

LIMITES RACIONALES

$$[1] (1) \lim_{x \rightarrow 2} \frac{x^2 + 5x - 12}{3x - 5} = \frac{4 + 10 - 12}{3 \cdot 2 - 5} = \frac{2}{1} = 2$$

$$(2) \lim_{x \rightarrow \infty} \frac{3x^2 + 5x}{7x - 3} = \frac{\infty}{\infty} \text{ Indeterminación}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{3x^2}{x^2} + \frac{5x}{x^2}}{\frac{7x}{x^2} - \frac{3}{x^2}} = \lim_{x \rightarrow \infty} \frac{3 + \frac{5}{x}}{\frac{7}{x} - \frac{3}{x^2}} = \frac{3 + 0^+}{0^+ - 0} = \frac{3}{0^+} = +\infty$$

Alternativa: dividir por el menor de los términos de mayor grado

$$\lim_{x \rightarrow \infty} \frac{\frac{3x^2}{x} - \frac{5x}{x}}{\frac{7x}{x} - \frac{3}{x}} = \lim_{x \rightarrow \infty} \frac{3x - 5}{7 - \frac{3}{x}} = \frac{+\infty - 5}{7 - 0^+} = +\infty$$

$$(3) \lim_{x \rightarrow \infty} \frac{12x^3 - 9x}{4x^3 + 8} = \frac{\infty - \infty}{\infty + 8} = \frac{\infty}{\infty} \text{ Indeterminación}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{12x^3}{x^3} - \frac{9x}{x^3}}{\frac{4x^3}{x^3} + \frac{8}{x^3}} = \lim_{x \rightarrow \infty} \frac{12 - \frac{9}{x^2}}{4 + \frac{8}{x^3}} = \frac{12 - 0^+}{4 - 0^+} = \frac{12}{4} = 3$$

$$(4) \lim_{x \rightarrow \infty} \frac{5x - 8}{10x^3 + 8} = \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{5x}{x^3} - \frac{8}{x^3}}{\frac{10x^3}{x^3} + \frac{8}{x^3}} = \lim_{x \rightarrow \infty} \frac{\frac{5}{x^2} - \frac{8}{x^3}}{10 + \frac{8}{x^3}} = \frac{0^+ - 0^+}{10 + 0^+} = \frac{0}{10} = 0$$

$$(5) \lim_{x \rightarrow \infty} \frac{(x+3)^2 + (x-5)^2}{2(x-1)(x+3)} = \frac{\infty + \infty}{2 \cdot \infty \cdot \infty} = \frac{\infty}{\infty} \text{ Indeterminación}$$

$$\lim_{x \rightarrow \infty} \frac{x^2 + 6x + 9 + x^2 - 10x + 25}{2(x^2 + 3x - x - 3)} = \lim_{x \rightarrow \infty} \frac{2x^2 - 4x + 34}{2x^2 + 4x - 12} =$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^2} - \frac{4x}{x^2} + \frac{34}{x^2}}{\frac{2x^2}{x^2} + \frac{4x}{x^2} - \frac{12}{x^2}} = \lim_{x \rightarrow \infty} \frac{2 - \frac{4}{x} + \frac{34}{x^2}}{2 + \frac{4}{x} - \frac{12}{x^2}} = \frac{2 - 0 + 0}{2 + 0 - 0} = 1$$

$$(6) \lim_{x \rightarrow \infty} \frac{(x+5)(x-5) + 3x^4}{(x+2)^2} = \frac{\infty + \infty}{\infty} = \frac{\infty}{\infty} \text{ Indet.}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^2 - 25 + 3x^4}{x^2 + 2x + 4} &= \lim_{x \rightarrow \infty} \frac{\frac{3x^4}{x^4} + \frac{x^2}{x^4} - \frac{25}{x^4}}{\frac{x^2}{x^4} + \frac{2x}{x^4} + 4} = \\ &= \lim_{x \rightarrow \infty} \frac{3 + \frac{1}{x^2} - \frac{25}{x^4}}{\frac{1}{x^2} + \frac{2}{x^3} + 4} = \frac{3 + 0^+ - 0^+}{0^+ + 0^+ + 4} = \frac{3}{0^+} = +\infty \end{aligned}$$

$$(7) \lim_{x \rightarrow 2} \frac{4x}{x^2 - 4} = \frac{8}{0} \text{ Indet. } \begin{matrix} +\infty \\ -\infty \end{matrix} ?$$

Estudiamos límites laterales

$$\lim_{x \rightarrow 2^-} \frac{4x}{x^2 - 4} = \frac{8^-}{0^-} = -\infty$$

$$\lim_{x \rightarrow 2^+} \frac{4x}{x^2 - 4} = \frac{8^+}{0^+} = +\infty$$

$$(8) \lim_{x \rightarrow 0} \frac{(x+1)^2 - 1}{x} = \frac{0}{0} \text{ Indeterminación}$$

Factorizamos y simplificamos

$$\lim_{x \rightarrow 0} \frac{x^2 + 2x + 1 - 1}{x} = \lim_{x \rightarrow 0} \frac{x(x+2)}{x} = \lim_{x \rightarrow 0} x+2 = 0+2 = \underline{2}$$

$$(9) \lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{2x^2 - 2x - 4} = \frac{4 + 6 - 10}{8 - 4 - 4} = \frac{0}{0} \text{ Indet.}$$

