓN



3.- a) $[2, 7/2] = \{x \in \mathbb{R} / 2 \leq x \leq \frac{7}{2}\}$

b) $[-2, 3/5) = \{x \in \mathbb{R} / -2 \leq x < 3/5\}$

c) $(-\infty, 5) = \{x \in \mathbb{R} / x < 5\}$

d) $[-\frac{2}{3}, +\infty) = \{x \in \mathbb{R} / x \geq -\frac{2}{3}\}$

4.- a) $[-2, 7] = \{x \in \mathbb{R} / -2 \leq x \leq 7\}$

b) $[13, +\infty) = \{x \in \mathbb{R} / x \geq 13\}$

c) $(-\infty, 0) = \{x \in \mathbb{R} / x < 0\}$

d) $(-3, 0] = \{x \in \mathbb{R} / -3 < x \leq 0\}$

e) $[\frac{3}{2}, 6) = \{x \in \mathbb{R} / \frac{3}{2} \leq x < 6\}$

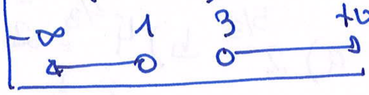
f) $(0, +\infty) = \{x \in \mathbb{R} / 0 < x\}$

5.- (a) $|2x-1| = 5 \Rightarrow$ $2x-1 = 5 \Rightarrow x = 3$; $2x-1 = -5 \Rightarrow x = -2$

Son 2 puntos aislados

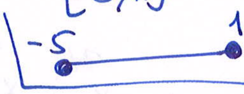
(b) $|x-2| > 1 \Rightarrow$ $x-2 > 1 \Rightarrow x > 3$
o bien ó
$x-2 < -1$ $x < 1$

$(-\infty, 1) \cup (3, +\infty)$

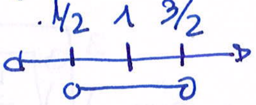


(c) $|x+2| \leq 3 \Rightarrow -3 \leq x+2 \leq 3 \Rightarrow -3-2 \leq x \leq 3-2$

$-5 \leq x \leq 1$ $[-5, 1]$



(d) $E(1; 1/2)$ = Entorno de centro 1 y radio $\frac{1}{2}$



$|x-1| < 1/2$

$-1/2 < x-1 < 1/2 \Rightarrow -\frac{1}{2}+1 < x < \frac{1}{2}+1 \Rightarrow \frac{1}{2} < x < \frac{3}{2}$

Intervalo $(\frac{1}{2}, \frac{3}{2})$

6.- a) $|x| < 7 \Rightarrow -7 < x < 7 \Rightarrow (-7, 7)$

b) $|x| \geq 5 \Rightarrow \begin{cases} x \geq 5 \\ \text{ó} \\ x \leq -5 \end{cases} \Rightarrow (-\infty, -5] \cup [5, +\infty)$

c) $|2x| < 8 \Rightarrow -8 < 2x < 8 \Rightarrow -4 < x < 4$ $(-4, 4)$

d) $|x-1| \leq 6 \Rightarrow -6 \leq x-1 \leq +6 \Rightarrow -5 \leq x \leq 7$ $[-5, 7]$

e) $|x+2| > 9 \Rightarrow \begin{cases} x+2 > 9 \Rightarrow x > 7 \\ \text{ó} \quad\quad \text{ó} \\ x+2 < -9 \Rightarrow x < -11 \end{cases}$ $(-\infty, -11) \cup (7, +\infty)$

f) $|x-5| \geq 1 \Rightarrow \begin{cases} x-5 \geq 1 \\ \text{ó} \\ x-5 \leq -1 \end{cases} \Leftrightarrow \begin{cases} x \geq 6 \\ \text{ó} \\ x \leq 4 \end{cases}$ $(-\infty, 4] \cup [6, +\infty)$

7.- a) $E(0; 4) = (-4, 4)$
$|x| < 4$

(b) $E(-1; 2) = (-3, 1)$
$|x+1| < 2$

(c) $E(2'5; 4'25) =$
$|x-2'5| < 4'25$

d) $E(-5; 2/3) = (-17/3, -13/3)$

8.- a) $(-3,1) = E(-1;2)$

El centro es el punto medio entre los extremos: $\frac{-3+1}{2} = -1$

El radio es la distancia del centro a uno de los extremos; en este caso es 2. También puede calcularse como la amplitud del intervalo dividido por 2: $\frac{1-(-3)}{2} = 2$.

