Integrales de Funciones Algebraicas

✳1 --

$$\int x^6 \, dx = \frac{x^{6+1}}{6+1} + C = \frac{x^7}{7} + C$$

✳2 --

$$\int 5x^4 \, dx = 5 \int x^4 dx = \frac{5 \, x^{4+1}}{4+1} + C = \frac{5 \, x^5}{5} + C = x^5 + C$$

✳3 --

$$\int bx^3 \, dx = b \int x^3 dx = \frac{b \, x^{3+1}}{3+1} + C = \frac{b \, x^4}{4} + C$$

✳4 --

$$\int \sqrt{3}x^2 \, dx = \sqrt{3} \int x^2 dx = \frac{\sqrt{3}x^{2+1}}{2+1} + C = \frac{\sqrt{3}x^3}{3} + C$$

✳5 --

$$\int a \, dx = a \int dx = ax + C$$

✳6 --

$$\int \frac{3dx}{4} = \frac{3}{4} \int dx = \frac{3}{4}x + C$$

✱7 --

$$\int \frac{dx}{3} = \frac{1}{3} \int dx = \frac{1}{3}x + C$$

✱8 --

$$\int \sqrt[3]{x} = \int x^{\frac{1}{3}} dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{4}\sqrt[3]{x^4} + C$$

✱9 --

$$\int 5\sqrt[4]{x} = 5\int x^{\frac{1}{4}} dx = \frac{5x^{\frac{1}{4}+1}}{\frac{1}{4}+1} + C = \frac{5x^{\frac{5}{4}}}{\frac{5}{4}} + C = 4\sqrt[4]{x^5} + C$$

✱10 --
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$$\int \frac{dx}{x^3} = \int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + C = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$$

✱11 --
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$$\int \frac{5dx}{x^4} = 5\int x^{-4} dx = \frac{5x^{-4+1}}{-4+1} + C = \frac{5x^{-3}}{-3} + C = -\frac{5}{3x^3} + C$$

✱12 --
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$$\int \frac{4dx}{x} = 4\int \frac{dx}{x} = 4\ln x + C$$

✱13 --
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$$\int \frac{dx}{\sqrt[4]{x}} = \int x^{-\frac{1}{4}} dx = \frac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1} + C = \frac{x^{\frac{3}{4}}}{\frac{3}{4}} + C = \frac{4}{3}x^{\frac{3}{4}} + C = \frac{4}{3}\sqrt[4]{x^3} + C$$

✳14 ---
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$$\int \frac{6dx}{\sqrt[3]{x}} = 6 \int x^{-\frac{1}{3}} \, dx = \frac{6x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + C = \frac{6x^{\frac{2}{3}}}{\frac{2}{3}} + C = \frac{18}{2} x^{\frac{2}{3}} + C = 9\sqrt[3]{x^2} + C$$

✳15 ---
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$$\int \sqrt[5]{x^3} \, dx = \int x^{\frac{3}{5}} \, dx = \frac{x^{\frac{3}{5}+1}}{\frac{3}{5}+1} + C = \frac{x^{\frac{8}{5}}}{\frac{8}{5}} + C = \frac{5}{8} x^{\frac{8}{5}} + C = \frac{5}{8} \sqrt[5]{x^8} + C$$

✳16 ---
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$$\int \frac{a \, dx}{\sqrt[3]{x^2}} = a \int \frac{dx}{x^{\frac{2}{3}}} = a \int x^{-\frac{2}{3}} \, dx = \frac{ax^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + C = \frac{ax^{\frac{1}{3}}}{\frac{1}{3}} + C = 3a \sqrt[3]{x} + C$$

✳17 ---
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$$\int \frac{5dx}{2x} = \frac{5}{2} \int \frac{dx}{x} = \frac{5}{2} In|x| + C$$

✳18 ---
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$$\int \sqrt{bx} \, dx = \sqrt{b} \int x^{\frac{1}{2}} dx = \frac{\sqrt{b}x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{\sqrt{b}x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2\sqrt{b}}{3} \sqrt{x^3} + C = \frac{2}{3}\sqrt{bx^3} + C$$

