

EJERCICIOS DE POLINOMIOS

1 Realiza la suma de los siguientes polinomios:

$$a) \quad p(x) = x^5 + x^4 - 4x^3 + 6x^2 + x - 7$$

$$q(x) = x^6 + 2x^4 + x^2 + 5$$

$$b) \quad p(x) = 9x^5 - 2x^4 + 12x^3 + x^2 - x + 10$$

$$q(x) = -x^5 + 5x^4 - 12x^3 - 2x^2 + x - 15$$

$$c) \quad p(x) = -5x^4 + 6x^3 - 2x^2 + 3x + 8$$

$$q(x) = 2x^4 - 3x^3 + 2x^2 - 4$$

$$d) \quad p(x) = 3x^4 + x^3 - 2x^2 + x - 14$$

$$q(x) = 6x^4 - 8x^3 + 2x^2 - 3x$$

$$r(x) = 2x + 14$$

$$e) \quad p(x) = -x^6 + 4x^5 - 2x^4 - 7x^3 + 6x^2 + x - 2$$

$$q(x) = 3x^6 + 2x^5 - x^3 + 2x^2 - 2x + 5$$

$$r(x) = -2x^6 - 6x^5 + 2x^4 + 8x^3 - 8x^2 + x - 3$$

$$f) \quad p(x) = x^4 - 3x^3 + x^2 - 7x + 11$$

$$q(x) = 2x^5 - 3x^4 + x^3 - x^2 - 7$$

$$r(x) = -3x^5 + 2x^4 - 5x^3 + 8x^2 + 3x - 4$$

Solución

$$a) \quad \left. \begin{array}{l} p(x) = x^5 + x^4 - 4x^3 + 6x^2 + x - 7 \\ q(x) = x^6 + 2x^4 + x^2 + 5 \end{array} \right\}$$

$$p(x) + q(x) = x^6 + x^5 + 3x^4 - 4x^3 + 7x^2 + x - 2$$

$$b) \quad \left. \begin{array}{l} p(x) = 9x^5 - 2x^4 + 12x^3 + x^2 - x + 10 \\ q(x) = -x^5 + 5x^4 - 12x^3 - 2x^2 + x - 15 \end{array} \right\}$$

$$p(x) + q(x) = 8x^5 + 3x^4 - x^2 - 5$$

$$c) \quad \left. \begin{array}{l} p(x) = -5x^4 + 6x^3 - 2x^2 + 3x + 8 \\ q(x) = 2x^4 - 3x^3 + 2x^2 - 4 \end{array} \right\}$$

$$p(x) + q(x) = -3x^4 + 3x^3 + 3x + 4$$

$$d) \left. \begin{aligned} p(x) &= 3x^4 + x^3 - 2x^2 + x - 14 \\ q(x) &= 6x^4 - 8x^3 + 2x^2 - 3x \\ r(x) &= 2x + 14 \end{aligned} \right\}$$

$$\begin{aligned} p(x) + q(x) + r(x) &= [p(x) + q(x)] + r(x) = \\ &= [9x^4 - 7x^3 - 2x - 14] + 2x + 14 = 9x^4 - 7x^3 \end{aligned}$$

$$e) \left. \begin{aligned} p(x) &= -x^6 + 4x^5 - 2x^4 - 7x^3 + 6x^2 + x - 2 \\ q(x) &= 3x^6 + 2x^5 - x^3 + 2x^2 - 2x + 5 \\ r(x) &= -2x^6 - 6x^5 + 2x^4 + 8x^3 - 8x^2 + x - 3 \end{aligned} \right\}$$

$$\begin{aligned} p(x) + q(x) + r(x) &= [p(x) + q(x)] + r(x) = \\ &= [2x^6 + 6x^5 - 2x^4 - 8x^3 + 8x^2 - x + 3] + \\ &\quad + [-2x^6 - 6x^5 + 2x^4 + 8x^3 - 8x^2 + x - 3] = 0 \end{aligned}$$