Factorizamos y simplificamos.

$$\begin{array}{r|rrr} 1 & 1 & 3 & -10 \\ 2 & & 2 & 10 \\ \hline & 1 & 5 & 0 \end{array} \quad \begin{array}{r|rrr} 2 & 2 & -2 & -4 \\ 2 & & 4 & 4 \\ \hline & 2 & 2 & 0 \end{array}$$

$$\lim_{x \rightarrow 2} \frac{(x-2)(x+5)}{(x-2)(2x+2)} = \lim_{x \rightarrow 2} \frac{x+5}{2x+2} = \frac{7}{6}$$

$$(10) \lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \frac{0}{0} \text{ Indeterminación}$$

Factorizamos y simplificamos

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$$\lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+1)}{(x-1)} = \lim_{x \rightarrow 1} x^2+x+1 = 1+1+1 = \underline{\underline{3}}$$

$$(11) \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 7x + 10} = \frac{4 - 10 + 6}{4 - 14 + 10} = \frac{0}{0} \text{ Indet.}$$

Factorizamos y simplificamos

$$\begin{array}{r|rrr} 1 & 1 & -5 & 6 \\ & & 2 & -6 \\ \hline & 1 & -3 & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 & 1 & -7 & 10 \\ & & 2 & -10 \\ \hline & 1 & -5 & 0 \end{array}$$

$$\lim_{x \rightarrow 2} \frac{(x-2)(x-3)}{(x-2)(x-5)} = \lim_{x \rightarrow 2} \frac{x-3}{x-5} = \frac{-1}{-3} = \frac{1}{3}$$

$$(12) \lim_{x \rightarrow 2} \frac{x^3 - 6x^2 + x + 14}{x^3 + x^2 - 12} = \frac{2^3 - 6 \cdot 2^2 + 2 + 14}{2^3 + 2^2 - 12} = \frac{8 - 24 + 16}{8 + 4 - 12} = \frac{0}{0} \text{ Ind.}$$

Usamos Ruffini para factorizar (y luego simplificamos)

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 1 & 14 \\ & & 2 & -8 & -14 \\ \hline & 1 & -4 & -7 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & 0 & -12 \\ & & 2 & 6 & 12 \\ \hline & 1 & 3 & 6 & 0 \end{array}$$

$$\lim_{x \rightarrow 2} \frac{(x-2)(x^2-4x-7)}{(x-2)(x^2+3x+6)} = \lim_{x \rightarrow 2} \frac{x^2-4x-7}{x^2+3x+6} = \frac{4-8-7}{4+12+6} = \frac{-11}{22} = -\frac{1}{2}$$

LÍMITES IRRACIONALES (Ficha 7. Ej. 2)

$$(1) \lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x-3} = \frac{2-2}{0} = \frac{0}{0} \text{ Indet.}$$

Multiplicamos num y den. por el conjugado

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)} &= \lim_{x \rightarrow 3} \frac{x+1-2^2}{(x-3)(\sqrt{x+1}+2)} = \\ &= \lim_{x \rightarrow 3} \frac{(x-3)}{(x-3)(\sqrt{x+1}+2)} = \lim_{x \rightarrow 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{2+2} = \frac{1}{4} \end{aligned}$$

$$(2) \lim_{x \rightarrow 0} \frac{1-\sqrt{1-x^2}}{x} = \frac{0}{0} \text{ Indet.}$$

Multiplicamos num. y den. por el conjugado y simplificamos.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{(1-\sqrt{1-x^2})(1+\sqrt{1-x^2})}{x(1+\sqrt{1-x^2})} &= \lim_{x \rightarrow 0} \frac{1-(1-x^2)}{x(1+\sqrt{1-x^2})} = \\ &= \lim_{x \rightarrow 0} \frac{x^2}{x(1+\sqrt{1-x^2})} = \lim_{x \rightarrow 0} \frac{x}{1+\sqrt{1-x^2}} = \frac{0}{2} = 0 \end{aligned}$$

$$(3) \lim_{x \rightarrow 1} \frac{\sqrt{3x+1}-2}{\sqrt{x}-1} = \frac{\sqrt{4}-2}{\sqrt{1}-1} = \frac{0}{0} \text{ Indet}$$

Multiplicamos num y den. por el conjugado y simplificamos.

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{(\sqrt{3x+1}-2)(\sqrt{3x+1}+2)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{3x+1}+2)(\sqrt{x}+1)} &= \lim_{x \rightarrow 1} \frac{(3x+1-2^2)(\sqrt{x}+1)}{(x-1)(\sqrt{3x+1}+2)} = \\ &= \lim_{x \rightarrow 1} \frac{3(x-1)(\sqrt{x}+1)}{(x-1)(\sqrt{3x+1}+2)} = \lim_{x \rightarrow 1} \frac{3(\sqrt{x}+1)}{\sqrt{3x+1}+2} = \frac{6}{4} = \frac{3}{2} \end{aligned}$$

$$(4) \lim_{x \rightarrow 3} \frac{1-\sqrt{x-2}}{x^2-9} = \frac{0}{0} \text{ Indet.}$$

Factorizamos, Multiplicamos num y denom. por el conjugado y simplif.