✳19 ---
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$$\int \left(\frac{5}{\sqrt[3]{x}} - 4\sqrt[3]{x} \right) dx = 5 \int x^{-\frac{1}{3}} dx - 4 \int x^{\frac{1}{3}} \, dx = \frac{5x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} - \frac{4x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{5x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{4x^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{15}{2} \sqrt[3]{x^2} - 3 \sqrt[3]{x^4} + C$$

＊20

$$\int \left(\frac{3}{x^5} - \frac{2}{x^2} - \frac{6}{x}\right] dx = 3\int x^{-5} dx - 2\int x^{-2}\, dx - 6\int x^{-1}\, dx = \frac{3x^{-5+1}}{-5+1} - \frac{2x^{-2+1}}{-2+1} - 6\, In\, x + C$$

$$= \frac{3x^{-4}}{-4} - \frac{2x^{-1}}{-1} - 6\, In\, x + C = -\frac{3}{4x^4} + \frac{2}{x} - 6\, In\, x + C$$

＊21

$$\int \sqrt[3]{at}\, dt = \sqrt[3]{a}\int t^{\frac{1}{3}} dt = \frac{\sqrt[3]{a} \cdot t^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{\sqrt[3]{a} \cdot t^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{4}\sqrt[3]{a}\sqrt[3]{t^4} + C = \frac{3}{4}\sqrt[3]{at^4} + C$$

＊22

$$\int \sqrt{6t}\, dt = \sqrt{6}\int t^{\frac{1}{2}} dt = \frac{\sqrt{6} \cdot t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{\sqrt{6} \cdot t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}\sqrt{6}\sqrt{t^3} + C = \frac{2}{3}\sqrt{6t^3} + C$$

＊23

$$\int (8x^5 - 5x^4 - 9x^3 - 6x^2 - 2x - 3)\, dx = 8\int x^5 dx - 5\int x^4\, dx - 9\int x^3\, dx - 6\int x^2\, dx - 2\int x\, dx - 3\int dx$$

$$= \frac{8x^{5+1}}{5+1} - \frac{5x^{4+1}}{4+1} - \frac{9x^{3+1}}{3+1} - \frac{6x^{2+1}}{2+1} - \frac{2x^{1+1}}{1+1} - 3x + C$$

$$= \frac{8x^6}{6} - \frac{5x^5}{5} - \frac{9x^4}{4} - \frac{6x^3}{3} - \frac{2x^2}{2} - 3x + C$$

$$= \frac{4}{3}x^6 - x^5 - \frac{9}{4}x^4 - 2x^3 - x^2 - 3x + C$$

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$$\int (ax^3 - bx^2 - cx + d)dx = a \int x^3 dx - b \int x^2 dx - c \int x \, dx \, d \int dx = \frac{ax^{3+1}}{3+1} - \frac{bx^{2+1}}{2+1} - \frac{cx^2}{2} + \, dx + C$$

$$= \frac{ax^4}{4} - \frac{bx^3}{3} - \frac{cx^2}{2} + \, dx + C$$

*25

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$$\int \left(\frac{x^2}{\sqrt{a^2 + b^2}} - \frac{3x}{\sqrt{a}} - 5\sqrt{b} \right) dx = \frac{1}{\sqrt{a^2 + b^2}} \int x^2 dx - \frac{3}{\sqrt{a}} \int x \, dx - 5\sqrt{b} \int dx$$

$$= \frac{1}{\sqrt{a^2 + b^2}} \cdot \frac{x^3}{3} - \frac{3}{\sqrt{a}} \cdot \frac{x^2}{2} - 5\sqrt{b} \, x + C = \frac{x^3}{3\sqrt{a^2 + b^2}} - \frac{3x^2}{2\sqrt{a}} - 5\sqrt{b}x + C$$

*26

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$$\int \left(\frac{x^4 - 6x^3 - 7x}{x} \right) dx = \int \frac{x^4}{x} dx - 6 \int \frac{x^3}{x} dx - 7 \int \frac{x}{x} dx =$$

$$= \int x^3 \, dx - 6 \int x^2 dx - 7 \int dx = \frac{x^{3+1}}{3+1} - \frac{6x^{2+1}}{2+1} - 7x + C = \frac{x^4}{4} - 2x^3 - 7x + C$$