$$f) \left. \begin{aligned} p(x) &= x^4 - 3x^3 + x^2 - 7x + 11 \\ q(x) &= 2x^5 - 3x^4 + x^3 - x^2 - 7 \\ r(x) &= -3x^5 + 2x^4 - 5x^3 + 8x^2 + 3x - 4 \end{aligned} \right\}$$

$$\begin{aligned} p(x) + q(x) + r(x) &= [p(x) + q(x)] + r(x) = \\ &= [2x^5 - 2x^4 - 2x^3 - 7x + 4] + [-3x^5 + 2x^4 - 5x^3 + 8x^2 + 3x - 4] = \\ &= -x^5 - 7x^3 + 8x^2 - 4x \end{aligned}$$

2 Realiza la resta de los siguientes polinomios:

$$a) \begin{aligned} p(x) &= x^6 + 2x^5 - 3x^4 + x^3 + 4x^2 + 4x - 4 \\ q(x) &= -x^6 + 2x^5 - 5x^4 + x^3 + 2x^2 + 3x - 8 \end{aligned}$$

$$b) \begin{aligned} p(x) &= -3x^3 + 7x^2 - 3x - 2 \\ q(x) &= 5x^3 + 5x^2 + 5x + 5 \end{aligned}$$

$$c) \begin{aligned} p(x) &= x^4 + 4x^3 - 2x^2 + 7x + 10 \\ q(x) &= -2x^4 + 5x^3 - 8x^2 + 3x + 11 \end{aligned}$$

$$d) \begin{aligned} p(x) &= -x^5 + 5x^3 + 4x^2 - x + 1 \\ q(x) &= x^4 + 9x^3 - 3x^2 + x - 1 \end{aligned}$$

$$e) \begin{aligned} p(x) &= -7x^3 + x^2 - 12x - 2 \\ q(x) &= -6x^3 + 3x^2 - 13x + 15 \end{aligned}$$

$$f) \begin{aligned} p(x) &= x^4 + 3x^3 - 3x^2 + 2x + 14 \\ q(x) &= -x^5 - 2x^4 + 3x^3 - 3x + 14 \end{aligned}$$

Solución

$$a) \left. \begin{aligned} p(x) &= x^6 + 2x^5 - 3x^4 + x^3 + 4x^2 + 4x - 4 \\ q(x) &= -x^6 + 2x^5 - 5x^4 + x^3 + 2x^2 + 3x - 8 \end{aligned} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= x^6 + 2x^5 - 3x^4 + x^3 + 4x^2 + 4x - 4 - \\ &\quad - [-x^6 + 2x^5 - 5x^4 + x^3 + 2x^2 + 3x - 8] \end{aligned}$$

$$p(x) - q(x) = 2x^6 + 2x^4 + 2x^2 + x + 4$$

$$b) \left. \begin{aligned} p(x) &= -3x^3 + 7x^2 - 3x - 2 \\ q(x) &= 5x^3 + 5x^2 + 5x + 5 \end{aligned} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= -3x^3 + 7x^2 - 3x - 2 - [5x^3 + 5x^2 + 5x + 5] \end{aligned}$$

$$p(x) - q(x) = -8x^3 + 2x^2 - 8x - 7$$

$$c) \left. \begin{aligned} p(x) &= x^4 + 4x^3 - 2x^2 + 7x + 10 \\ q(x) &= -2x^4 + 5x^3 - 8x^2 + 3x + 11 \end{aligned} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= x^4 + 4x^3 - 2x^2 + 7x + 10 - [-2x^4 + 5x^3 - 8x^2 + 3x + 11] \end{aligned}$$

