Ejercicios de derivadas

1. Halla la derivada de la función $f(x) = \dfrac{2}{x+1}$ en el punto $x = 3$, aplicando la definición dederivada.

1º $f(a) \qquad \Rightarrow f(3) = \dfrac{2}{3+1} = \dfrac{2}{4} = \dfrac{1}{2}$

2º $f(a+h) \qquad \Rightarrow f(3+h) = \dfrac{2}{3+h+1} = \dfrac{2}{4+h}$

3º $f(a+h) - f(a) \qquad \Rightarrow f(3+h) - f(3) = \dfrac{2}{4+h} - \dfrac{1}{2} = \dfrac{4 - 1.(4+h)}{2(4+h)} = \dfrac{-h}{2(4+h)}$

4º $f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h} \qquad \Rightarrow f'(3) = \lim\limits_{h \to 0} \dfrac{\frac{-h}{2(4+h)}}{h} = \lim\limits_{h \to 0} \dfrac{-h}{2h(4+h)} = \lim\limits_{h \to 0} \dfrac{-1}{2(4+h)} = \dfrac{-1}{8}$

2. Calcula la derivada de las siguientes funciones :

a) $f(x) = \text{sen}\, x^2$ $\qquad f(x) = \text{sen}\, x^2 \quad \to f = x^2 \to f' = 2x \qquad \Rightarrow f'(x) = 2x \cos x^2$ (seno)

b) $f(x) = \text{sen}^2 x$ $\qquad f(x) = (\text{sen}x)^2 \quad \to f = \text{sen}x \to f' = \cos x \quad \Rightarrow f'(x) = 2\text{sen}x \cdot \cos x$ (potencial)

c) $f(x) = (3x^2 - 2)^5$ $\qquad f(x) = (3x^2 - 2)^5 \to f = 3x^2 - 2 \to f' = 6x \quad \Rightarrow f'(x) = 30x\,(3x^2 - 2)^4$

d) $f(x) = \sqrt[3]{x^2 - 3}$ $\qquad f(x) = \sqrt[3]{x^2 - 3} \quad \to f = x^2 - 3 \quad \to f' = 2x \qquad \Rightarrow f'(x) = \dfrac{2x}{3\sqrt[3]{(x^2-3)^2}}$

e) $f(x) = e^{3x+2}$ $\qquad f(x) = e^{3x+2} \quad \to f = 3x+2 \quad \to f' = 3 \qquad \Rightarrow f'(x) = 3 \cdot e^{3x+2}$

f) $\log_3(4x+1)$ $\qquad \log_3(4x+1) \quad \to f = 4x+1 \quad \to f' = 4 \qquad \Rightarrow f'(x) = \dfrac{4}{(4x+1).\ln 3}$

g) $f(x) = \sqrt{x^2 - 3x}$ $\qquad f(x) = \sqrt{x^2 - 3x} \quad \to f = x^2 - 3x \quad f' = 2x - 3 \qquad \Rightarrow f'(x) = \; = \dfrac{2x-3}{2\sqrt{x^2-3x}}$

h) $f(x) = \text{sen}^2(2x^3 + 2x)$ $\quad f(x) = \left[\underbrace{\text{sen}(2x^3 + 2x)}_{f}\right]^2$

1. potencial $\to f'(x) = 2 \cdot \left[\text{sen}(2x^3 + 2x)\right] \cdot f'$......$f'(x) = 2 \cdot \left[\text{sen}(2x^3 + 2x)\right] \cdot \cos(2x^3 + 2x) \cdot (6x^2 + 2)$

2. seno $\to f'(x) = \cos(2x^3 + 2x) \cdot f'$.................... $f'(x) = \cos(2x^3 + 2x) \cdot (6x^2 + 2)$ esto es f´de 1.

3. suma $\to f'(x) = 6x^2 + 2$ esto es f´de 2. Completamos.