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{(1-\sqrt{x-2})(1+\sqrt{x-2})}{(x-3)(x+3)(1+\sqrt{x-2})} &= \lim_{x \rightarrow 3} \frac{1-(x-2)}{(x-3)(x+3)(1+\sqrt{x-2})} = \\ &= \lim_{x \rightarrow 3} \frac{-(x-3)}{(x-3)(x+3)(1+\sqrt{x-2})} = \lim_{x \rightarrow 3} \frac{-1}{(x+3)(1+\sqrt{x-2})} = \frac{-1}{6 \cdot 3} = \\ &= -\frac{1}{18} \end{aligned}$$

$$(5) \lim_{x \rightarrow \infty} (\sqrt{x^2-2} - x) = \infty - \infty \text{ Indet.}$$

Multipliquemos y dividimos por el conjugado.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2-2} - x)(\sqrt{x^2-2} + x)}{(\sqrt{x^2-2} + x)} &= \lim_{x \rightarrow \infty} \frac{x^2-2-x^2}{(\sqrt{x^2-2} + x)} = \\ &= \lim_{x \rightarrow \infty} \frac{-2}{(\sqrt{x^2-2} + x)} = \lim_{x \rightarrow \infty} \frac{-2}{\infty + \infty} = \frac{-2}{\infty - \infty} = 0 \end{aligned}$$

$$(6) \lim_{x \rightarrow \infty} (\sqrt{x+3} - \sqrt{x+2}) = \infty - \infty \text{ Indet.}$$

Multipliquemos y dividimos por el conjugado.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{(\sqrt{x+3} - \sqrt{x+2})(\sqrt{x+3} + \sqrt{x+2})}{(\sqrt{x+3} + \sqrt{x+2})} &= \lim_{x \rightarrow \infty} \frac{x+3-(x+2)}{\sqrt{x+3} + \sqrt{x+2}} = \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x+3} + \sqrt{x+2}} = \frac{1}{\infty + \infty} = 0. \end{aligned}$$

$$(7) \lim_{x \rightarrow \infty} (\sqrt{x^2+x} - x) = \infty - \infty \text{ Indet.}$$

Multipliquemos y dividimos por el conjugado.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+x} - x)(\sqrt{x^2+x} + x)}{(\sqrt{x^2+x} + x)} &= \lim_{x \rightarrow \infty} \frac{x^2+x-x^2}{\sqrt{x^2+x} + x} = \\ &= \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+x} + x} = \frac{\infty}{\infty} \text{ Indet.} \end{aligned}$$

Dividimos por la mayor potencia de x .

$$\lim_{x \rightarrow \infty} \frac{\frac{x}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{x}{x^2}} + \frac{x}{x}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} = \frac{1}{2}$$

$$(8) \lim_{x \rightarrow \infty} (\sqrt{x^2+1} - \sqrt{x^2-1}) = \infty - \infty \text{ Indet.}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2+1} - \sqrt{x^2-1})(\sqrt{x^2+1} + \sqrt{x^2-1})}{(\sqrt{x^2+1} + \sqrt{x^2-1})} &= \lim_{x \rightarrow \infty} \frac{x^2+1-(x^2-1)}{(\sqrt{x^2+1} + \sqrt{x^2-1})} = \\ &= \lim_{x \rightarrow \infty} \frac{2}{\sqrt{x^2+1} + \sqrt{x^2-1}} = \frac{2}{\infty + \infty} = 0 \end{aligned}$$

$$(9) \lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 3} + 6x}{2x - 5} = \frac{\infty + \infty}{\infty} \text{ Indet.}$$

Dividimos num y den por la potencia de x de mayor grado.

$$\lim_{x \rightarrow \infty} \frac{\sqrt{\frac{4x^2}{x^2} - \frac{3}{x^2}} + \frac{6x}{x}}{\frac{2x}{x} - \frac{5}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{4 - \frac{3}{x^2}} + 6}{2 - \frac{5}{x}} = \frac{2 + 6}{2} = \frac{8}{2} = 4.$$

$$(10) \lim_{x \rightarrow \infty} \frac{10x + 1}{\sqrt{25x^2 + 7}} = \frac{\infty}{\infty} \text{ Indet.}$$

Dividimos num y denom. por la m x. potencia de x.

$$\lim_{x \rightarrow \infty} \frac{\frac{10x}{x} + \frac{1}{x}}{\sqrt{\frac{25x^2}{x^2} + \frac{7}{x^2}}} = \lim_{x \rightarrow \infty} \frac{10 + \frac{1}{x}}{\sqrt{25 + \frac{7}{x^2}}} = \frac{10}{5} = 2.$$

$$(11) \lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 1} + 4}{6 + \sqrt{x^2 - 3}} = \frac{\infty + 4}{6 + \infty} = \frac{\infty}{\infty} \text{ Indet.}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{\frac{9x^2}{x^2} + \frac{1}{x^2}} + \frac{4}{x}}{\frac{6}{x} + \sqrt{\frac{x^2}{x^2} - \frac{3}{x^2}}} = \lim_{x \rightarrow \infty} \frac{\sqrt{9 + \frac{1}{x^2}} + \frac{4}{x}}{\frac{6}{x} + \sqrt{1 - \frac{3}{x^2}}} = \frac{3 + 0^+}{0^+ + 1} = 3$$

$$(12) \lim_{x \rightarrow \infty} \frac{5x + \sqrt{x^2 + 3}}{8x + \sqrt{16x^2 - 1}} = \frac{\infty}{\infty} \text{ Indet.}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{5x}{x} + \sqrt{\frac{x^2}{x^2} + \frac{3}{x^2}}}{\frac{8x}{x} + \sqrt{\frac{16x^2}{x^2} - \frac{1}{x^2}}} = \lim_{x \rightarrow \infty} \frac{5 + \sqrt{1 + \frac{3}{x^2}}}{8 + \sqrt{16 - \frac{1}{x^2}}} = \frac{5 + 1}{8 + 4} = \frac{6}{12} = \frac{1}{2}$$