(b) $(-5,4) = E\left(-\frac{1}{2}; \frac{9}{2}\right) = E(-0'5; 4'5)$

$\frac{-5+4}{2} = -1/2$ (Centro) $\quad \frac{4-(-5)}{2} = \frac{9}{2}$ (Radio)

(c) $(-2,2) = E(0;2)$ $\quad$ (d) $(-1,2) = E\left(\frac{1}{2}; 3/2\right)$ $\quad$ (e) $(1'3; 2'9) = E(2'1; 0'8)$

(f) $(-2'2, 0'2) = E(-1; 1'2)$

9.- a) $2^{5/3}$ b) $4^{-2/3} = 2^{-4/3}$ c) $a^3 \cdot a^{-\frac{2}{3}} = a^{3-\frac{2}{3}} = a^{7/3}$ d) $2^{\frac{3}{5}} \cdot 2^{\frac{2}{3}} = 2^{19/15}$

e) $3^{2/3} : 3^{1/2} = 3^{\frac{2}{3}-\frac{1}{2}} = 2^{1/6}$ $\quad$ f) $5^{1/2} \cdot 5^{1/4} \cdot 5^{1/8} \cdot 5^{1/16} = 5^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}} = 5^{15/16}$

o bien $\sqrt{\sqrt{5^2 \cdot 5\sqrt{5\sqrt{5}}}} = \sqrt[4]{\sqrt{5^6 5\sqrt{5}}} = \sqrt[8]{\sqrt{5^4 \cdot 5}} = \sqrt[16]{5^{15}}$

(g) $\frac{x^{3/4} \cdot x^{2/3}}{x^{1/2}} = x^{\frac{3}{4}+\frac{2}{3}-\frac{1}{2}} = x^{\frac{9}{12}+\frac{8}{12}-\frac{6}{12}} = x^{11/12}$

(h) $\sqrt[3]{5} \cdot \sqrt[3]{\sqrt{1/25}} = 5^{1/3} \cdot 25^{-1/9} = 5^{\frac{1}{3}} \cdot 5^{-\frac{2}{9}} = 5^{+1/9}$ o bien $\sqrt[3]{\sqrt[3]{\frac{5^3}{25}}} = \sqrt[9]{5} = 5^{1/9}$

(i) $\sqrt[3]{2} \cdot \sqrt[4]{8} = 2^{1/3} \cdot 2^{3/4} = 2^{\frac{4+9}{12}} = 2^{13/12}$

$8 = 2^3$

10.- a) $\sqrt{27} \cdot \sqrt{15} \cdot \sqrt{40} = \sqrt{3^3} \cdot \sqrt{3 \cdot 5} \cdot \sqrt{2^3 \cdot 5} = \sqrt{3^4 \cdot 2^3 \cdot 5^2} = 3^2 \cdot 2 \cdot 5 \cdot \sqrt{2} = 90\sqrt{2}$

b) $\frac{\sqrt[4]{3^3} \cdot \sqrt[6]{3}}{\sqrt{3}} = \frac{\sqrt[12]{3^9} \cdot \sqrt[12]{3^2}}{\sqrt[12]{3^6}} = \sqrt[12]{\frac{3^{11}}{3^6}} = \sqrt[12]{3^5}$

mcm$(4,6,2) = 12$

c) $\frac{\left(\sqrt[3]{a^2}\right)^4 \cdot \left(a^2 \sqrt{a}\right)^3}{\sqrt[6]{a^5}} = \frac{\sqrt[3]{a^8} \cdot a^6 \sqrt{a^3}}{\sqrt[6]{a^5}} = \frac{\sqrt[6]{a^{16}} \cdot \sqrt[6]{a^{36}} \cdot \sqrt[6]{a^9}}{\sqrt[6]{a^5}} = \sqrt[6]{a^{56}} =$