*27

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$$\int \left(\frac{3}{\sqrt[5]{x^2}} - \frac{2}{\sqrt[5]{x}} \right) dx = 3 \int x^{-\frac{2}{5}} dx - 2 \int x^{-\frac{1}{5}} dx = \frac{3x^{-\frac{2}{5}+1}}{-\frac{2}{5}+1} - \frac{2x^{-\frac{1}{5}+1}}{-\frac{1}{5}+1} + C = \frac{3x^{\frac{3}{5}}}{\frac{3}{5}} - \frac{2x^{\frac{4}{5}}}{\frac{4}{5}} + C$$

$$= 5x^{\frac{3}{5}} - \frac{5}{2}x^{\frac{4}{5}} = 5\sqrt[5]{x^3} - \frac{5}{2}\sqrt[5]{x^4} + C$$

✳28 --

$$\int \left(\frac{4}{\sqrt[3]{x}} - \frac{5}{\sqrt[4]{x}}\right) dx = 4 \int x^{-\frac{1}{3}} dx - 5 \int x^{-\frac{1}{4}} dx = \frac{4x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} - \frac{5x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1} + C = \frac{4x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{5x^{\frac{3}{4}}}{\frac{3}{4}} + C$$

$$= 6x^{\frac{2}{3}} - \frac{20}{3}x^{\frac{3}{4}} + C = 6\sqrt[3]{x^2} - \frac{20}{3}\sqrt[4]{x^3} + C$$

✳29 --

$$\int \left(y^{\frac{5}{2}} - 5y^{\frac{4}{3}} - 2y^{\frac{1}{4}} - \sqrt{y}\right) dy = \int y^{\frac{5}{2}} dy - 5 \int y^{\frac{4}{3}} dy - \int y^{\frac{1}{2}} dy = \frac{y^{\frac{5}{2}+1}}{\frac{5}{2}+1} - \frac{5y^{\frac{4}{3}+1}}{\frac{4}{3}+1} - \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{y^{\frac{7}{2}}}{\frac{7}{2}} - \frac{5y^{\frac{7}{3}}}{\frac{7}{3}} - \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{7}y^{\frac{7}{2}} - \frac{15}{7}y^{\frac{7}{3}} - \frac{2}{3}y^{\frac{3}{2}} + C = \frac{2}{7}\sqrt{y^7} - \frac{15}{7}\sqrt[3]{y^7} - \frac{2}{3}\sqrt{y^3} + C$$

✳30 --

$$\int \left(\frac{y^{\frac{7}{2}} - y^{\frac{5}{3}} - y^{\frac{1}{4}}}{y^2}\right) dy = \int \frac{y^{\frac{7}{2}}}{y^2} dy - \int \frac{y^{\frac{5}{3}}}{y^2} dy - \int \frac{y^{\frac{1}{4}}}{y^2} dy = \int y^{\frac{7}{2}-2} dy - \int y^{\frac{5}{3}-2} dy - \int y^{\frac{1}{4}-2} dy$$

$$= \frac{y^{\frac{7}{2}-1}}{\frac{7}{2}-1} - \frac{y^{\frac{5}{3}-1}}{\frac{5}{3}-1} - \frac{y^{\frac{1}{4}-1}}{\frac{1}{4}-1} + C = \frac{y^{\frac{5}{2}}}{\frac{5}{2}} - \frac{y^{\frac{2}{3}}}{\frac{2}{3}} - \frac{y^{-\frac{3}{4}}}{-\frac{3}{4}} + C = \frac{2}{5}y^{\frac{5}{2}} - \frac{3}{2}y^{\frac{2}{3}} + \frac{4}{3}y^{-\frac{3}{4}} + C$$

$$= \frac{2}{5}\sqrt{y^5} - \frac{3}{2}\sqrt[3]{y^2} + \frac{4}{3\sqrt[4]{y^3}} + C$$