5CHA 8.

$$(1) \lim_{x \rightarrow \infty} \left(\frac{x^2}{2x-1} - \frac{x^2+1}{2x+1} \right) = \infty - \infty = \text{Indet.}$$

Haremos la operaci n para convertirla en una racional t pica.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^2(2x+1) - (x^2+1)(2x-1)}{(2x-1)(2x+1)} &= \lim_{x \rightarrow \infty} \frac{2x^3 + x^2 - (2x^3 - x^2 + 2x - 1)}{4x^2 - 1} \\ &= \lim_{x \rightarrow \infty} \frac{2x^2 - 2x + 1}{4x^2 - 1} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2}}{\frac{4x^2}{x^2} - \frac{1}{x^2}} = \frac{2 - 0 + 0}{4 - 0} = \frac{1}{2} \end{aligned}$$

LIMITES EXPONENTIALES (Ficha 8, Ej. 2)

$$(1) \lim_{x \rightarrow \infty} \left(\frac{4x+1}{2x-1} \right)^{x+2} = \lim_{x \rightarrow \infty} \left(\frac{4x+1}{2x-1} \right)^{\frac{1}{\frac{1}{x+2}}} = 2^{\infty} = \underline{\underline{\infty}}$$

$$(2) \lim_{x \rightarrow \infty} \left(\frac{3x+2}{x-1} \right)^{\frac{1}{x}} = \lim_{x \rightarrow \infty} \left(\frac{3x+2}{x-1} \right)^{\frac{1}{\frac{1}{x+2}}} = 3^{0^+} = \underline{\underline{1}}$$

$$(3) \lim_{x \rightarrow \infty} \left(\frac{x^2+3}{2x-1} \right)^{4x} = \lim_{x \rightarrow \infty} \left(\frac{x^2+3}{2x-1} \right)^{\frac{1}{\frac{1}{4x}}} = \infty^{\infty} = \underline{\underline{\infty}}$$

$$(4) \lim_{x \rightarrow \infty} \left(\frac{x+5}{4x-1} \right)^{3x} = \lim_{x \rightarrow \infty} \left(\frac{x+5}{4x-1} \right)^{\frac{1}{\frac{1}{3x}}} = \left(\frac{1}{4} \right)^{\infty} = \underline{\underline{0}}$$

$$(5) \lim_{x \rightarrow \infty} \left(\frac{5x+3}{x^2-2} \right)^{6x+2} = \lim_{x \rightarrow \infty} \left(\frac{5x+3}{x^2-2} \right)^{\frac{1}{\frac{1}{6x+2}}} = 0^{\infty} = \underline{\underline{0}}$$

$$(6) \lim_{x \rightarrow \infty} \left(\frac{x-7}{2x^2+1} \right)^{\frac{4x+1}{2x}} = \lim_{x \rightarrow \infty} \left(\frac{x-7}{2x^2+1} \right)^{\frac{1}{\frac{1}{\frac{4x+1}{2x}}}} = 0^2 = \underline{\underline{0}}$$

$$(7) \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x+3} \right)^{4x} = \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x+3} \right)^{\frac{1}{\frac{1}{4x}}} = 1^{\infty} \text{ Indet}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{\frac{x+3}{5}} \right)^{\frac{4x \cdot \frac{x+3}{5}}{\frac{x+3}{5}}} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{1}{\frac{x+3}{5}} \right)^{\frac{x+3}{5}} \right]^{\frac{20x}{x+3}} = e^{\underline{\underline{20}}}$$

$$(8) \lim_{x \rightarrow \infty} \left(1 + \frac{6}{5x-1} \right)^{\frac{2x}{x+1}} = \lim_{x \rightarrow \infty} \left(1 + \frac{6}{5x-1} \right)^{\frac{1}{\frac{1}{\frac{2x}{x+1}}}} = 1^2 = \underline{\underline{1}}$$

$$(9) \lim_{x \rightarrow \infty} \left(1 + \frac{3}{7x+4} \right)^{\frac{2x^2}{x+1}} = \lim_{x \rightarrow \infty} \left(1 + \frac{3}{7x+4} \right)^{\frac{1}{\frac{1}{\frac{2x^2}{x+1}}}} = 1^{\infty} \text{ Indet.}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{\frac{7x+4}{3}} \right)^{\frac{\frac{2x^2}{x+1} \cdot \frac{7x+4}{3}}{\frac{7x+4}{3}}} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{1}{\frac{7x+4}{3}} \right)^{\frac{7x+4}{3}} \right]^{\frac{6x^2}{(x+1)(7x+4)}} = e^{\underline{\underline{\infty}}}$$

$$(10) \lim_{x \rightarrow \infty} \left(\frac{x-2}{x+1} \right)^{2x} = \lim_{x \rightarrow \infty} \left(\frac{x-2}{x+1} \right)^{\frac{1}{\frac{1}{2x}}} = 1^{\infty} \text{ Indet.}$$