3. Halla las funciones derivadas de las siguientes funciones:

a) $\mathbf{f(x)=\frac{1}{7x+1}} \Rightarrow f'(x)=\frac{0\cdot(7x+1)-1\cdot 7}{(7x+1)^2} \Rightarrow f'(x)=-\frac{7}{(7x+1)^2}$

b) $\mathbf{f(x)=x^{2/3}} \Rightarrow f'(x)=\frac{2}{3}x^{\frac{2}{3}-1} \Rightarrow f'(x)=\frac{2}{3}x^{-\frac{1}{3}} \Rightarrow f'(x)=\frac{2}{3\sqrt[3]{x}}$

c) $\mathbf{f(x)=x^2\cdot x^{\frac{1}{3}}}\quad f(x)=x^{\frac{7}{3}} \Rightarrow f'(x)=\frac{7}{3}x^{\frac{7}{3}-1} \Rightarrow f'(x)=\frac{7}{3}x^{\frac{4}{3}} \Rightarrow f'(x)=\frac{7\sqrt[3]{x^4}}{3}$

d) $\mathbf{f(x)=\left(x-\sqrt{1-x^2}\right)^2} \Rightarrow f'(x)=2\left(x-\sqrt{1-x^2}\right)\left(1+\frac{2x}{2\sqrt{1-x^2}}\right) \Rightarrow f'(x)=2\left(x-\sqrt{1-x^2}\right)\left(1+\frac{x}{\sqrt{1-x^2}}\right)$

e) $\mathbf{f(x)=\frac{e^{2x}}{x^2}} \Rightarrow f'(x)=\frac{\left(2\cdot e^{2x}\right)\cdot x^2-\left(e^{2x}\cdot 2x\right)}{x^4} \Rightarrow f'(x)=\frac{2x\cdot e^{2x}\cdot(x-1)}{x^4} \Rightarrow f'(x)=\frac{2\cdot e^{2x}\cdot(x-1)}{x^3}$

f) $\mathbf{f(x)=x\cos 2x} \Rightarrow f'(x)=1.\cos 2x+x(-\text{sen}2x)2 \Rightarrow f'(x)=\cos 2x-2x\,\text{sen}2x$

g) $\mathbf{f(x)=\ln\cos x} \Rightarrow f'(x)=\frac{-\text{sen}\,x}{\cos x} \Rightarrow f'(x)=-\text{tg}\,x$

h) $\mathbf{f(x)=\sqrt{\frac{1-x}{1+x}}} \Rightarrow 1.\ f'(x)=\frac{\frac{-2}{(1+x)^2}}{2\sqrt{\frac{1-x}{1+x}}} \Rightarrow f'(x)=-\frac{1}{(1+x)^2\sqrt{\frac{1-x}{1+x}}}$

Derivamos: $\frac{1-x}{1+x}=\frac{-1(1+x)-(1-x)\cdot 1}{(1+x)^2} \Rightarrow \frac{-1-x-1+x}{(1+x)^2}=\frac{-2}{(1+x)^2}$ lo colocamos en f´de 1.

i) $\mathbf{f(x)=e^{x^2}\cdot\text{tg}\,x} \Rightarrow f'(x)=2x\cdot e^{x^2}\cdot\text{tg}\,x+e^{x^2}\cdot\frac{1}{\cos^2 x} \Rightarrow$

$f'(x)=e^{x^2}\left(2x\cdot\text{tg}\,x+\frac{1}{\cos^2 x}\right) \Rightarrow f'(x)=e^{x^2}\left(2x\cdot\text{tg}\,x+1+\text{tg}^2 x\right)$

$$f(x)=\left(1+3x^4\right)^5 \Rightarrow f'(x)=60\,x^3\left(1+3x^4\right)^4$$

$$f(x)=\left(1+x+x^2\right)^3 \Rightarrow f'(x)=3(2x+1)\left(1+x+x^2\right)^2$$

$$f(x)=\frac{1}{\left(x^2-1\right)^4} \Rightarrow f'(x)=-8x\left(x^2-1\right)^{-5}$$

$$f(x)=\frac{1}{x-1}+\frac{2}{(x-1)^2}+\frac{3}{(x-1)^3} \Rightarrow f'(x)=-(x-1)^{-2}-4(x-1)^{-3}-9(x-1)^{-4}$$

$$f(x)=\sqrt{1-x^2} \Rightarrow f'(x)=\frac{-x}{\sqrt{1-x^2}}$$

$$f(x)=\sqrt[3]{2+5x^2} \Rightarrow f'(x)=\frac{10x}{3\sqrt[3]{\left(2+5x^2\right)^2}}$$