$$p(x) - q(x) = 3x^4 - x^3 + 6x^2 + 4x - 1$$

$$d) \left. \begin{aligned} p(x) &= -x^5 + 5x^3 + 4x^2 - x + 1 \\ q(x) &= x^4 + 9x^3 - 3x^2 + x - 1 \end{aligned} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= -x^5 + 5x^3 + 4x^2 - x + 1 - [x^4 + 9x^3 - 3x^2 + x - 1] \end{aligned}$$

$$p(x) - q(x) = -x^5 - x^4 - 4x^3 + 7x^2 - 2x + 2$$

$$e) \left. \begin{aligned} p(x) &= -7x^3 + x^2 - 12x - 2 \\ q(x) &= -6x^3 + 3x^2 - 13x + 15 \end{aligned} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= -7x^3 + x^2 - 12x - 2 - [-6x^3 + 3x^2 - 13x + 15] \end{aligned}$$

$$p(x) - q(x) = -x^3 - 2x^2 + x - 17$$

$$f) \quad p(x) = x^4 + 3x^3 - 3x^2 + 2x + 14$$

$$q(x) = -x^5 - 2x^4 + 3x^3 - 3x + 14$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= x^4 + 3x^3 - 3x^2 + 2x + 14 - [-x^5 - 2x^4 + 3x^3 - 3x + 14] \end{aligned}$$

$$p(x) - q(x) = x^5 + 3x^4 - 3x^2 + 5x$$

3 Realiza el producto de los siguientes polinomios:

$$a) \quad p(x) = x^4 + 2x^3 - x^2 + 3x + 1$$

$$q(x) = 2x$$

$$b) \quad p(x) = -x^5 + x^4 - x^3 + x^2 - x + 1$$

$$q(x) = -5x^4$$

$$c) \quad p(x) = 2x^6 + 3x^4 + x^2 - 6$$

$$q(x) = x^3 + x$$

$$d) \quad p(x) = x^2 + 2x + 3$$

$$q(x) = -x^2 + x + 4$$

Solución

$$a) \quad \left. \begin{aligned} p(x) &= x^4 + 2x^3 - x^2 + 3x + 1 \\ q(x) &= 2x \end{aligned} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) &= [x^4 + 2x^3 - x^2 + 3x + 1] \cdot 2x = \\ &= x^4 \cdot 2x + 2x^3 \cdot 2x - x^2 \cdot 2x + 3x \cdot 2x + 1 \cdot 2x = \\ &= 2x^5 + 4x^4 - 2x^3 + 6x^2 + 2x \end{aligned}$$

$$b) \quad \left. \begin{aligned} p(x) &= -x^5 + x^4 - x^3 + x^2 - x + 1 \\ q(x) &= -5x^4 \end{aligned} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) &= [-x^5 + x^4 - x^3 + x^2 - x + 1] \cdot (-5x^4) = \\ &= 5x^9 - 5x^8 + 5x^7 - 5x^6 + 5x^5 - 5x^4 \end{aligned}$$

$$c) \left. \begin{aligned} p(x) &= 2x^6 + 3x^4 + x^2 - 6 \\ q(x) &= x^3 + x \end{aligned} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) &= [2x^6 + 3x^4 + x^2 - 6] \cdot [x^3 + x] = \\ &= [2x^6 + 3x^4 + x^2 - 6] \cdot x^3 + [2x^6 + 3x^4 + x^2 - 6] \cdot x = \\ &= [2x^9 + 3x^7 + x^5 - 6x^3] + [2x^7 + 3x^5 + x^3 - 6x] = \\ &= 2x^9 + 5x^7 + 4x^5 - 5x^3 - 6x \end{aligned}$$

$$d) \left. \begin{aligned} p(x) &= x^2 + 2x + 3 \\ q(x) &= -x^2 + x + 4 \end{aligned} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) &= [x^2 + 2x + 3] \cdot [-x^2 + x + 4] = \\ &= [x^2 + 2x + 3] \cdot (-x^2) + [x^2 + 2x + 3] \cdot x + [x^2 + 2x + 3] \cdot 4 = \\ &= -x^4 - 2x^3 - 3x^2 + x^3 + 2x^2 + 3x + 4x^2 + 8x + 12 = \\ &= -x^4 - x^3 + 3x^2 + 11x + 12 \end{aligned}$$