Aplica la fórmula.

$$\lim_{x \rightarrow \infty} \underbrace{\left(\frac{x-2}{x+1} \right)}_{f(x)}^{2x \cdot \frac{1}{g(x)}} = e^{\lim_{x \rightarrow \infty} g(x) [f(x) - 1]} = e^{\lim_{x \rightarrow \infty} \frac{-6x}{x+1}} = e^{\underline{\underline{6}}}$$

$$g(x) \cdot [f(x) - 1] = 2x \cdot \left[\frac{x-2}{x+1} - 1 \right] = 2x \cdot \left[\frac{x-2-(x+1)}{x+1} \right] =$$

$$= 2x \cdot \left(\frac{-3}{x+1} \right) = \frac{-6x}{x+1}$$

$$(11) \lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x^2-2} \right)^{5x+1} = 1^\infty [\text{Indet}] = e^{\lim_{x \rightarrow \infty} g(x) \cdot [f(x)-1]}$$

$$g(x) \cdot [f(x)-1] = (5x+1) \cdot \left(\frac{x^2+1}{x^2-2} - 1 \right) = (5x+1) \left[\frac{x^2+1-(x^2-2)}{x^2-2} \right] =$$

$$= (5x+1) \cdot \left[\frac{3}{x^2-2} \right] = \frac{15x+3}{x^2-2}$$

$$\lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x^2-2} \right)^{5x+1} = e^{\lim_{x \rightarrow \infty} \frac{15x+3}{x^2-2}} = e^0 = \underline{1}$$

$$(12) \lim_{x \rightarrow \infty} \left(\frac{5x^2+7}{5x^2+3} \right)^{2x^3-3} = 1^\infty [\text{Indet}] = e^{\lim_{x \rightarrow \infty} g(x) \cdot [f(x)-1]}$$

$$g(x) [f(x)-1] = (2x^3-3) \cdot \left(\frac{5x^2+7}{5x^2+3} - \frac{5x^2+3}{5x^2+3} \right) =$$

$$= (2x^3-3) \cdot \left[\frac{4}{5x^2+3} \right] = \frac{8x^3-12}{5x^2+3}$$

$$\lim_{x \rightarrow \infty} \left(\frac{5x^2+7}{5x^2+3} \right)^{2x^3-3} = e^{\lim_{x \rightarrow \infty} \frac{8x^3-12}{5x^2+3}} = e^\infty = \underline{\underline{\infty}}$$

[3] Hallar "a" para que se verifique:

$$\lim_{x \rightarrow \infty} \left(\frac{4x+5}{4x+3} \right)^x = \lim_{x \rightarrow \infty} \left(\frac{4x^2+1}{4x^2+\pi} \right)^{ax^2}$$

$\underset{1^\infty}{\lim_{x \rightarrow \infty} \left(\frac{4x+5}{4x+3} \right)^x}$
 $\underset{1^\infty}{\lim_{x \rightarrow \infty} \left(\frac{4x^2+1}{4x^2+\pi} \right)^{ax^2}}$

Calculamos por separado e igualamos al final.

$$\lim_{x \rightarrow \infty} \left(\frac{4x+5}{4x+3} \right)^x = e^{\lim_{x \rightarrow \infty} x \cdot \left(\frac{4x+5-(4x+3)}{4x+3} \right)} = e^{\lim_{x \rightarrow \infty} \frac{2x}{4x+3}} = e^{\frac{1}{2}}$$

$$\lim_{x \rightarrow \infty} \left(\frac{4x^2+1}{4x^2+\pi} \right)^{ax^2} = e^{\lim_{x \rightarrow \infty} ax^2 \left(\frac{4x^2+1-(4x^2+\pi)}{4x^2+\pi} \right)} = e^{\lim_{x \rightarrow \infty} \frac{ax^2(1-\pi)}{4x^2+\pi}} = e^{\frac{a(1-\pi)}{4}}$$

$$\frac{1}{2} = \frac{a(1-\pi)}{4} \quad 4 = 2a(1-\pi) \quad \boxed{a = \frac{4}{2(1-\pi)}}$$

[4] Hallar k para que $\lim_{x \rightarrow \infty} \left(\frac{x+3}{x} \right)^{kx+5} = e^2$

$$\lim_{x \rightarrow \infty} \left(\frac{x+3}{x} \right)^{kx+5} = 1^\infty = e^{\lim_{x \rightarrow \infty} (kx+5) \left(\frac{x+3-x}{x} \right)} = e^{\lim_{x \rightarrow \infty} \frac{3kx+15}{x}} = e^{3k}$$

$$2 = 3k \quad \Leftrightarrow \quad \boxed{k = \frac{2}{3}}$$

CONTINUIDAD (Ficha 9)

2) $f(x) = \begin{cases} 2x+2 & \text{si } x \leq 0 \rightarrow \text{Recta, continua} \\ x^2-3x+2 & \text{si } x > 0 \rightarrow \text{Parábola, continua} \end{cases}$

Estudiar continuidad en $x=0$ y representarla posteriormente.

