

Hallar la derivada de la función $f(x) = 3x^2$ en el punto $x = 2$.

$$\begin{aligned}f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{3(2+h)^2 - 3 \cdot 2^2}{h} = \\&= \lim_{h \rightarrow 0} \frac{3(4 + 4h + h^2) - 12}{h} = \lim_{h \rightarrow 0} \frac{3h^2 + 12h}{h} = \\&= \lim_{h \rightarrow 0} (3h + 12) = \mathbf{12}\end{aligned}$$

Calcular la derivada de la función $f(x) = x^2 + 4x - 5$ en $x = 1$.

$$\begin{aligned}f'(1) &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \\&= \lim_{h \rightarrow 0} \frac{(1+h)^2 + 4(1+h) - 5 - (1^2 + 4 \cdot 1 - 5)}{h} = \\&= \lim_{h \rightarrow 0} \frac{1 + 2h + h^2 + 4 + 4h - 5}{h} = \lim_{h \rightarrow 0} \frac{h^2 + 6h}{h} = \\&= \lim_{h \rightarrow 0} (h + 6) = \mathbf{6}\end{aligned}$$

Calcular derivada de $f(x) = x^2 - x + 1$ en $x = -1$, $x = 0$ y $x = 1$.

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \\&= \lim_{h \rightarrow 0} \frac{(x+h)^2 - (x+h) + 1 - (x^2 - x + 1)}{h} = \\&= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - x - h + 1 - x^2 + x - 1}{h} = \\&= \lim_{h \rightarrow 0} \frac{2hx + h^2 - h}{h} = \lim_{h \rightarrow 0} \frac{h(2x + h - 1)}{h} = \\&= \lim_{h \rightarrow 0} (2x + h - 1) = \mathbf{2x - 1}\end{aligned}$$

$f'(-1)$, $f'(0)$ y $f'(1)$.

$$f'(-1) = 2(-1) - 1 = \mathbf{-3}$$

$$f'(0) = 2(0) - 1 = \mathbf{-1}$$

$$f'(1) = 2(1) - 1 = \mathbf{1}$$

Calcular derivada de $f(x) = 2x^2 - 6x + 5$ en $x = -5$.

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 6(x+h) + 5 - (2x^2 - 6x + 5)}{h} = \\&= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 6x - 6h + 5 - 2x^2 + 6x - 5}{h} = \\&= \lim_{h \rightarrow 0} \frac{4xh - 6h + 2h^2}{h} = \lim_{h \rightarrow 0} \frac{h(4x - 6 + 2h)}{h} = 4x - 6 \\f'(-5) &= 4(-5) - 6 = -26\end{aligned}$$

Calcular derivada de $f(x) = x^3 + 2x - 5$ en $x = 1$.

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^3 + 2(x+h) - 5 - (x^3 + 2x - 5)}{h} = \\&= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 2x + 2h - 5 - x^3 - 2x + 5}{h} = \\&= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3 + 2h}{h} = \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2 + 2)}{h} = 3x^2 + 2 \\f'(1) &= 3(1)^2 + 2 = 5\end{aligned}$$

Calcular derivada de $f(x) = \frac{1}{x}$ en $x = 2$.

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h} = \lim_{h \rightarrow 0} \frac{\frac{-h}{x^2 + xh}}{h} = \\&= \lim_{h \rightarrow 0} \left(-\frac{1}{x^2 + xh} \right) = -\frac{1}{x^2} \\f'(2) &= -\frac{1}{2^2} = -\frac{1}{4}\end{aligned}$$

Calcular derivada de $f(x) = \sqrt{x}$ en $x = 3$.

$$f'(3) = \lim_{h \rightarrow 0} \frac{\sqrt{3+h} - \sqrt{3}}{h} = \lim_{h \rightarrow 0} \frac{(\sqrt{3+h} - \sqrt{3})(\sqrt{3+h} + \sqrt{3})}{h(\sqrt{3+h} + \sqrt{3})} =$$

$$\lim_{h \rightarrow 0} \frac{(3+h) - 3}{h(\sqrt{3+h} + \sqrt{3})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{3+h} + \sqrt{3})} =$$

$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{3+h} + \sqrt{3}} = \frac{1}{\sqrt{3} + \sqrt{3}} = \frac{1}{2\sqrt{3}}$$

Calcular derivada de $f(x) = \frac{x}{x-1}$ en $x = 2$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{x+h}{x+h-1} - \frac{x}{x-1}}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\frac{x^2 - x + hx - h - x^2 - hx + x}{(x+h-1)(x-1)}}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\frac{-h}{(x+h-1)(x-1)}}{h} = \lim_{h \rightarrow 0} \frac{-1}{(x+h-1)(x-1)} = \frac{-1}{(x-1)^2}$$

$$f'(2) = \frac{-1}{(2-1)^2} = -1$$