

MATEMÁTICAS 1º BACH. C. N. Y S.  
Trigonometría y Complejos

4 de diciembre de 2006

- 1) Demostrar que:  $\cos x + \operatorname{sen} x \operatorname{tg} x = \sec x$
- 2) Resolver la ecuación:  $2 \operatorname{tg} x \sec x - \operatorname{tg} x = 0$
- 3) Resolver un triángulo del que conocemos:  $a = 13$ ,  $b = 5$  y  $C = 100^\circ$
- 4) Hallar  $(-2 + 2i)^6$ , dando los resultados en polar, trigonométrica, binómica y cartesiana.
- 5) Hallar todos los complejos que son resultados de  $\sqrt[5]{-1}$

①  $\cos x + \operatorname{sen} x \operatorname{tg} x = \cos x + \operatorname{sen} x \frac{\operatorname{sen} x}{\cos x} = \cos x + \frac{\operatorname{sen}^2 x}{\cos x} = \frac{\cos^2 x + \operatorname{sen}^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

②  $2 \operatorname{tg} x \sec x - \operatorname{tg} x = 0 \Rightarrow \operatorname{tg} x (2 \sec x - 1) = 0 \Rightarrow \begin{cases} \operatorname{tg} x = 0 \\ 2 \sec x - 1 = 0 \end{cases}$   
 Si  $\operatorname{tg} x = 0 \Rightarrow \boxed{x = 0^\circ + 180^\circ k, k \in \mathbb{Z}}$   
 Si  $2 \sec x - 1 = 0 \Rightarrow 2 \sec x = 1 \Rightarrow \sec x = \frac{1}{2} \Rightarrow \frac{1}{\cos x} = \frac{1}{2} \Rightarrow \cos x = 2$   $\left\{ \begin{array}{l} \text{No es} \\ \text{posible} \end{array} \right.$

③ 2 lados y ángulo comprendido  $\rightarrow$  T. coseno y solución única.  
 $c = \sqrt{a^2 + b^2 - 2ab \cos C} = \sqrt{169 + 25 - 130 \cos 100^\circ} = 15\sqrt{2}$   
 $b^2 = a^2 + c^2 - 2ac \cos B \Rightarrow 2ac \cos B = a^2 + c^2 - b^2 \Rightarrow \cos B = \frac{a^2 + c^2 - b^2}{2ac}$   
 $\Rightarrow B = 19,55^\circ = 19^\circ 32' 52,2'' \Rightarrow \boxed{A = 180^\circ - 100^\circ - B = 60,45^\circ = 60^\circ 27' 25''}$

④  $z = -2 + 2i \Rightarrow (z)(= \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}) \operatorname{tg} x = \frac{2}{2} = 1 \Rightarrow x = 45^\circ$   
 $\left. \begin{array}{l} \text{2º cuadrante} \\ \text{2º cuadrante} \end{array} \right\} \Rightarrow \alpha = 135^\circ$   
 $\Rightarrow z = (2\sqrt{2})_{135^\circ} \Rightarrow \boxed{z^6 = [(2\sqrt{2})_{135^\circ}]^6 = (2\sqrt{2})_{135^\circ \cdot 6}^6 = [2^6 \cdot (\sqrt{2})^6]_{810^\circ} = (2^6 \cdot 2^3)_{90^\circ} = (2^9)_{90^\circ} = \boxed{512_{90^\circ}} \text{ Polar. } \boxed{z^6 = 512(\cos 90^\circ + i \operatorname{sen} 90^\circ)} \text{ Trigonométrica.}$   
 $\boxed{z^6 = 512(0 + i(+1)) = \boxed{+512i}} \text{ Binómica } \boxed{z^6 = (0, +512)} \text{ Cartesiana}$

⑤  $z = -1 = 1_{180^\circ} \Rightarrow \sqrt[5]{-1} = \sqrt[5]{1_{180^\circ}} \cdot \text{Tiene 5 resultados, de módulo } \sqrt[5]{1} = 1$   
 y argumentos:  $\beta_1 = \frac{180}{5} = 36^\circ \Rightarrow \boxed{1_{36^\circ}}$   
 $\beta_2 = \frac{180}{5} + \frac{360}{5} = 108^\circ \Rightarrow \boxed{1_{108^\circ}}$   
 $\beta_3 = \frac{180}{5} + \frac{360}{5} \cdot 2 = 180^\circ \Rightarrow \boxed{1_{180^\circ} = -1}$   
 $\beta_4 = \frac{180}{5} + \frac{360}{5} \cdot 3 = 252^\circ \Rightarrow \boxed{1_{252^\circ}}$   
 $\beta_5 = \frac{180}{5} + \frac{360}{5} \cdot 4 = 324^\circ \Rightarrow \boxed{1_{324^\circ}}$