$x=0$

1) $f(0) = 2 \cdot 0 + 2 = 2$

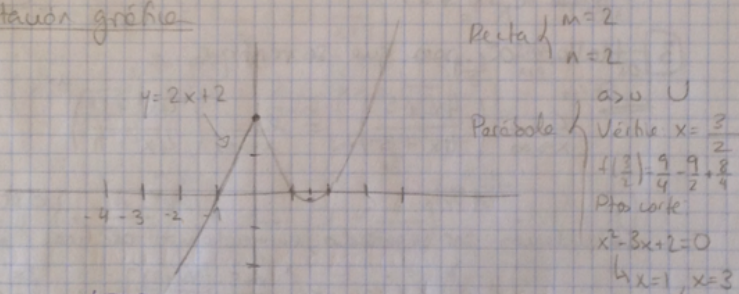
2) $\lim_{x \rightarrow 0} f(x) ?$

$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} 2x+2 = 2$ (1ª rama) $\Rightarrow \lim_{x \rightarrow 0} f(x) = 2$

$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2-3x+2 = 2$ (2ª rama)

3) Como $f(0) = 2 = \lim_{x \rightarrow 0} f(x) \Rightarrow f(x)$ es continua en $x=0$

Representación gráfica



3) Sea $f(x) = \begin{cases} \sqrt{3x+1} & \text{si } x < 3 \\ \frac{x^2-2x-3}{x-3} & \text{si } x > 3 \end{cases}$

Estudiar la continuidad en $x=3$, clasificar la discontinuidad.

$x=3$

1) $f(3) \Rightarrow$ No es continua

2) $\lim_{x \rightarrow 3} f(x) ?$ $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} \sqrt{3x+1} = 4$ (1ª rama)

$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{x^2-2x-3}{x-3} = \frac{0}{0}$ (2ª rama)

Es una discontinuidad evitable. Bastaría con dar el valor $f(3)=4$ para que f fuese continua en $x=3$.

$= \lim_{x \rightarrow 3^+} \frac{(x-3)(x+1)}{(x-3)} = \lim_{x \rightarrow 3^+} x+1 = 4$

4) $f(x) = \frac{x^2-9}{x^3-27}$ no está definida en $x=3 \Rightarrow$ No es continua.
 ¿Qué valor se le debe dar a $f(3)$ para que f fuese continua?

Dom $f = \mathbb{R} \setminus \{3\} \Rightarrow 1) \nexists f(3)$

2) Veamos si $\lim_{x \rightarrow 3} f(x)$

$$\lim_{x \rightarrow 3} \frac{x^2-9}{x^3-27} = \frac{0}{0} = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{(x-3)(x^2+3x+9)} = \frac{6}{9+9+9} = \frac{6}{27}$$

Factorizamos con Ruffini

1	0	0	-27
3	9	9	27
1	3	9	0

3) Bastaría con definir $f(3) = \frac{6}{27}$ y así $f(3) = \lim_{x \rightarrow 3} f(x)$
 f sería continua en $x=3$.

5) Estudiar si $f(x) = \frac{x^2-9}{x^2-1}$ es continua en $x=1$ y $x=-1$.

Si no, analizar el tipo de discontinuidad.

Dom $f = \mathbb{R} \setminus \{-1, 1\} \Rightarrow f$ no es continua en $x=1$ ni en $x=-1$

$x=1$
 $\lim_{x \rightarrow 1} f(x) = ?$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2-9}{x^2-1} = \frac{1-9}{0} = \frac{-8}{0} \text{ Indet. } \begin{matrix} -\infty \\ +\infty \end{matrix} ?$$

Estudiamos límites laterales.

$$\lim_{x \rightarrow 1^-} \frac{x^2-9}{x^2-1} = \frac{-8}{0^-} = +\infty \quad \lim_{x \rightarrow 1^+} \frac{x^2-9}{x^2-1} = \frac{-8}{0^+} = -\infty$$

Es una discontinuidad no evitable de salto infinito.

$x=-1$

$$\lim_{x \rightarrow -1} \frac{x^2-9}{x^2-1} = \frac{1-9}{0} = \frac{-8}{0} \text{ Indet. } \begin{matrix} +\infty? \\ -\infty? \end{matrix}$$

Estudiamos límites laterales

$$\lim_{x \rightarrow -1^-} \frac{x^2-9}{x^2-1} = \frac{-8}{0^+} = -\infty \quad \lim_{x \rightarrow -1^+} \frac{x^2-9}{x^2-1} = \frac{-8}{0^-} = +\infty$$

Es una discontinuidad no evitable de salto infinito.