✳31 --

$$\int \sqrt[3]{t}(5t^2 - 3t + 2)dt = \int \left(5t^{\frac{1}{3}+2} - 3t^{\frac{1}{3}+1} + 2t^{\frac{1}{3}}\right) dt = 5 \int t^{\frac{1}{3}+2} dt - 3 \int t^{\frac{1}{3}+1} dt + 2 \int t^{\frac{1}{3}} dt$$

$$= \frac{5t^{\frac{1}{3}+3}}{\frac{1}{3}+3} - \frac{3t^{\frac{1}{3}+2}}{\frac{1}{3}+2} + \frac{2t^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{5t^{\frac{10}{3}}}{\frac{10}{3}} - \frac{3t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{2t^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{15t^{\frac{10}{3}}}{10} - \frac{9t^{\frac{7}{3}}}{7} + \frac{6t^{\frac{4}{3}}}{4} + C = \frac{3}{2}\sqrt[3]{t^{10}} - \frac{9}{7}\sqrt[3]{t^7} + \frac{3}{2}\sqrt[3]{t^4} + C$$

✱32 --
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$$\int \sqrt[3]{7t}\, dt = \sqrt[3]{7} \int t^{\frac{1}{3}}\, dt = \frac{\sqrt[3]{7}t^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{\sqrt[3]{7}t^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3\sqrt[3]{7t^4}}{4} + C$$

✱33 --
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$$\int (3x+4)^6\, dx = \frac{1}{3}\int 3(3x+4)^6\, dx = \frac{1}{3}\int u^6\, du = \frac{1}{3}\cdot\frac{u^{6+1}}{6+1} + C = \frac{1}{21}u^7 + C = \frac{1}{21}(3x+4)^7$$

$$u = 3x + 4$$
$$du = 3dx$$

✱34 --
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$$\int (ax^2 - b)^5 x\, dx = \frac{1}{2a}\int (ax^2 - b)^5\, 2ax\; dx = \frac{1}{2a}\int u^5\, du = \frac{1}{2a}\cdot\frac{u^{5+1}}{5+1} + C = \frac{1}{12a}u^6 + C = \frac{1}{12a}(ax^2 - b)^6 + C$$

$$u = ax^2 - b$$
$$du = 2ax\, dx$$

✱35 --
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$$\int t^2(t^3 - 4)^2\, dt = \frac{1}{3}\int 3t^2(t^3 - 4)^2\, dt = \frac{1}{3}\int u^2\, du = \frac{1}{3}\cdot\frac{u^{2+1}}{2+1} + C = \frac{u^3}{9} + C = \frac{(t^3 - 4)^3}{9} + C$$

$$u = t^3 - 4$$
$$du = 3t^2\, dt$$

✱36 --
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$$\int (a - by)^4\, dy = -\frac{1}{b}\int -(a - by)^4\, b\, dy = -\frac{1}{b}\int u^4\, du = -\frac{1}{b}\cdot\frac{u^{4+1}}{4+1} + C = -\frac{u^5}{5b} + C = -\frac{(a - by)^5}{5b} + C$$

$$u = a - by$$
$$du = -bdy$$

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$$\int (t^2 - 6)^2 \, dt = \int (t^4 - 12t^2 + 36) \, dt = \int t^4 dt - 12 \int t^2 dt + 36 \int dt$$

$$= \frac{t^{4+1}}{4+1} - \frac{12 \, t^{2+1}}{2+1} + 36t + C = \frac{t^5}{5} - 4t^3 + 36t + C$$

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$$\int x \, (x + 4)^2 dx = \int x(x^2 + 8x + 16) \, dx = \int (x^3 + 8x^2 + 16x) \, dx = \frac{x^{3+1}}{3+1} + \frac{8 \, x^{2+1}}{2+1} + \frac{16x^2}{2} + C$$

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$$\int x^2 (x + 1)^3 \, dx = \int x^2 (x^3 + 3x^2 + 3x + 1) \, dx = \int (x^5 + 3x^4 + 3x^3 + x^2) \, dx$$

$$= \frac{x^{5+1}}{5+1} + \frac{3 \, x^{4+1}}{4+1} + \frac{3 \, x^{3+1}}{3+1} + \frac{x^{2+1}}{2+1} + C$$