

EJERCICIOS DE DERIVADAS

Deriva las siguientes funciones. Simplifica los cálculos cuando sea posible.

1. a) $y = 5x^2 - 3x + 6$ b) $y = -3x^4 + 2x^3 - 5x^2 - 3$

c) $y = x^4 - 4x^2 + 2x$ d) $y = \frac{2}{3}x^3 - \frac{1}{4}x^2 + 3x$

Solución:

a) $y' = 10x - 3$

b) $y' = -12x^3 + 6x^2 - 10x$

c) $y' = 4x^3 - 8x + 2$

d) $y' = 2x^2 - \frac{1}{2}x + 3$

2. a) $y = \frac{5}{4}x^4 - \frac{2}{3}x^3 + 7x$ b) $y = \frac{3x^4}{4} - \frac{2x^3}{3} + 7x$

c) $y = \frac{3x^4 - 3x}{4}$ d) $y = \frac{3}{4}(x^4 - 3x)$

e) $y = \frac{1}{3}x^2 - \frac{5}{7}x + 3$ f) $y = \frac{3x^4}{4} - 7x^3 + \frac{x^2}{2} + \frac{1}{5}$

Solución:

a) $y' = 3x^3 - 2x^2 + 7$

b) $y' = 3x^3 - 2x^2 + 7$

c) $y' = \frac{12x^3 - 3}{4}$

d) $y' = \frac{3}{4}(4x^3 - 3) = 3x^3 - \frac{9}{4}$

e) $y' = \frac{2}{3}x - \frac{5}{7}$

f) $y' = x^3 - 21x^2 + x$

3. a) $y = (x^2 + 3x)(2x^2 - 3)$ b) $y = (-2x^3 + 2x)(x^2 + 5x - 1)$

c) $y = 2(x^2 + 3)(x^2 - 5x)$ d) $y = -(x^2 - 3x + 5)(2x + 4)$

Solución:

a) $y' = (2x + 3)(2x^2 - 3) + (x^2 + 3x)(4x) = 8x^3 + 18x^2 - 6x - 9$

b) $y' = (-6x^2 + 2)(x^2 + 5x - 1) + (-2x^3 + 2x)(2x + 5) = -10x^4 - 40x^3 + 12x^2 + 20x - 2$

c) $y' = 2(2x)(x^2 - 5x) + 2(x^2 + 3)(2x - 5) = 8x^3 - 30x^2 + 12x - 30$

d) $y' = -(2x - 3)(2x + 4) - (x^2 - 3x + 5) \cdot 2 = -6x^2 + 4x + 2$

4. a) $y = (x - 3)^5$

b) $y = (3x - 2)^5$

c) $y = (x^2 + 2)^3$

d) $y = 2(4x - 7)^3$

e) $y = -2(5x^2 + 1)^3$

f) $y = 5(6x^2 + 2x - 1)^3$

Solución:

- a) $y' = 5(x-3)^4$
 b) $y' = 5(3x-2)^4 \cdot 3 = 15(3x-2)^4$
 c) $y' = 3(x^2+2)^2 \cdot 2x = 6x(x^2+2)^2$
 d) $y' = 6(4x-7)^2 \cdot 4 = 24(4x-7)^2$
 e) $y' = -6(5x^2+1)^2 \cdot 10x = -60x(5x^2+1)^2$
 f) $y' = 15(6x^2+2x-1)^2(12x+2)$

5. a) $y = \frac{2x-3}{5x}$ b) $y = \frac{2x}{x^2+3}$
 c) $y = \frac{2}{4x^2-3x}$ d) $y = \frac{3x}{x^2-1}$
 e) $y = \frac{x^2}{x^2-1}$ f) $y = \frac{1-x}{3x^3+x}$

Solución:

a) $y' = \frac{2 \cdot 5x - (2x-3) \cdot 5}{(5x)^2} = \frac{15}{25x^2} = \frac{3}{5x^2}$
 b) $y' = \frac{2(x^2+3) - 2x \cdot 2x}{(x^2+3)^2} = \frac{-2x^2+6}{(x^2+3)^2}$
 c) $y' = \frac{-2 \cdot (8x-3)}{(4x^2-3x)^2} = \frac{6-16x}{(4x^2-3x)^2}$
 d) $y' = \frac{3(x^2-1) - 3x \cdot 2x}{(x^2-1)^2} = \frac{-3x^2-3}{(x^2-1)^2}$
 e) $y' = \frac{2x(x^2-1) - x^2 \cdot 2x}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$
 f) $y' = \frac{-(3x^3+x) - (1-x)(9x^2+1)}{(3x^3+x)^2} = \frac{6x^3-9x^2-1}{(3x^3+x)^2}$