[6] Sea $f(x) = \frac{x^2 + 2x - 3}{x^2 + x - 2}$. Hallar dominio y el valor del límite en los puntos donde f no está definida.

$$\text{Dom } f = \mathbb{R} \setminus \{x^2 + x - 2 = 0\} = \mathbb{R} \setminus \{-2, 1\}$$

$$x^2 + x - 2 = 0 \quad x = \frac{-1 \pm \sqrt{1 + 4 \cdot 2 \cdot 1}}{2} = \frac{-1 \pm 3}{2} = \begin{cases} x_1 = -2 \\ x_2 = 1 \end{cases}$$

$$\boxed{x = -2}$$

$$1. \lim_{x \rightarrow -2} \frac{x^2 + 2x - 3}{x^2 + x - 2} = \frac{4 - 4 - 3}{0} = \frac{-3}{0} = \text{Indet.} \begin{cases} \infty \\ -\infty \end{cases} ?$$

$$2. \lim_{x \rightarrow -2} \frac{x^2 + 2x - 3}{(x+2)(x-1)} = \frac{-3}{0^+} = -\infty \quad 2. \lim_{x \rightarrow -2} \frac{x^2 + 2x - 3}{\underbrace{(x+2)(x-1)}} = \frac{-3}{0^-} = +\infty$$

Es una discontinuidad de salto infinito.

$$\boxed{x = 1}$$

$$1. \lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x^2 + x - 2} = \frac{1 + 2 - 3}{0} = \frac{0}{0} = \text{Indet.}$$

Factorizamos y simplificamos $\rightarrow$ Ruffini

$$1. \lim_{x \rightarrow 1} \frac{(x-1)(x+3)}{(x+2)(x-1)} = \frac{4}{3}$$

$$\begin{array}{r|rrr} 1 & 1 & 2 & -3 \\ & 1 & 3 & 0 \end{array}$$

Es una discontinuidad evitable. Bastaría con dar a función el valor $f(1) = \frac{4}{3}$ para que fuera continua.

Ficha 10. Continuidad con parámetros

[1] Calcular a para que $f(x) = \begin{cases} x+1 & \text{si } x \leq 1 \\ 3-ax^2 & \text{si } x > 1 \end{cases}$ sea continua en $x=1$.

$$1) f(1) = 1 + 1 = 2.$$

$$2) \lim_{x \rightarrow 1} f(x)?$$

$$1. \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x+1 = 2$$

$$2. \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 3-ax^2 = 3-a$$

$$2 = 3-a \Rightarrow a = 1$$

3) Si $a=1 \Rightarrow f(x) = 2 = \lim_{x \rightarrow 1} f(x)$ y f es continua en $x=1$

2

$$f(x) = \begin{cases} \frac{x^2+3x}{x} & \text{si } x < 0 \\ K & \text{si } x = 0 \\ 2x+3 & \text{si } x > 0 \end{cases}$$

Calcular K para que f sea continua en x=0

1) $f(x) = K$

2) $\exists \lim_{x \rightarrow 0} f(x) ? \Leftrightarrow \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$

$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x^2+3x}{x} = \frac{0}{0} = \text{Indet} =$

Factorizamos y simplificamos

$\lim_{x \rightarrow 0^-} \frac{x(x+3)}{x} = \lim_{x \rightarrow 0^-} x+3 = 3 \quad \Rightarrow \exists \lim_{x \rightarrow 0^-} f(x) = 3$

$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 2x+3 = 3$

3) Para que sea continua en x=0 $\lim_{x \rightarrow 0} f(x) = 3 = f(0) = K$

$\Rightarrow K = 3$

3) Calcula a y b para que $f(x) = \begin{cases} x^2+2x-1 & \text{si } x < 0 \\ ax+b & \text{si } 0 \leq x < 1 \\ 2 & \text{si } x \geq 1 \end{cases}$ sea continua y dibuje la gráfica

f(x) es una función a trozos continua en cada rama:

1ª rama es una parábola, 2ª rama es una recta, 3ª rama escte.

Por tanto sólo se debe estudiar la continuidad en los puntos donde se unen las ramas: $x=0, x=1$

$x=0$ 1) $f(0) = ax+b \Rightarrow f(0) = -1 = \lim_{x \rightarrow 0} f(x)$ cont. $x=0$

2) $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2+2x-1 = -1 \Rightarrow b = -1$

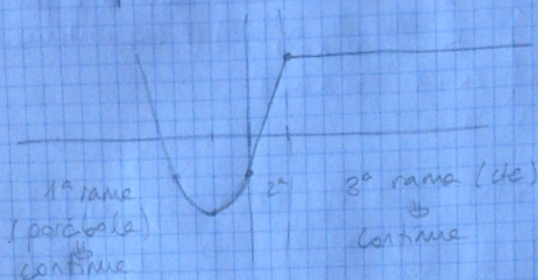
$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} ax+b = b$

$x=1$ 1) $f(1) = 2$

2) $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} ax-1 = a-1 \Rightarrow a-1=2 \Rightarrow a=3$

$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2 = 2$

Para $a=3$ y $b=-1$, $f(x)$ es continua en todo $\mathbb{R}$



Parábola

$$V: x = \frac{-2}{2} = -1$$

$$V: (-1, -2)$$

$$x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{4+4}}{2}$$

4) Calcular a y b para que f sea continua en todo su dominio:

$$f(x) = \begin{cases} \frac{3x+b}{x-1} & \text{si } x \leq 0 \rightarrow \text{Definido en } \mathbb{R} \setminus \{1\} \checkmark \\ \frac{4}{x-2} & \text{si } 0 < x < 1 \rightarrow \text{Definido en } \mathbb{R} \setminus \{2\} \checkmark \\ \frac{x^2+1}{ax} & \text{si } x \geq 1 \rightarrow \text{Definido en } \mathbb{R} \setminus \{0\} \checkmark \end{cases}$$

Por tanto don $f: \mathbb{R} \rightarrow \mathbb{R}$ ya que los puntos que anulan los denominadores caen fuera de las ramas.