$$f(x)=\frac{1}{\sqrt[3]{\left(x^3-2\right)^2}} \Rightarrow f'(x)=-2x^2\left(x^3-2\right)^{-5/3}$$

$$f(x)=\left(5x^3+1\right)^3\cdot\left(x^2+x+1\right)^4 \Rightarrow$$

$$f'(x)=45x^2\left(5x^3+1\right)^2\left(x^2+x+1\right)^4+4(2x+1)\left(5x^3+1\right)^3\left(x^2+x+1\right)^3$$

$$f(x)=(5-3\cos x)^4 \Rightarrow f'(x)=12\,\mathrm{sen}\,x\,(5-3\cos x)^3$$

$$f(x)=\mathrm{sen}\,x+\mathrm{sen}^2x+\mathrm{sen}^3x \Rightarrow f'(x)=\cos x\left(1+2\,\mathrm{sen}\,x+3\,\mathrm{sen}^2x\right)$$

$$f(x)=\frac{1}{\mathrm{arc\,tg}x} \Rightarrow f'(x)=\frac{-1}{\left(1+x^2\right)(\mathrm{arc\,tg}\,x)^2}$$

$$f(x)=\mathrm{sen}^3x-\cos^3x \Rightarrow f'(x)=3\,\mathrm{sen}x\cos x\,(\mathrm{sen}x+\cos x)$$

$$f(x)=\frac{1}{3\cos^3x}-\frac{1}{\cos x} \Rightarrow f'(x)=\mathrm{sen}x\left[\frac{1}{\cos^4x}-\frac{1}{\cos^2x}\right]$$

$$f(x)=\mathrm{sen}\left(x^2\right) \Rightarrow f'(x)=2x\cos\left(x^2\right)$$

$$f(x)=(1+\mathrm{sen}\,5x)^4 \Rightarrow f'(x)=20\cos 5x\,(1+\mathrm{sen}5x)^3$$

$$f(x)=\sqrt{x\,e^x+x} \Rightarrow f'(x)=\frac{e^x+xe^x+1}{2\sqrt{xe^x+x}}$$

$$f(x)=\sqrt[3]{2^x+x} \Rightarrow f'(x)=\frac{2^x\mathrm{Ln}2+1}{3\sqrt[3]{\left(2^x+x\right)^2}}$$

$$f(x)=\mathrm{Ln}\,(\mathrm{Ln}\,x) \Rightarrow f'(x)=\frac{1}{x\,\mathrm{Ln}x}$$

$$f(x)=\mathrm{Arc}\cos\sqrt{x} \Rightarrow f'(x)=\frac{-1}{2\sqrt{x-x^2}}$$

$$f(x)=\mathrm{Arc\,sen}\left(\frac{1}{x^2}\right) \Rightarrow f'(x)=\frac{-2}{x\sqrt{x^4-1}}$$

$$f(x)=\frac{1+\cos 2x}{1-\cos 2x} \Rightarrow f'(x)=\frac{-4\,\mathrm{sen}\,2x}{(1-\cos 2x)^2}$$

$$f(x)=\mathrm{Arc\,tg}\left(\frac{1}{x}\right) \Rightarrow f'(x)=\frac{-1}{1+x^2}$$

$$f(x)=\frac{1+\operatorname{sen}^2 x}{1+\cos^2 x} \Rightarrow f'(x)=\frac{6\ \operatorname{sen}x\cos x}{\left(1+\cos^2 x\right)^2}$$

$$f(x)=x^2\cdot e^{x^6} \Rightarrow f'(x)=x\cdot e^{x^6}\left(2+3x^3\right)$$

$$f(x)=e^{\sqrt{x^2+1}} \Rightarrow f'(x)=\frac{x\cdot e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$$

$$f(x)=\left[\frac{x^2+x+1}{x^3-6}\right]^5 \Rightarrow f'(x)=5\cdot\left(\frac{x^2+x+1}{x^3-6}\right)^4\cdot\frac{-x^4-2x^3-3x^2-12x-6}{\left(x^3-6\right)^2}$$