6. a) $y = \frac{1}{2x}$ b) $y = \frac{-3}{x^2}$
 c) $y = \frac{2}{5x^3}$ d) $y = \frac{-1}{x^2-2x}$
 e) $y = \frac{2}{x^3-5x}$ f) $y = \frac{1}{(x^2-2)^3}$

Solución:

a) $y' = \frac{-1}{2x^2}$
 b) $y' = \frac{6}{x^3}$
 c) $y' = \frac{-6}{5x^4}$

$$d) y' = \frac{2x-2}{(x^2-2x)^2}$$

$$e) y' = \frac{-2(3x^2-5)}{(x^3-5x)^2} = \frac{10-6x^2}{(x^3-5x)^2}$$

$$f) y' = \frac{-6x}{(x^2-2)^4}$$

Deriva y simplifica:

$$7. a) y = \sqrt{3x^2}$$

$$b) y = \sqrt{2x^3}$$

$$c) y = \sqrt{5x+2}$$

$$d) y = \sqrt{3x^2+5x}$$

Solución:

$$a) y' = \frac{6x}{2\sqrt{3x^2}} = \frac{3x}{\sqrt{3x^2}} = \sqrt{3}$$

$$b) y' = \frac{6x^2}{2\sqrt{2x^3}} = \frac{3x}{\sqrt{2x^3}} = \frac{3}{2}\sqrt{2x}$$

$$c) y' = \frac{5}{2\sqrt{5x+2}}$$

$$d) y' = \frac{6x+5}{2\sqrt{3x^2+5x}}$$

$$8. a) y = \sqrt{3x^2-4x+1}$$

$$b) y = \sqrt{x^4+4x}$$

$$c) y = \sqrt{(1+5x)^3}$$

$$d) y = \sqrt{1-2x+3x^2}$$

Solución:

$$a) y' = \frac{6x-4}{2\sqrt{3x^2-4x+1}} = \frac{3x-2}{\sqrt{3x^2-4x+1}}$$

$$b) y' = \frac{4x^3+4}{2\sqrt{x^4+4x}} = \frac{2x^3+2}{\sqrt{x^4+4x}}$$

$$c) y' = \frac{3(1+5x)^2 \cdot 5}{2\sqrt{(1+5x)^3}} = \frac{15(1+5x)}{2\sqrt{1+5x}} = \frac{15}{2}\sqrt{1+5x}$$

$$d) y' = \frac{-2+6x}{2\sqrt{1-2x+3x^2}} = \frac{-1+3x}{\sqrt{1-2x+3x^2}}$$

$$9. a) y = 2\sqrt{4x-5}$$

$$b) y = x\sqrt{3x}$$

$$c) y = \frac{3}{7}\sqrt{x^2-x}$$

$$d) y = x\sqrt{x^2-3}$$

Solución:

$$a) y' = 2 \cdot \frac{4}{2\sqrt{4x-5}} = \frac{4}{\sqrt{4x-5}}$$

$$b) y' = \frac{3\sqrt{3}}{2} \sqrt{x}$$

$$c) y' = \frac{3}{7} \cdot \frac{2x-1}{2\sqrt{x^2-x}} = \frac{6x-3}{14\sqrt{x^2-x}}$$

$$d) y' = \sqrt{x^2-3} + x \cdot \frac{2x}{2\sqrt{x^2-3}} = \sqrt{x^2-3} + \frac{x^2}{\sqrt{x^2-3}}$$

$$10. a) y = (3x^2 - x)^{1/3} \quad b) y = (3x - x^2)^{1/3}$$

$$c) y = \sqrt[3]{3x - x^2} \quad d) y = \sqrt[4]{2x - 3x^2}$$

Solución:

$$a) y' = \frac{1}{3} (3x^2 - x)^{-2/3} (6x - 1)$$

$$b) y' = \frac{1}{3} (3x - x^2)^{-2/3} (3 - 2x) = \frac{3 - 2x}{3\sqrt[3]{(3x - x^2)^2}}$$

$$c) \text{ (Es la misma que antes: } y = \sqrt[3]{3x - x^2} = (3x - x^2)^{1/3}$$

$$y' = \frac{2 - 6x}{4\sqrt[4]{(2x - 3x^2)^3}} = \frac{1 - 3x}{2\sqrt[4]{(2x - 3x^2)^3}}$$

$$11. a) y = \frac{1}{\sqrt{x}} \quad b) y = \frac{\sqrt{3x^2 - 2x}}{5x}$$

$$c) y = \sqrt{\frac{3x^2 - 2x}{2}} \quad d) y = \sqrt{\frac{2x - 3}{x^2}}$$

Solución:

$$a) y' = \frac{-1/2\sqrt{x}}{(\sqrt{x})^2} = \frac{-1}{2\sqrt{x^3}} \quad (\text{También: } y = \frac{1}{\sqrt{x}} = x^{-1/2} \Rightarrow y' = -\frac{1}{2} x^{-3/2})$$