1ª rama 2ª rama 3ª rama $\Rightarrow$ Estudiamos $x=0$ $x=1$
Continua Cont. Continua

$x=0$ ① $f(0) = \frac{3 \cdot 0 + b}{0 - 1} = -b$ $f(0) = -2 = \lim_{x \rightarrow 0} f(x)$ ③

② $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{3x+b}{x-1} = \frac{b}{-1} = -b$ $\left\{ \begin{array}{l} \text{1ª rama} \\ \text{2ª rama} \end{array} \right.$ $\left\{ \begin{array}{l} b=2 \\ \text{f cont. en } x=0 \end{array} \right.$

$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{4}{x-2} = \frac{4}{-2} = -2$

$x=1$ ① $f(1) = \frac{1^2+1}{a \cdot 1} = \frac{2}{a}$

② $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \frac{4}{x-2} = \frac{4}{1-2} = -4$ $\Rightarrow -4 = \frac{2}{a} \Rightarrow$

$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{x^2+1}{ax} = \frac{2}{a}$ $\Rightarrow \left\{ \begin{array}{l} a = -\frac{1}{2} \\ 2 \end{array} \right.$

③ $\lim_{x \rightarrow 1} f(x) = -4 = f(1) = \frac{2}{-\frac{1}{2}} = -4 \checkmark \Rightarrow f \text{ cont. en } x=1$

5. Calcula todas las asíntotas de las siguientes funciones. Representa las asíntotas e intenta esbozar la gráfica de las funciones.

a) $f(x) = \frac{x+1}{x-2}$

b) $f(x) = \frac{x^2+1}{x^2-1}$

c) $f(x) = \frac{x^2-x+2}{x}$

d) $f(x) = \frac{x^3}{x^2-1}$

e) $f(x) = \sqrt{x^2-9}$

f) $f(x) = \ln(x^2-4)$

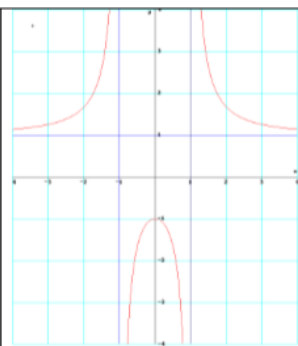
g) $f(x) = \frac{x}{\ln(x)}$

h) $f(x) = e^{-x^2}$

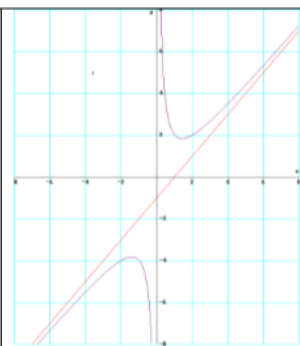
i) $f(x) = \operatorname{cosec}(x)$



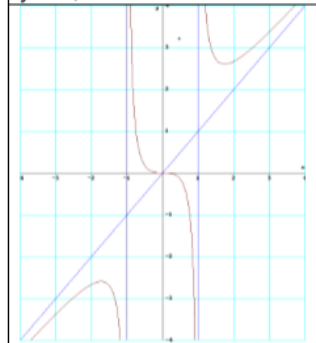
$y = 1, x = 2$



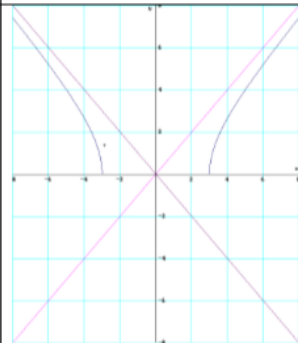
$y = 1, x = 1, x = -1$



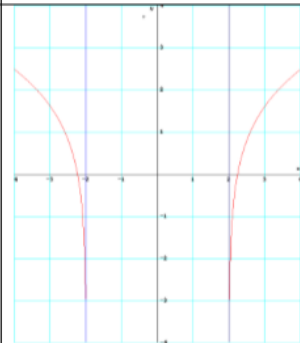
$x = 0, y = x - 1$



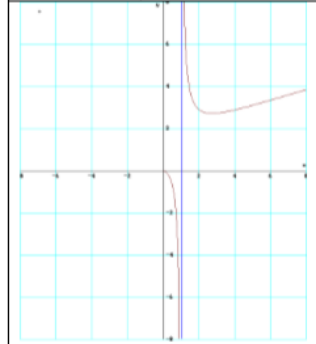
$x = 1, x = -1, y = x$



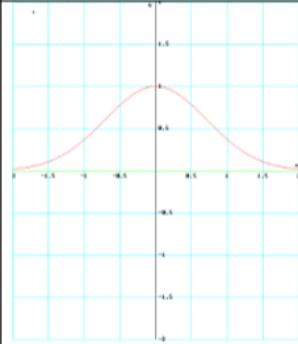
$y = x, y = -x$



$x = 2, x = -2$



$x = 1$



$y = 0$



$X = k\pi, \text{ para todo } k \text{ entero}$