$= a^9 \sqrt[6]{a^2}$

$= a^9 \cdot \sqrt[3]{a}$

11.- a) $4\sqrt{27}\cdot 5\sqrt{6} = 20\cdot\sqrt{3^3\cdot 23} = 20\cdot 3^2\sqrt{2} = 180\sqrt{2}$

b) $2\sqrt{\frac{4}{3}}\cdot\sqrt{\frac{27}{8}} = 2\sqrt{\frac{2^2\cdot 3^3}{3\cdot 2^3}} = 2\sqrt{\frac{3^2}{2}} = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

c) $\sqrt{2}\cdot\sqrt{\frac{1}{8}} = \sqrt{2}\cdot\frac{1}{\sqrt{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

d) $(\sqrt[3]{12})^2 = \sqrt[3]{12^2} \underset{12=2^2\cdot 3}{=} \sqrt[3]{2^4 3^2} = 2\sqrt[3]{2\cdot 9} = 2\sqrt[3]{18}$

e) $(\sqrt[6]{32})^3 = \sqrt[6]{32^3} \underset{\uparrow}{=} \sqrt{32} = \sqrt{2^5} = 2^2\sqrt{2} = 4\sqrt{2}$.

simplificando el radical

f) $\sqrt[3]{24} : \sqrt[3]{3} = \sqrt[3]{8} = 2$

g) $\sqrt[3]{2}\cdot\sqrt{3} = \sqrt[6]{2^2}\cdot\sqrt[6]{3^3} = \sqrt[6]{4\cdot 27} = \sqrt[6]{108}$

h) $\sqrt[3]{a}\cdot\sqrt[3]{\frac{1}{a}}\cdot\sqrt{a} = \sqrt[3]{\frac{a}{a}}\cdot\sqrt{a} = \sqrt[3]{1}\cdot\sqrt{a} = \sqrt{a}$.

i) $\left(\frac{\sqrt[6]{32}}{\sqrt{8}}\right)^3 = \left(\frac{\sqrt[6]{32}}{\sqrt[6]{8^2}}\right)^3 = \left(\sqrt[6]{\frac{1}{2}}\right)^3 = \sqrt[6]{\left(\frac{1}{2}\right)^3} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

j) $\sqrt[3]{2\sqrt{3}} : \sqrt{\sqrt[3]{4}} = \sqrt[3]{\sqrt{2^2\cdot 3}} : \sqrt{\sqrt[3]{2^2}} = \sqrt[6]{2^2\cdot 3} : \sqrt[6]{2^2} = \sqrt[6]{3}$

(k) $\sqrt[3]{3\sqrt{3}}\cdot\sqrt[4]{\underset{3^2}{\underset{\shortparallel}{9}}\sqrt{3}}\cdot\sqrt{3\cdot\sqrt[4]{3}} = \sqrt[3]{\sqrt{3^2 3}}\cdot\sqrt[4]{\sqrt{3^4 3}}\cdot\sqrt{\sqrt[4]{3^4\cdot 3}} =$

$= \sqrt[6]{3^3}\cdot\sqrt[8]{3^5}\cdot\sqrt[8]{3^5} = \sqrt{3}\cdot\sqrt[8]{3^{10}} = \sqrt[8]{3^4}\cdot\sqrt[8]{3^{10}} = \sqrt[8]{3^{14}} = \sqrt[4]{3^7} = 3\sqrt[4]{3^3}$

l) $\sqrt[3]{\frac{a^4}{b^5}\cdot\sqrt[4]{\frac{b^3}{a}}\cdot\sqrt{a^2 b}} = \sqrt[3]{\sqrt[4]{\frac{a^{16}\cdot b^3}{b^{20}\cdot a}}\cdot\sqrt[4]{a^4 b^2}} = \sqrt[3]{\sqrt[4]{\frac{a^{20}\cdot b^5}{b^{20}\cdot a}}} =$

$= \sqrt[12]{\frac{a^{19}}{b^{15}}} = \frac{a}{b}\sqrt[12]{\frac{a^7}{b^3}}$

12.- a) $\sqrt[6]{8} + \sqrt[4]{4} - 7\sqrt{72} = \sqrt[6]{2^3} + \sqrt[4]{2^2} - 7\sqrt{2^3\cdot 3^2} = \sqrt{2} + \sqrt{2} - 7\cdot 2\cdot 3\sqrt{2} =$

$= \sqrt{2} + \sqrt{2} - 42\sqrt{2} = -40\sqrt{2}$

b) $\sqrt{75} - \frac{\sqrt{18}}{3} + \frac{3\sqrt{12}}{4} - \sqrt{\frac{2}{25}} =$

$= \sqrt{3\cdot5^2} - \frac{\sqrt{3^2\cdot2}}{3} + \frac{3\sqrt{2^2\cdot3}}{4} - \frac{\sqrt{2}}{5} = 5\sqrt{3} - \frac{3\sqrt{2}}{3} + \frac{3\cdot2\sqrt{3}}{4} - \frac{\sqrt{2}}{5} =$