4. Sean los polinomios:

$$M(x) = 3x^2 - 5x - 3; \quad N(x) = \frac{1}{2}x^2 + \frac{3}{4}x + 1; \quad K(x) = x^2 - \frac{1}{3}x + \frac{2}{3}$$

Calcula:

a) $2M(x) + 4N(x) + 3K(x)$

b) $M(x) - 2N(x)$

c) $M(x) + 3N(x) - K(x)$

$$M(x) = 3x^2 - 5x - 3; \quad N(x) = \frac{1}{2}x^2 + \frac{3}{4}x + 1; \quad K(x) = x^2 - \frac{1}{3}x + \frac{2}{3}$$

$$\left. \begin{aligned} a) \quad 2M(x) &= 6x^2 - 10x - 6 \\ 4N(x) &= 2x^2 + 3x + 4 \\ 3K(x) &= 3x^2 - x + 2 \end{aligned} \right\} \rightarrow 2M(x) + 4N(x) + 3K(x) = 11x^2 - 8x$$

b) $2N(x) = x^2 + \frac{3}{2}x + 2$

$$-2N(x) = -x^2 - \frac{3}{2}x - 2$$

$$M(x) - 2N(x) = 2x^2 - \frac{13}{2}x - 5$$

$$\left. \begin{aligned} c) \quad 3N(x) &= \frac{3}{2}x^2 + \frac{9}{4}x + 3 \\ -K(x) &= -x^2 + \frac{1}{3}x - \frac{2}{3} \end{aligned} \right\}$$

$$M(x) + 3N(x) - K(x) = \frac{7}{2}x^2 - \frac{29}{12}x - \frac{2}{3}$$

5. Calcula el cociente y el resto en cada una de estas divisiones:

a) $(x^5 + 7x^3 - 5x + 1) : (x^3 + 2x)$

b) $(x^3 - 5x^2 + x) : (x^2 - 1)$

a) $(x^5 + 7x^3 - 5x + 1) : (x^3 + 2x)$

$$\begin{array}{r} x^5 + 7x^3 - 5x + 1 \quad \overline{) x^3 + 2x} \\ -x^5 - 2x^3 \\ \hline 5x^3 - 5x \\ -5x^3 - 10x \\ \hline -15x + 1 \end{array} \quad \begin{array}{l} C(x) \\ R(x) \end{array}$$

b) $(x^3 - 5x^2 + x) : (x^2 - 1)$

$$\begin{array}{r} x^3 - 5x^2 + x \quad \overline{) x^2 - 1} \\ -x^3 + x \\ \hline -5x^2 + 2x \\ 5x^2 - 5 \\ \hline 2x - 5 \end{array} \quad \begin{array}{l} C(x) \\ R(x) \end{array}$$

6. 1. Utilizando la regla de Ruffini, halla el cociente y el resto de estas divisiones.

a) $(3x^4 - 2x^2 + 5x - 2) : (x - 2)$

b) $(-x^4 + 2x^3 - 3x + 1) : (x + 1)$

c) $(x^3 - 27) : (x - 3)$

a) $(3x^4 - 2x^2 + 5x - 2) : (x - 2)$

$$\begin{array}{r|rrrrr} 2 & 3 & 0 & -2 & 5 & -2 \\ & & 6 & 12 & 20 & 50 \\ \hline & 3 & 6 & 10 & 25 & 48 \end{array}$$

$$\begin{array}{l} C(x) = 3x^3 + 6x^2 + 10x + 25 \\ R = 48 \end{array}$$

$$b) (-x^4 + 2x^3 - 3x + 1) : (x + 1)$$

-1	-1	2	0	-3	1
		1	-3	3	0
	-1	3	-3	0	1

$$C(x) = -x^3 + 3x^2 - 3x$$

$$R = 1$$

$$c) (x^3 - 27) : (x - 3)$$

3	1	0	0	-27
	3	9	27	
	1	3	9	0

$$C(x) = x^2 + 3x + 9$$

$$R = 0$$

7.

