1º.- Dados Los números complejos $z_1= 3+i$ y $z_2=1-i$ calcular:

a) $z_1+ z_2$

b) $z_1 . z_2$

c) z_1 / z_2

d) $z_1^2 . z_2^2$

2º.- Escribir en forma binómica los siguientes números complejos:

a) $2_{270º}$

b) $1_{180º}$

c) $3_{-270º}$

d) $5_{0º}$

e) $6_{2700º}$

3º.- Calcular:

$$\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)^4\left(-\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\right)^4(1+i)^{10}$$

4º.- Hallar a y b para que se cumpla la siguiente igualdad:

$$\frac{a+19i}{-5+bi}=3-2i$$

5º.- Calcular dando los resultados en forma binómica:

$$\sqrt[4]{-\frac{1}{2}-i\frac{\sqrt{3}}{2}}$$

1º Dados los nºs complejos $z_1 = 3+i$ y $z_2 = 1-i$ Calcular:

a) $z_1 + z_2$ b) $z_1 \cdot z_2$ c) $\frac{z_1}{z_2}$ d) $z_1^2 \cdot z_2^2$

a) $z_1 + z_2 = (3+i) + (1-i) = (3+1) + (i-i) = 4+0 = 4$

b) ~~$(3+i)(1-i) = 3 - 3i + 3i - i^2 = 4 + i0$~~

$(3+i)(1-i) = 3 - 3i + i - i^2 = 3+1 - 3i + i = 4 - 2i$ ($i^2 = -1$)

c) $\frac{3+i}{1-i} = \frac{(3+i)(1+i)}{(1-i)(1+i)} = \frac{3+3i+i+i^2}{1^2 - i^2} = \frac{2+4i}{2} = 1 + 2i$

d) $z_1^2 = (3+i)^2 = 3^2 + i^2 + 6i = 9 - 1 + 6i = 8 + 6i$

$z_2^2 = (1-i)^2 = 1 + i^2 - 2i = 1 - 1 - 2i = -2i$

$z_1^2 z_2^2 = (8+6i)(-2i) = -16i - 12i^2 = 12 - 16i$

2º a) $2_{270º} = -2i$ b) $1_{180º} = -1$ c) $3_{-270º} = 3_{90º} = 3i$ d) $5_{0º} = 5$

3º Calcular $\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^4 \left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)^4 (1+i)^{10} = x$

$z_1 = \left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^4 = 1_{135º}$

$r = \sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{\frac{4}{4}} = 1$

$\alpha = \text{arctg}\left(\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}\right) = -1 = 135º$

$z_2 = \left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right) = 1_{225º}$

$s = \sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = 1$ $\beta = \text{arctg}\left(\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}\right) = \text{arctg } 1 = 225º$

$z_3 = (1+i) = \sqrt{2}_{45º}$

$t = \sqrt{1^2+1^2} = \sqrt{2}$ $\alpha = \text{arctg}\left(\frac{1}{1}\right) = 45º$

$\left(1_{135º}\right)^4 \cdot \left(1_{225º}\right)^4 \cdot \left[\left(\sqrt{2}\right)_{45º}\right]^{10} = 1_{540} \cdot 1_{900} \cdot 2^5_{45º} = (1 \cdot 1 \cdot 2^5)_{540+900+45º} =$

$= 32_{45º} = 32(\cos 45º + i\,\text{sen}\,45º) = \boxed{16\sqrt{2} + 16\sqrt{2}\,i}$

$540 + 900 + 45 = 1485$; $1485 : 360 = 4$, resto 045

4° Hallar a y b para que se cumpla la igualdad: $\frac{a+19i}{-5+bi} = 3-2i$

$a+19i = (3-2i)(-5+bi) = -15+3bi+10i-2b = (-15-2b)+i(10+3b)$

$a = -15-2b$
$19 = 10+3b \rightarrow 3b = 9$ $\boxed{b=3}$

$\boxed{a = -15-2\cdot 3 = -15-6 = -21}$

5° Calcular dando los resultados en forma binómica: $\sqrt[4]{-\frac{1}{2}-i\frac{\sqrt{3}}{2}}$

$z = -\frac{1}{2}-i\frac{\sqrt{3}}{2}$ (III cuadrante) $\quad r = \sqrt{\left(-\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{\frac{4}{4}} = 1$

$\alpha = \text{arctg}\left(\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right) = \text{arctg}\sqrt{3} = 240°$

$\sqrt[4]{1_{240°}} = \left(\sqrt[4]{1}\right)_{\frac{240°}{4}+k\cdot\frac{360}{4}} = 1_{60°+k\cdot 90°} =$

$k = 0, 1, 2, 3$

$$= \begin{cases} 1_{60°} = 1\cdot(\cos 60° + i \sin 60°) = \frac{1}{2}+i\frac{\sqrt{3}}{2} \\ 1_{150°} = 1(\cos 150° + i\sin 150°) = -\frac{\sqrt{3}}{2}+i\frac{1}{2} \\ 1_{240°} = 1(\cos 240° + i\sin 240°) = -\frac{1}{2}-i\frac{\sqrt{3}}{2} \\ 1_{330°} = 1(\cos 330° + i\sin 330°) = \frac{\sqrt{3}}{2}-i\frac{1}{2} \end{cases}$$

$150° = 180° - 30°$
$240 = 180° + 60$
$330° = 360 - 30°$