$= 5\sqrt{3} - \sqrt{2} + \frac{3\sqrt{3}}{2} - \frac{\sqrt{2}}{5} = \frac{10\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} - \frac{5\sqrt{2}}{5} - \frac{\sqrt{2}}{5} = \boxed{\frac{13\sqrt{3}}{2} - \frac{6\sqrt{2}}{5}}$

(c) $3\sqrt{12} - 5\sqrt{27} + \sqrt{243} - \frac{1}{5}\sqrt{75} = 3\sqrt{2^2\cdot3} - 5\sqrt{3^3} + \sqrt{3^5} - \frac{1}{5}\sqrt{3\cdot5^2} =$

$= 6\sqrt{3} - 15\sqrt{3} + 9\sqrt{3} - \sqrt{3} = -\sqrt{3}$.

(d) $2\sqrt[3]{16} + 3\sqrt[3]{128} - 5\sqrt[3]{54} = 2\sqrt[3]{2^4} + 3\sqrt[3]{2^7} - 5\sqrt[3]{2\cdot3^3} =$

$= 2\cdot2\sqrt[3]{2} + 3\cdot2^2\cdot\sqrt[3]{2} - 5\cdot3\sqrt[3]{2} = 4\sqrt[3]{2} + 12\sqrt[3]{2} - 15\sqrt[3]{2} = \sqrt[3]{2}$

(e) $\frac{4}{5}\sqrt{8} - \sqrt{50} + \frac{7}{2}\sqrt{18} - \frac{3}{4}\sqrt{98} = \frac{4\cdot2}{5}\sqrt{2} - 5\sqrt{2} + \frac{7}{2}\cdot3\sqrt{2} - \frac{3}{4}\cdot7\sqrt{2} =$

$= \left(\frac{8}{5} - 5 + \frac{21}{2} - \frac{21}{4}\right)\sqrt{2} = \frac{37}{20}\cdot\sqrt{2}$

(f) $5\sqrt{4x} - 3\sqrt{36x} + 3\sqrt{25x} - 4\sqrt{9x} = 10\sqrt{x} - 18\sqrt{x} + 15\sqrt{x} - 12\sqrt{x} =$

$= -5\sqrt{x}$

(g) $3\sqrt{8x^3} - 4\sqrt{\underbrace{72}_{2^3\cdot3^2}x^3} + 2\sqrt{\underbrace{32}_{2^5}x^3} = 6x\sqrt{x} - 24x\sqrt{x} + 8x\sqrt{x} = -10x\sqrt{x}$

(h) $6\sqrt[3]{x^7} + x^2\cdot\sqrt[3]{x} - 5x\sqrt[3]{x^4} - 3x^2\cdot\sqrt[3]{27x} = 6x^2\sqrt[3]{x} + x^2\sqrt[3]{x} - 5x\,x\sqrt[3]{x} - 3x^2\cdot3\sqrt[3]{x} = 6x^2\cdot\sqrt[3]{x} + x^2\cdot\sqrt[3]{x} - 5x^2\cdot\sqrt[3]{x} - 9x^2\sqrt[3]{x} = -7x^2\cdot\sqrt[3]{x}$

13.- (a) $(2\sqrt{3} + \sqrt{5})^2 = (2\sqrt{3})^2 + (\sqrt{5})^2 + 2\cdot2\sqrt{3}\cdot\sqrt{5} = 4\cdot3 + 5 + 4\sqrt{15} = 17 + 4\sqrt{15}$

(b) $2\sqrt{6}\cdot(2\sqrt{5} - \sqrt{2})^2 = 2\sqrt{6}\,(4\cdot5 + 2 - 4\sqrt{10}) = 2\sqrt{6}\cdot(22 - 4\sqrt{10}) =$

$= 44\sqrt{6} - 8\sqrt{\underbrace{60}_{2^2\cdot3\,5}} = 44\sqrt{6} - 16\sqrt{15}$

(c) $(\sqrt{2}+1)^2\cdot\sqrt{3} = (2+1+2\sqrt{2})\cdot\sqrt{3} = (3+2\sqrt{2})\sqrt{3} = 3\sqrt{3} + 2\sqrt{6}$

d) $\left[(\sqrt{2}-1)^2-1\right]\cdot\sqrt{2} = \left[2+1-2\sqrt{2}-1\right]\cdot\sqrt{2} = (2-2\sqrt{2})\cdot\sqrt{2} = 2\sqrt{2}-2\sqrt{4} =$
$= \underline{2\sqrt{2}-4}$