$$b) y' = \frac{\frac{6x-2}{2\sqrt{3x^2-2x}} \cdot 5x - (\sqrt{3x^2-2x}) \cdot 5}{25x^2} \Rightarrow y' = \frac{\frac{6x-2}{2\sqrt{3x^2-2x}} \cdot 5x - (\sqrt{3x^2-2x}) \cdot 5}{5x^2}$$

$$c) y = \frac{\sqrt{3x^2-2x}}{\sqrt{2}} \Rightarrow y' = \frac{6x-2}{2\sqrt{2}\sqrt{3x^2-2x}} = \frac{3x-1}{\sqrt{6x^2-4x}}$$

$$d) y' = \frac{\frac{2 \cdot x^2 - (2x-3) \cdot 2x}{x^4}}{2\sqrt{\frac{2x-3}{x^2}}} = \frac{-x+3}{x^2\sqrt{2x-3}}$$

Deriva:

$$12. a) y = 2^{x^2-1}$$

$$b) y = 3^{2x-x^2}$$

$$c) y = e^{-x+3}$$

$$d) y = 2e^{3-5x}$$

$$e) y = xe^x$$

$$f) y = x^2e^{1-5x}$$

g) $y = 3xe^{-x^2}$

h) $y = (2x+1)e^{2x+1}$

Solución:

a) $y' = 2x \cdot 2^{x^2-1} \ln 2$

b) $y' = (2-2x) \cdot 3^{2x-x^2} \ln 3$

c) $y' = -e^{-x+3}$

d) $y' = -10e^{3-5x}$

e) $y' = e^x + xe^x = (1+x)e^x$

f) $y = 2x \cdot e^{1-5x} - 5x^2 e^{1-5x} = (2x-5x^2)e^{1-5x}$

g) $y' = 3e^{-x^2} - 6x^2 e^{-x^2} = (3-6x^2)e^{-x^2}$

h) $y' = 2e^{2x+1} + (2x+1) \cdot 2e^{2x+1} = (4x+4)e^{2x+1}$

13. a) $y = e^{\sqrt{x}}$

b) $y = \sqrt{e^x}$

c) $y = \frac{e^x}{x}$

d) $y = \frac{x}{e^x}$

e) $y = \frac{3e^x}{2x+1}$

f) $y = \frac{xe^x}{1-x}$

Solución:

a) $y' = \frac{1}{2\sqrt{x}} e^{\sqrt{x}}$

b) $y = \sqrt{e^x} = e^{x/2} \Rightarrow y' = \frac{1}{2} e^{x/2}$

c) $y' = \frac{e^x \cdot x - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$

d) $y' = \frac{e^x - xe^x}{e^{2x}} = \frac{1-x}{e^x}$

e) $y' = \frac{3e^x(2x+1) - 3e^2 \cdot 2}{(2x+1)^2} = \frac{(6x-3)e^x}{(2x+1)^2}$

f) $y' = \frac{(e^x + xe^x)(1-x) - xe^x(-1)}{(1-x)^2} = \frac{(1+x-x^2)e^x}{(1-x)^2}$

Deriva y simplifica (piensa si puedes utilizar las propiedades de los logaritmos):

14. a) $y = \log(3x)$

b) $y = \log(3x^2)$

c) $y = \log(3x)^2$

d) $y = (\log(3x))^2$

e) $y = \log(x^2 + 3x)$

f) $y = \log(3x+4)^7$

g) $y = \log\left(\frac{2x-1}{x^2}\right)$

h) $y = \frac{\log(2x-1)}{\log x^2}$

Solución:

a) $y = \log(3x) = \log 3 + \log x \Rightarrow y' = \frac{1}{x} \log e$

$$\text{b) } y = \log(3x^2) = \log 3 + 2\log x \Rightarrow y' = \frac{2}{x} \log e$$

$$\text{c) } y = \log(3x)^2 = 2\log(3x) = 2\log 3 + 2\log x \Rightarrow y' = \frac{2}{x} \log e$$

$$\text{d) } y' = 2(\log(3x)) \cdot \frac{3}{3x} \cdot \log e = \frac{2\log(3x)\log e}{x}$$

$$\text{e) } y' = \frac{2x+3}{x^2+3x} \log e$$

$$\text{f) } y = \log(3x+4)^7 = 7\log(3x+4) \Rightarrow y' = 7 \cdot \frac{3}{3x+4} = \frac{21}{3x+4}$$

$$\text{g) } y = \log\left(\frac{2x-1}{x^2}\right) = \log(2x-1) - \log x^2 \Rightarrow y' = \left(\frac{2}{2x-1} - \frac{2}{x}\right) \log e$$

$$\text{h) } y' = \frac{\frac{2}{2x-1} \log(x^2) - \log(2x-1) \cdot \frac{2}{x}}{(\log x^2)^2} \log e$$

