1.- Calcular las siguientes integrales inmediatas **(0, 75 puntos cada integral)**

a) $\int \frac{e^x}{1+e^{2x}}dx$ b) $\int \frac{sen(artg(x-1))}{x^2 \quad 2x+2}dx$

c) $\int \frac{\ln(x \quad 2)}{(x \quad 2)}dx$ d) $\int \frac{6Cos(Ln5x)}{x}dx$

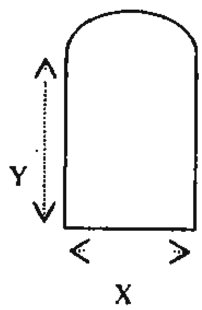
2.- Calcular las siguientes integrales·

a) $\int (2x \quad 3)sen(4x)dx$ **(1 punto)**

b) $\int \frac{2x^2+3x \quad 1}{x^2 \quad 9}dx$ **(1,5 puntos)**

c) $\int \frac{x \quad 5}{x^2+2x+6}dx$ **(1,5 puntos)**

3.- Se considera una ventana como la que se indica en la figura (la parte inferior es rectangular; la superior, una semicircunferencia) El perímetro de la ventana mide 6 m: Hallar las dimensiones x e y del rectángulo para que la superficie de la ventana sea maxima. (Expresar los resultados en función de π). **(1,5 puntos)**



4.- Calcula los valores de a, b, c y d para que la función $f(x) - ax^3 + bx^2 + cx + d$ tenga un máximo relativo en $x_0 = 1$, un punto de inflexión en (2, 0) y su recta tangente en $x_0 = 0$ tenga de pendiente 2. **(1,5 puntos)**

(A)

1º a) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{e^x}{1+(e^x)^2}\,dx = \boxed{\operatorname{arctg}(e^x)+C}$

$u = e^x$

$u' = e^x$

b) $\displaystyle\int \frac{\operatorname{sen}(\operatorname{arctg}(x-1))}{x^2+2x+2}\,dx = \boxed{-\cos(\operatorname{arctg}(x-1))+C}$

$u = \operatorname{arctg}(x-1)$

$u' = \dfrac{1}{1+(x-1)^2} = \dfrac{1}{x^2-2x+2}$

c) $\displaystyle\int \frac{\ln(x-2)}{x-2}\,dx = \int \frac{1}{x-2}\ln(x-2)\,dx = \boxed{\frac{\ln^2(x-2)}{2}+C}$

d) $\displaystyle\int 6\,\frac{\cos(\ln 5x)}{x}\,dx = 6\int \frac{1}{x}\cos\ln(5x)\,dx = \boxed{6\operatorname{sen}(\ln(5x))+C}$

$u = \ln 5x$

$u' = \dfrac{5}{5x} = \dfrac{1}{x}$

2) a) $\displaystyle\int (2x-3)\operatorname{sen}(4x)\,dx = \frac{-(2x-3)\cos(4x)}{4} + \frac{1}{2}\int \cos(4x)\,dx$

$u = 2x-3 \qquad du = 2dx$

$dv = \operatorname{sen}(4x)\,dx \qquad v = -\frac{1}{4}\cos(4x)$

$= \boxed{\dfrac{-(2x-3)\cos(4x)}{4} + \dfrac{1}{8}\operatorname{sen}(4x)+C}$

b) $\displaystyle\int \frac{2x^2+3x-1}{x^2-9}\,dx = \int 2 + \frac{3x+17}{x^2-9}\,dx = 2x + \int \frac{3x+17}{x^2-9}\,dx =$

$$\begin{array}{r|l} \cancel{2x^2}+3x-1 & x^2-9 \\ \cancel{-2x^2}\quad\;\; +18 & 2 \\ \hline 3x+17 & \end{array}$$