(e) $(\sqrt{3}+2\sqrt{2})(\sqrt{2}-\sqrt{3})\cdot\sqrt{3} = (\sqrt{6}-3+2\cdot 2-2\sqrt{6})\cdot\sqrt{3} =$
$= (1-\sqrt{6})\cdot\sqrt{3} = \sqrt{3}-\sqrt{18} = \underline{\sqrt{3}-3\sqrt{2}}$

(f) $(2+\sqrt{2})^2-(2+\sqrt{2})(2-\sqrt{2}) =$
$= (4+2+4\sqrt{2})-(4-2) = 6+4\sqrt{2}-2 = \underline{4+4\sqrt{2}} = 4(1+\sqrt{2})$

(g) $(1+\sqrt{2})(1-\sqrt{2})+(2+\sqrt{2})(2-\sqrt{2}) =$
$= (1-2)+(4-2) = -1+2 = 1$

h) $(\sqrt{72}-\sqrt{20}-\sqrt{2})\cdot(\sqrt{2}+2\sqrt{8}-7\sqrt{2}) =$
$(6\sqrt{2}-2\sqrt{5}-\sqrt{2})\cdot(\sqrt{2}+4\sqrt{2}-7\sqrt{2}) =$
$(5\sqrt{2}-2\sqrt{5})\cdot(-2\sqrt{2}) = -10\cdot 2+4\sqrt{10} = -20+4\sqrt{10}$

15.- a) $\frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$ (b) $\frac{5}{2\sqrt{5}} = \frac{5\cdot\sqrt{5}}{2\sqrt{5}\cdot\sqrt{5}} = \frac{5\sqrt{5}}{2\cdot 5} = \frac{\sqrt{5}}{2}$

(c) $\frac{\sqrt[5]{2}}{\sqrt[5]{3^3}} = \frac{\sqrt[5]{2}\cdot\sqrt[5]{3^2}}{\sqrt[5]{3^3}\cdot\sqrt[5]{3^2}} = \frac{\sqrt[5]{2\cdot 3^2}}{\sqrt[5]{3^5}} = \frac{\sqrt[5]{18}}{3}$

(d) $\frac{3}{2+\sqrt{2}} = \frac{3(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = \frac{3\cdot(2-\sqrt{2})}{2^2-(\sqrt{2})^2} = \frac{3(2-\sqrt{2})}{4-2} = \frac{3(2-\sqrt{2})}{2}$

(e) $\frac{\sqrt{3}}{3-2\sqrt{3}} = \frac{\sqrt{3}\cdot(3+2\sqrt{3})}{(3-2\sqrt{3})(3+2\sqrt{3})} = \frac{\sqrt{3}(3+2\sqrt{3})}{9-4\cdot 3} = \frac{3\sqrt{3}+2\cdot 3}{9-12} = \frac{3(\sqrt{2}+2)}{-3}$

$= \underline{-\sqrt{2}-2}$

(f) $\frac{\sqrt{5}-2}{\sqrt{5}+2} = \frac{(\sqrt{5}-2)^2}{(\sqrt{5}+2)(\sqrt{5}-2)} = \frac{5+4-4\sqrt{5}}{5-4} = \frac{9-4\sqrt{5}}{1} = 9-4\sqrt{5}$

(g) $\frac{\sqrt{3}+1}{\sqrt{2}} = \frac{(\sqrt{3}+1)\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$

(h) $6\sqrt[3]{x^7}+x^2\cdot\sqrt[3]{x}-5x\sqrt[3]{x^4}-3x^2\sqrt[3]{27x} =$
$6x^2\sqrt[3]{x}+x^2\sqrt[3]{x}-5x^2\sqrt[3]{x}-\underbrace{3x^2\cdot 3}_{9x^2}\sqrt[3]{x} = \underline{-7x^2\sqrt[3]{x}}$

13.-

(a) $\left(2\sqrt{3}+\sqrt{5}\right)^2 = \left(2\sqrt{3}\right)^2 + \left(\sqrt{5}\right)^2 + 2\cdot 2\sqrt{3}\cdot\sqrt{5} = 12+5+4\sqrt{15} = 17+4\sqrt{15}$