ACTIVIDADES

9. Realiza las siguientes multiplicaciones de polinomios:

a) $x^2y \cdot xy^2$

b) $3a^2b \cdot 4ab^3c$

c) $4x^3y^2 \cdot 6xy^4$

d) $4ax^2 \cdot 5a^4x^3$

10. Realiza las siguientes multiplicaciones de polinomios:

a) $4x^2(2x^2 + 3x - 4)$

b) $3x(9x^3 - 6x + 2)$

c) $5x^3(8x - 12x - 3)$

11. Con $F(x) = 4x + 5$, $G(x) = 3x^2 - 2x + 1$ y $H(x) = 3x - 7$, calcula:

a) $F(x) \cdot G(x)$

b) $F(x) \cdot H(x)$

c) $G(x) \cdot H(x)$

12. Saca factor común en las siguientes expresiones:

a) $x^6 - x^4$

b) $6a^7 - 3a^3 + 9a^2$

c) $12x^5y^6z^3 - 6x^3y^5z^4 + 18x^4y^4z^5$

Solución:

9.

a) $x^2y \cdot xy^2 = x^3y^3$

b) $3a^2b \cdot 4ab^3c = 12a^3b^4c$

c) $4x^3y^2 \cdot 6xy^4 = 24x^4y^6$

d) $4ax^2 \cdot 5a^4x^3 = 20a^5x^5$

10.

a) $4x^2(2x^2 + 3x - 4) = 8x^4 + 12x^3 - 16x^2$

b) $3x(9x^3 - 6x + 2) = 27x^4 - 18x^2 + 6x$

c) $5x^3(8x - 12x - 3) = 5x^3(-4x - 3) = -20x^4 - 15x^3$

11.

a)

$$F(x) \cdot G(x) = (4x + 5)(3x^2 - 2x + 1) =$$

$$12x^3 - 8x^2 + 4x + 15x^2 - 10x + 5 = 12x^3 + 7x^2 - 6x + 5$$

b) $F(x) \cdot H(x) = (4x + 5)(3x - 7) = 12x^2 - 28x + 15x - 35 = 12x^2 - 13x - 35$

c)

$$G(x) \cdot H(x) = (3x^2 - 2x + 1)(3x - 7) =$$

$$9x^3 - 21x^2 - 6x^2 + 14x + 3x - 7 = 9x^3 - 27x^2 + 17x - 7$$

12.

a) $x^6 - x^4 = x^4(x^2 - 1)$

b) $6a^7 - 3a^3 + 9a^2 = 3a^2(2a^5 - a + 3)$

c) $12x^5y^6z^3 - 6x^3y^5z^4 + 18x^4y^4z^5 = 6x^3y^4z^3(2x^2y^2 - yz + 3xz^2)$

8.

ACTIVIDADES

13. Desarrolla las siguientes identidades notables:

a) $(9x - 4)^2$

c) $(x - 8)^2$

e) $(x^2 + 6)^2$

b) $(4x + 5)^2$

d) $(4x - 7) \cdot (4x + 7)$

f) $(x + 1) \cdot (x - 1)$

14. Desarrolla las siguientes identidades notables:

a) $(x + 1) - (x - 1)^2$

c) $(x + 2) - (x - 1) \cdot (x + 1)$

b) $(2x + y)^2 - (y - 2x)^2$

d) $(x - 2x) \cdot (x + 2) + (x - 3x)^2$

Solución:

13.