$$f(x)=\left[\frac{\operatorname{sen}x+\cos x}{\operatorname{sen}x-\cos x}\right]^3 \Rightarrow f'(x)=\frac{-6}{1-2\operatorname{sen}x\cos x}\cdot\left(\frac{\operatorname{sen}x+\cos x}{\operatorname{sen}x-\cos x}\right)^2$$

$$f(x)=x\cdot e^{-1/x^2} \Rightarrow f'(x)=e^{-1/x^2}\left(1+\frac{2}{x^2}\right)$$

$$f(x)=\sec\left(x^2\right)+\operatorname{cosec}\left(x^2\right) \Rightarrow f'(x)=2x\left(\operatorname{tg}\left(x^2\right)\sec\left(x^2\right)-\cot\left(x^2\right)\operatorname{cosec}\left(x^2\right)\right)$$

$$f(x)=\frac{1-\cos\left(x^2\right)}{1+\cos\left(x^2\right)} \Rightarrow f'(x)=\frac{4x\operatorname{sen}x^2}{\left(1+\cos\left(x^2\right)\right)^2}$$

$$f(x)=\left(1+e^{\operatorname{sen}x}\right)^3 \Rightarrow f'(x)=3\ e^{\operatorname{sen}x}\cos x\left(1+e^{\operatorname{sen}x}\right)^2$$

$$f(x)=\operatorname{Ln}\left(\sqrt{1+e^x}-1\right)-\operatorname{Ln}\left(\sqrt{1+e^x}+1\right) \Rightarrow f'(x)=\frac{1}{\sqrt{1+e^x}}$$

$$f(x)=\operatorname{Ln}\frac{x}{\sqrt{x^2+9}} \Rightarrow f'(x)=\frac{9}{x\cdot\left(x^2+9\right)}$$

$$f(x)=\operatorname{Ln}\sqrt{\frac{1-\cos x}{1+\cos x}} \Rightarrow f'(x)=\operatorname{cosec}x$$

$$f(x)=\operatorname{Arc\,tg}\left(x^2-1\right) \Rightarrow f'(x)=\frac{2x}{x^4-2x^2+2}$$

$$f(x)=\operatorname{Arc\,tg}\left(\frac{1-x^2}{1+x^2}\right) \Rightarrow f'(x)=\frac{-2x}{1+x^4}$$

$$f(x)=\operatorname{Arc\,sen}(1+x)+\sqrt{2x-x^2} \Rightarrow f'(x)=\frac{-x}{\sqrt{2x-x^2}}$$

$$f(x)=\frac{\operatorname{Arc\,cos}x}{\sqrt{1-x^2}} \Rightarrow f'(x)=\frac{x\operatorname{Arc\,cos}x-\sqrt{1-x^2}}{\left(1-x^2\right)\sqrt{1-x^2}}$$

$$f(x)=\operatorname{Ln}\left(\cos\frac{x-1}{x}\right) \Rightarrow f'(x)=\frac{1}{x^2}\cdot\operatorname{tg}\left(\frac{x-1}{x}\right)$$

$$f(x)=\sqrt{x^2+1}-\operatorname{Ln}\frac{1+\sqrt{x^2+1}}{x} \Rightarrow f'(x)=\frac{x^2+1+\sqrt{x^2+1}}{x\left(1+\sqrt{x^2+1}\right)}=\frac{\sqrt{1+x^2}}{x}$$

$$f(x)=\operatorname{Ln}\frac{1+\sqrt{\operatorname{sen}x}}{1-\sqrt{\operatorname{sen}x}} \Rightarrow f'(x)=\frac{\cos x}{\sqrt{\operatorname{sen}x}\left(1-\operatorname{sen}x\right)}$$

$$f(x)=\operatorname{Ln}\left(\operatorname{Ln}\left(\operatorname{Ln}\left(\operatorname{Ln}x\right)\right)\right) \Rightarrow f'(x)=\frac{1}{\left(\operatorname{Ln}\left(\operatorname{Ln}\left(\operatorname{Ln}x\right)\right)\right)\left(\operatorname{Ln}\left(\operatorname{Ln}x\right)\right)\left(\operatorname{Ln}x\right)x}$$

$$f(x)=\left(x^2\right)^x \Rightarrow f(x)=x^{2x} \Rightarrow f'(x)=2\cdot x^{2x}\left(\operatorname{Ln}x+1\right)$$