(b) $2\sqrt{6}\cdot\left(2\sqrt{5}-\sqrt{2}\right)^2 = 2\sqrt{6}\cdot\left[\left(2\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2 - 2\cdot 2\sqrt{5}\cdot\sqrt{2}\right] =$

$= 2\sqrt{6}\cdot\left[20+2-4\sqrt{10}\right] = 2\sqrt{6}\cdot\left(22-4\sqrt{10}\right) = 44\sqrt{6} - 8\sqrt{60} =$

$= 44\sqrt{6} - 18\cdot 2\sqrt{15}$

(c) $\left(\sqrt{2}+1\right)^2\cdot\sqrt{3} = \left[\left(\sqrt{2}\right)^2 + 1^2 + 2\sqrt{2}\right]\cdot\sqrt{3} = \left[2+1+2\sqrt{2}\right]\cdot\sqrt{3} = \left[3+2\sqrt{2}\right]\cdot\sqrt{3}$

$= 3\sqrt{3}+2\sqrt{6}$

(d) $\left[\left(\sqrt{2}-1\right)^2-1\right]\cdot\sqrt{2} = \left[\left(\sqrt{2}\right)^2+1^2-2\sqrt{2}-1\right]\cdot\sqrt{2} = \left[3-2\sqrt{2}-1\right]\cdot\sqrt{2} =$

$= \left[2-2\sqrt{2}\right]\cdot\sqrt{2} = 2\sqrt{2}-4.$

(e) $\left(\sqrt{3}+2\sqrt{2}\right)\cdot\left(\sqrt{2}-\sqrt{3}\right)\cdot\sqrt{3} = \left(\sqrt{6}-\sqrt{3}^2+2\cdot\sqrt{2^2}-2\sqrt{6}\right)\cdot\sqrt{3} =$

$= \left(\sqrt{6}-3+4-2\sqrt{6}\right)\cdot\sqrt{3} = \left(1-\sqrt{6}\right)\cdot\sqrt{3} = \sqrt{3}-\sqrt{18} = \underline{\sqrt{3}-3\sqrt{2}}$

(f) $\left(2+\sqrt{2}\right)^2 - \left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right) = \left(2^2+4\sqrt{2}+2\right)-\left(4-2\right) = 6+4\sqrt{2}-2 =$

$= \underline{4+4\sqrt{2}}$

(g) $\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)+\left(2+\sqrt{2}\right)\cdot\left(2-\sqrt{2}\right) = \left(1-2\right)+\left(4-2\right) = -1+2 = 1$

h) $\left(\sqrt{72}-\sqrt{20}-\sqrt{2}\right)\cdot\left(\sqrt{2}+2\sqrt{8}-7\sqrt{2}\right) = \left(6\sqrt{2}-2\sqrt{5}-\sqrt{2}\right)\cdot\left(\sqrt{2}+4\sqrt{2}-7\sqrt{2}\right)$

$= \left(5\sqrt{2}+2\sqrt{5}\right)\cdot\left(-\sqrt{2}\right) = -5\cdot 2+2\sqrt{10} = -10+2\sqrt{10}$

14.- a) $\dfrac{3}{\sqrt{3}} = \dfrac{3\cdot\sqrt{3}}{\left(\sqrt{3}\right)^2} = \dfrac{\cancel{3}\sqrt{3}}{\cancel{3}} = \sqrt{3}$

b) $\dfrac{5}{2\sqrt{5}} = \dfrac{5\cdot\sqrt{5}}{2\sqrt{5}\sqrt{5}} = \dfrac{\cancel{5}\sqrt{5}}{2\cdot\cancel{5}} = \dfrac{\sqrt{5}}{2}$

c) $\dfrac{\sqrt[5]{2}}{\sqrt[5]{3^2}} = \dfrac{\sqrt[5]{2}\cdot\sqrt[5]{3^3}}{\sqrt[5]{3^2}\cdot\sqrt[5]{3^3}} = \dfrac{\sqrt[5]{2\cdot 3^3}}{3}$

d) $\dfrac{3}{2+\sqrt{2}} = \dfrac{3\cdot\left(2-\sqrt{2}\right)}{\left(2+\sqrt{2}\right)\cdot\left(2-\sqrt{2}\right)} = \dfrac{6-3\sqrt{2}}{4-2} = \dfrac{6-3\sqrt{2}}{2}$

e) $\frac{\sqrt{3}}{3-2\sqrt{3}} = \frac{\sqrt{3}\cdot(3+2\sqrt{3})}{(3-2\sqrt{3})\cdot(3+2\sqrt{3})} = \frac{3\sqrt{3}+2\cdot 3}{3^2-4\cdot 3} = \frac{3\sqrt{3}+6}{9-12} =$