Aplicamos el método de Hermite

$\dfrac{3x+17}{(x-3)(x+3)} = \dfrac{A}{x-3} + \dfrac{B}{x+3}$

$3x+17 = A(x+3)+B(x-3)$

$x=3 \rightarrow 26 = 6A \qquad A = \frac{13}{3}$

$x=-3 \rightarrow 8 = -6B \qquad B = -4/3$

$= \boxed{2x + \frac{13}{3}\ln|x-3| - \frac{4}{3}\ln|x+3| + C}$

$$\int \frac{x-5}{x^2+2x+6}dx = \frac{1}{2}\int \frac{2x-10}{x^2+2x+6}dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x+6} - \frac{12}{2}\int \frac{1}{x^2+2x+6}dx$$

$$\int \frac{1}{x^2+2x+6}dx = \int \frac{1}{5+(x+1)^2}dx = \frac{1}{5}\sqrt{5}\int \frac{1/\sqrt{5}}{1+\left(\frac{x+1}{\sqrt{5}}\right)^2}dx = \frac{\sqrt{5}}{5}\operatorname{arctg}\left(\frac{x+1}{\sqrt{5}}\right)$$

$$= \boxed{\frac{1}{2}\ln(x^2+2x+6) - \frac{6\sqrt{5}}{5}\operatorname{arctg}\left(\frac{x+1}{\sqrt{5}}\right) + C}$$

3) (1) $6 = \pi\cdot\frac{x}{2} + 2y + x$ (3) $y = \dfrac{6-\frac{\pi}{2}x - x}{2} = 3 - \frac{\pi}{4}x - \frac{x}{2}$

(2) $F(x,y) = \dfrac{\pi\left(\frac{x}{2}\right)^2}{2} + xy$

$$f(x) = \frac{\pi}{8}x^2 + x\left(3-\frac{\pi}{4}x-\frac{x}{2}\right) = \frac{\pi}{8}x^2 + 3x - \frac{\pi}{4}x^2 - \frac{x^2}{2} = \frac{\pi x^2 + 24x - 2\pi x^2 - 4x^2}{8} = \frac{1}{8}(24x - \pi x^2 - 4x^2)$$

$$f'(x) = \frac{1}{8}(24 - 2x(\pi+4)) = 0 \quad \boxed{x = \frac{12}{\pi+4}}$$

$f''(x) = \frac{1}{8}(-2(\pi+4)) < 0 \Rightarrow x = \frac{12}{\pi+4}$ es un máximo

$$y = 3 - \frac{\pi}{\cancel{4}}\left(\frac{\cancel{12}\,3}{\pi+4}\right) - \frac{1}{\cancel{2}}\left(\frac{\cancel{12}\,6}{\pi+4}\right) = 3 - \frac{3\pi-6}{\pi+4} = \frac{\cancel{3\pi}+12-\cancel{3\pi}+6}{\pi+4} = \boxed{\frac{18}{\pi+4}}$$

(4) $f(x) = ax^3+bx^2+cx+d$ $f'(x) = 3ax^2+2bx+c$ $f''(x) = 6ax+2b$

$x_0 = 1$ Max $f'(1) = 0$ $3a+2b+c=0$

$(2,0)$ P.I $\rightarrow f(2)=0$ $8a+4b+2c+d=0$

$\rightarrow f''(2)=0$ $12a+2b=0$

$$\begin{cases} 3a+2b=-2 \\ 12a+2b=0 \end{cases}$$

$9a = 2$

$\boxed{a = 2/9}$

$\boxed{b = -4/3}$

En $x_0=0$ $m=2$ $f'(0)=2$ $\boxed{c=2}$

$$d = -8a-4b-2c \Rightarrow d = -\frac{16}{9} - \frac{16}{3} - 4 = -\frac{4}{9}$$

$\boxed{d = -4/9}$

1.- Calcular las siguientes integrales inmediatas: **(0, 75 puntos cada integral)**

a) $\displaystyle\int \frac{\tan(x \quad 2)}{\cos^2(x \quad 2)}dx$

b) $\displaystyle\int 2^{\cos(\ln x)} \frac{sen(\ln x)}{x}dx$

c) $\displaystyle\int \frac{1}{x\sqrt{1-\ln^2 x}}dx$

d) $\displaystyle\int \quad \frac{3sen(\ln 2x)}{x}dx$

2.- Calcular las siguientes integrales:

a) $\displaystyle\int (3x \quad 4)\cos(5x)dx$ **(1 punto)**

b) $\displaystyle\int \frac{x^3+6x \quad 1}{x^3-1}dx$ **(1,5 puntos)**

c) $\displaystyle\int \frac{x \quad 2}{x^2+4x+7}dx$ **(1,5 puntos)**

3.- Calcula los valores de a, b, c y d para que la función $f(x) \quad ax^3 + bx^2 + cx + d$ tenga un máximo relativo en $x_0 \quad 2$, un punto de inflexión en (-1, 0) y su recta tangente en $x_0 \quad 0$ tenga de pendiente 3. **(1,5 puntos)**