a) $(9x - 4)^2 = 81x^2 - 72x + 16$

b) $(4x + 5)^2 = 16x^2 + 40x + 25$

c) $(x - 8)^2 = x^2 - 16x + 64$

d) $(4x - 7)(4x + 7) = 16x^2 - 49$

e) $(x^2 + 6)^2 = x^4 + 12x^2 + 36$

f) $(x + 1)(x - 1) = x^2 - 1$

14.

a) $(x+1) - (x-1)^2 = x+1 - (x^2 - 2x + 1) = x+1 - x^2 + 2x - 1 = -x^2 + 3x$

b)

$(2x+y)^2 - (y-2x)^2 = 4x^2 + 4xy + y^2 - (y^2 - 4xy + 4x^2) =$

$4x^2 + 4xy + y^2 - y^2 + 4xy - 4x^2 = 8xy$

c) $(x+2) - (x-1)(x+1) = x+2 - (x^2 - 1) = x+2 - x^2 + 1 = -x^2 + x + 3$

d) $(x-2x)(x+2) + (x-3x)^2 = -x(x+2) + (-2x)^2 = -x^2 - 2x + 4x^2 = 3x^2 - 2x$

9.

ACTIVIDADES

19. Realiza las siguientes divisiones de polinomios:

a) $(x^2 + 12x + 4) : (x - 2)$

b) $(x^3 - 1) : (x - 1)$

c) $(4x^4 + 4x^2 + 1) : (2x^2 + 2x + 1)$

d) $(3x^4 + 2x^3 + 5x - 17) : (x^2 - 2x - 1)$

e) $(9x^2 - 13x + 12) : (x - 3)$

f) $(2x^3 + 6x^2 - 7x + 2) : (2x^2 - 5)$

g) $(14x^4 - 15x^3 - 16x^2 + 17x + 5) : (2x^2 - x - 2)$

Solución:

a)

$$\begin{array}{r} x^2 + 12x + 4 \\ - x^2 + 2x \\ \hline 14x + 4 \\ - 14x + 28 \\ \hline 32 \end{array} \quad \begin{array}{r} x - 2 \\ \hline x + 14 \end{array}$$

32

b)

$$\begin{array}{r}
 x^3 + x^2 - 1 \\
 \underline{-x^3 + x^2} \\
 x^2 - 1 \\
 \underline{-x^2 + x} \\
 x - 1 \\
 \underline{-x + 1} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 x^2 - 1 \\
 \underline{x^2 + x + 1}
 \end{array}$$

c)

$$\begin{array}{r}
 4x^4 + 4x^2 + 1 \\
 \underline{-4x^4 - 4x^3 - 2x^2} \\
 -4x^3 + 2x^2 \\
 \underline{4x^3 + 4x^2 + 2x} \\
 6x^2 + 2x + 1 \\
 \underline{-6x^2 - 6x - 3} \\
 -4x - 2
 \end{array}
 \quad
 \begin{array}{r}
 2x^2 + 2x + 1 \\
 \underline{2x^2 - 2x + 3}
 \end{array}$$

d)

$$\begin{array}{r}
 3x^4 + 2x^3 + 5x - 17 \\
 \underline{-3x^4 + 6x^3 + 3x^2} \\
 8x^3 + 3x^2 + 5x - 17 \\
 \underline{-8x^3 + 16x^2 + 8x - 17} \\
 19x^2 + 13x - 17 \\
 \underline{-19x^2 + 38x + 19} \\
 51x + 2
 \end{array}
 \quad
 \begin{array}{r}
 x^2 - 2x - 1 \\
 \underline{3x^2 + 8x + 19}
 \end{array}$$

e)

$$\begin{array}{r}
 9x^2 - 13x + 12 \\
 \underline{-9x^2 + 27x} \\
 14x + 12 \\
 \underline{-14x + 42} \\
 54
 \end{array}
 \quad
 \begin{array}{r}
 x - 3 \\
 \underline{9x + 14}
 \end{array}$$