$= \frac{3(\sqrt{3}+2)}{-3} = -(\sqrt{3}+2) = -\sqrt{3}-2.$

f) $\frac{\sqrt{5}-2}{\sqrt{5}+2} = \frac{(\sqrt{5}-2)(\sqrt{5}+2)}{(\sqrt{5}+2)(\sqrt{5}-2)} = \frac{5-2\sqrt{5}-2\sqrt{5}+4}{5-4} = \frac{9-4\sqrt{5}}{1} = 9-4\sqrt{5}$

g) $\frac{\sqrt{3}+1}{\sqrt{2}} = \frac{(\sqrt{3}+1)\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$

h) $\frac{\sqrt{7}+1}{2\sqrt{7}+5} = \frac{(\sqrt{7}+1)(2\sqrt{7}-5)}{(2\sqrt{7}+5)(2\sqrt{7}-5)} = \frac{2\cdot 7-5\sqrt{7}+2\sqrt{7}-5}{4\cdot 7-25} = \frac{11-3\sqrt{7}}{-3}$

i) $\frac{1}{\sqrt{3+\sqrt{2}}} = \frac{\sqrt{3+\sqrt{2}}}{(\sqrt{3+\sqrt{2}})^2} = \frac{\sqrt{3+\sqrt{2}}}{3+\sqrt{2}} = \frac{(\sqrt{3+\sqrt{2}})(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} =$

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$= \frac{(\sqrt{3+\sqrt{2}})(3-\sqrt{2})}{9-2} = \frac{(\sqrt{3+\sqrt{2}})(3-\sqrt{2})}{5}$

k) $\frac{2-\sqrt{2}}{2\sqrt{3}} = \frac{(2-\sqrt{2})\cdot\sqrt{3}}{2\cdot\sqrt{3}\cdot\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{6}}{6}.$

15.- a) $\frac{3}{2\sqrt{5}} + \frac{2}{3-\sqrt{5}} = \frac{3\sqrt{5}}{10} + \frac{6+2\sqrt{5}}{9-5} = \frac{6\sqrt{5}+30+10\sqrt{5}}{20} = \frac{30+16\sqrt{5}}{20}$

$= \frac{30}{20} + \frac{16\sqrt{5}}{20} = \frac{3}{2} + \frac{4}{5}\sqrt{5}$

b) $\frac{2\sqrt{3}-3}{2\sqrt{3}+3} + \frac{12}{\sqrt{3}} = \frac{(2\sqrt{3}-3)(2\sqrt{3}-3)}{(2\sqrt{3}+3)(2\sqrt{3}-3)} + \frac{12\sqrt{3}}{(\sqrt{3})^2} =$

$= \frac{4\cdot 3-6\sqrt{3}-6\sqrt{3}+9}{12-9} + \frac{12\sqrt{3}}{3} = \frac{21-12\sqrt{3}}{3} + \frac{12\sqrt{3}}{3} = \frac{21}{3} = 7$

c) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} - \frac{3}{2\sqrt{6}} = \frac{(3\sqrt{2}-2\sqrt{3})^2}{(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}-2\sqrt{3})} - \frac{3\cdot\sqrt{6}}{2\cdot\sqrt{6}\cdot\sqrt{6}} =$

$= \frac{9\cdot 2+4\cdot 3-2\cdot 3\cdot 2\sqrt{6}}{9\cdot 2-4\cdot 3} - \frac{3\sqrt{6}}{12} = \frac{30-12\sqrt{6}}{6} - \frac{\sqrt{6}}{4} = \frac{60-24\sqrt{6}}{12} - \frac{3\sqrt{6}}{12} =$