4.- Las cinco caras de un estanque de base cuadrada tienen un área de 192m². Calcular las dimensiones para que el volumen sea máximo. **(1,5 puntos)**

B

1) a) $\int \frac{\tan(x-2)}{\cos^2(x-2)}\,dx = \frac{\tan^2(x-2)}{2} + C$

b) $\int e^{\cos(\ln x)}\,\frac{\sin(\ln x)}{x}\,dx = \frac{-e^{\cos(\ln x)}}{\ln e} + C$

c) $\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx = \arcsin(\ln x) + C$

d) $\int \frac{3\sin(\ln 2x)}{x} = -3\cos(\ln 2x) + C$

2) a) $\int (3x-4)\cos(5x)\,dx = \frac{(3x-4)}{5}\sin 5x + \frac{3}{25}\cos 5x + C$

$u = 3x-4 \qquad du = 3$

$dv = \cos 5x\,dx \qquad v = \int \cos 5x\,dx = \frac{1}{5}\sin 5x$

b) $\cancel{x^2} + 6x - 1 \;|\; x^2-1$ → 1

$-\cancel{x^2} \quad +1$

$6x$

$\int x + \frac{6x}{x^2-1}\,dx = \frac{x^2}{2} + \int \frac{6x}{x^2\ 1}$

$\frac{6x}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$= \boxed{\frac{x^2}{2} + 3\ln|x-1| + 3\ln|x+1| + C}$

$6x = A(x+1) + B(x-1)$

$x=1 \quad 6 = 2A \quad A=3$

$x=-1 \quad -6 = -2B \quad B=3$

c) $\int \frac{2x-4}{x^2+4x+7}\,dx \quad \frac{1}{2}\int \frac{2x+4}{x^2+4x+7}\,dx - \frac{8}{2}\int \frac{1}{x^2+4x+7}$

$= \boxed{\frac{1}{2}\ln|x^2+4x+7| - \frac{4\sqrt{3}}{3}\operatorname{arctg}\left(\frac{x+2}{\sqrt{3}}\right) + C}$

$x^2+4x+7 = 3 + (x+2)^2$

$\int \frac{1}{3+(x+2)^2}\,dx = \frac{1}{3}\int \frac{1}{1+\left(\frac{x+2}{\sqrt{3}}\right)^2}\,dx = \frac{\sqrt{3}}{3}\operatorname{arctg}\left(\frac{x+2}{\sqrt{3}}\right)$

3) $f(x) = ax^3 + bx^2 + cx + d$ $\quad f'(x) = 3ax^2 + 2bx + c$ $\quad f''(x) = 6ax + 2b$

Max $x_0 = 2$ $\quad f'(2) = 0$ $\quad 12a + 4b + c = 0$

PI $(-1, 0)$ $\to f(-1) = 0$ $\quad -a + b + c + d = 0$

$\to f''(-1) = 0$ $\quad -6a + 2b = 0$

$$\begin{cases} 12a + 4b = -3 \\ -6a + 2b \end{cases}$$

$f'(0) = 3$ $\quad \boxed{c = 3}$

$+12a + 4b = 3$

$-12a + 4b = 0$

$\boxed{b = -3/8}$

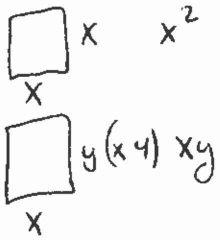
$-6a - \frac{6}{8} = 0$

$\boxed{a = -1/8}$

$d = a - b + c$

$d = -\frac{1}{8} + \frac{3}{8} + 3 = \frac{2}{8} + 3 = \frac{26}{8} = \frac{13}{4}$

④



① $x^2 + 4xy = 192$

② $F(x, y) = x^2 y$

③ $y = \frac{192 - x^2}{4x}$

④ $f(x) = x^2 \left(\frac{192 - x^2}{4x} \right) = \frac{1}{4} (192x - x^3)$

$f'(x) = \frac{1}{4} (192 - 3x^2)$ $\quad x = \pm \sqrt{64} = \pm 8$

$f''(x) = \frac{1}{4} (-6x)$

$\to f''(8) = < 0$ Max

$\to f''(-8) = > 0$ Min

$x = 8$

$y = \frac{192 - 64}{32} = 4$