$= \frac{60-27\sqrt{6}}{12} = \frac{30-9\sqrt{6}}{4} = 5-\frac{9}{4}\sqrt{6}$

$$(d)\ \frac{2-\sqrt{3}}{\sqrt{3}-\sqrt{2}}+\frac{8\sqrt{3}}{\sqrt{2}}-\frac{3\sqrt{2}}{\sqrt{3}}=\frac{(2-\sqrt{3})\cdot(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})\cdot(\sqrt{3}+\sqrt{2})}+\frac{8\sqrt{3}\cdot\sqrt{2}}{(\sqrt{2})^2}-\frac{3\sqrt{2}\cdot\sqrt{3}}{(\sqrt{3})^2}=$$

$$=\frac{2\sqrt{3}+2\sqrt{2}-\sqrt{3^2}-\sqrt{6}}{3-2}+\frac{8\sqrt{6}}{2}-\frac{3\sqrt{6}}{3}=2\sqrt{3}+2\sqrt{2}-3-\sqrt{6}+4\sqrt{6}-\sqrt{6}=$$

$$=2\sqrt{3}+2\sqrt{2}-3+2\sqrt{6}.$$

$$(e)\ \frac{1}{1-\sqrt{2}}-\frac{3+3\sqrt{2}}{\sqrt{2}-4}=\frac{1+\sqrt{2}}{(1-\sqrt{2})(1+\sqrt{2})}-\frac{(3+3\sqrt{2})(\sqrt{2}+4)}{(\sqrt{2}-4)(\sqrt{2}+4)}=$$

$$=\frac{1+\sqrt{2}}{1-2}-\frac{3\sqrt{2}+12+3\cdot\sqrt{2}^2+12\sqrt{2}}{2-16}=\frac{1+\sqrt{2}}{-1}-\frac{15\sqrt{2}+18}{-14}=$$

$$=\frac{-1-\sqrt{2}}{1}+\frac{15\sqrt{2}+18}{14}=\frac{-14-14\sqrt{2}+15\sqrt{2}+18}{14}=\frac{4+\sqrt{2}}{14}.$$

$$(f)\ \frac{5\sqrt{2}}{2\sqrt{3}}+\frac{\sqrt{2}}{\sqrt{3}-1}=\frac{5\cdot\sqrt{2}\cdot\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}}+\frac{\sqrt{2}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}=\frac{5\sqrt{6}}{6}+\frac{\sqrt{6}+\sqrt{2}}{3-1}=$$

$$\frac{5\sqrt{6}}{6}+\frac{3\sqrt{6}+3\sqrt{2}}{6}=\frac{8\sqrt{6}+3\sqrt{2}}{6}=\frac{8\sqrt{6}}{6}+\frac{3\sqrt{2}}{6}=\frac{4\sqrt{6}}{3}+\frac{\sqrt{2}}{2}.$$

$$(g)\ \frac{3+2\sqrt{2}}{6+6\sqrt{2}}+\frac{1}{\sqrt{8}}=\frac{(3+2\sqrt{2})(6-6\sqrt{2})}{(6+6\sqrt{2})(6-6\sqrt{2})}+\frac{\sqrt{8}}{8}=$$

$$=\frac{18-18\sqrt{2}+12\sqrt{2}-12\cdot 2}{36-36\cdot 2}+\frac{2\sqrt{2}}{8}=\frac{-6-6\sqrt{2}}{-36}+\frac{\sqrt{2}}{4}=\frac{1+\sqrt{2}}{6}+\frac{\sqrt{2}}{4}=$$

$$=\frac{2+2\sqrt{2}+3\sqrt{2}}{12}=\frac{5\sqrt{2}+2}{12}$$

$$h)\ \frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}=\frac{\frac{2-\sqrt{2}}{\cancel{2}}}{\frac{2+\sqrt{2}}{\cancel{2}}}=\frac{2-\sqrt{2}}{2+\sqrt{2}}=\frac{(2-\sqrt{2})^2}{(2+\sqrt{2})(2-\sqrt{2})}=\frac{4+2-4\sqrt{2}}{2}=$$

$$=\frac{6-4\sqrt{2}}{2}=3-2\sqrt{2}$$

$$(i)\ \frac{\frac{3}{4}+\frac{\sqrt{3}}{3}}{1-\frac{3}{4}\cdot\frac{\sqrt{3}}{3}}=\frac{\frac{9+4\sqrt{3}}{\cancel{12}}}{\frac{12-3\sqrt{3}}{\cancel{12}}}=\frac{9+4\sqrt{3}}{12-3\sqrt{3}}=\frac{144+75\sqrt{3}}{117}=\frac{48+25\sqrt{3}}{39}$$

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