$$15. \text{ a) } y = \ln(3x^2 - 2)$$

$$\text{b) } y = 3\ln(x^2 - 2)$$

$$\text{c) } y = \ln(x^2 - 2)^3$$

$$\text{d) } y = (\ln(2x^2 + 3))^2$$

$$\text{e) } y = x + \ln x$$

$$\text{f) } y = x \ln x$$

$$\text{g) } y = \frac{\ln x}{x}$$

$$\text{h) } y = \frac{1}{x} - \ln x$$

Solución:

$$\text{a) } y' = \frac{6x}{3x^2 - 2}$$

$$\text{b) } y' = 3 \cdot \frac{2x}{x^2 - 2} = \frac{12x}{x^2 - 2}$$

c) La expresión es equivalente a la anterior:

$$y = \ln(x^2 - 2)^3 = 3\ln(x^2 - 2) \Rightarrow y' = \frac{12x}{x^2 - 2}$$

$$\text{d) } y' = 2(\ln(2x^2 + 3)) \cdot \frac{4x}{2x^2 + 3} = \frac{8x \ln(2x^2 + 3)}{2x^2 + 3}$$

$$\text{e) } y' = 1 + \frac{1}{x}$$

$$\text{f) } y' = \ln x + 1$$

$$\text{g) } y' = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$$\text{h) } y' = -\frac{1}{x^2} - \frac{1}{x}$$

$$16. \text{ a) } y = \ln\left(\frac{x^2}{6}\right)$$

$$\text{b) } y = \frac{\ln x^2}{6}$$

c) $y = \frac{\ln x^2}{\ln 6}$

d) $y = \frac{6}{\ln x^2}$

Solución:

a) $y = \ln\left(\frac{x^2}{6}\right) = \ln x^2 - \ln 6 = 2 \ln x - \ln 6 \Rightarrow y' = \frac{2}{x}$

b) $y = \frac{\ln x^2}{6} = \frac{2}{6} \ln x \Rightarrow y' = \frac{1}{3x}$

c) $y = \frac{\ln x^2}{\ln 6} = \frac{2 \ln x}{\ln 6} \Rightarrow y' = \frac{2}{\ln 6} \cdot \frac{1}{x} = \frac{2}{x \ln 6}$

d) $y = \frac{6}{\ln x^2} = \frac{3}{\ln x} \Rightarrow y' = \frac{-3 \cdot \frac{1}{x}}{(\ln x)^2} = \frac{-3}{x(\ln x)^2}$

17. a) $y = \ln \sqrt{5x}$

b) $y = \sqrt{\ln 5x}$

c) $y = \ln(5\sqrt{x})$

d) $y = \ln(5 - \sqrt{x})$

Solución:

a) $y = \ln \sqrt{5x} = \ln(5x)^{1/2} = \frac{1}{2} \ln 5x = \frac{1}{2} \ln 5 + \frac{1}{2} \ln x \Rightarrow y' = \frac{1}{2x}$

b) $y' = \frac{1/x}{2\sqrt{\ln 5x}} = \frac{1}{2x\sqrt{\ln 5x}}$

c) $y = \ln(5\sqrt{x}) = \ln 5 + \ln \sqrt{x} = \ln 5 + \frac{1}{2} \ln x \Rightarrow y' = \frac{1}{2x}$

d) $y' = \frac{-1}{2\sqrt{x}} = \frac{-1}{10\sqrt{x} - 2x}$

Deriva:

18. a) $y = 7 \sin x$

b) $y = \sin 7x$

c) $y = \sin x^7$

d) $y = \sin^7 x$

Solución:

a) $y' = 7 \cos x$

b) $y' = 7 \cos 7x$

c) $y' = 7x^6 \cos x^7$

d) $y' = 7 \sin^6 x \cdot \cos x$

19. a) $y = -3 \cos x$

b) $y = \cos(-3x)$

c) $y = \cos x^{-3}$

d) $y = \cos^{-3} x$

Solución:

a) $y' = 3 \sin x$

b) $y' = 3 \sin(-3x)$

c) $y' = -3x^{-4} \cdot \sin x^{-3} = -\frac{3}{x^4} \cdot \sin \frac{1}{x^3}$

$$d) y' = -3\cos^{-4} x \cdot (-\sin x) = \frac{3\sin x}{\cos^4 x}$$

20. a) $y = 3\sin 2x - 5\cos x$ b) $y = x \sin 4x$
 c) $y = \cos x \cdot \sin(-3x)$ d) $y = \cos(-3x) \cdot \sin x$

Solución:

- a) $y' = 6\cos 2x + 5\sin x$
 b) $y' = \sin 4x + x \cdot 4\cos 4x = \sin 4x + 4x \cos 4x$
 c) $y' = -\sin x \cdot \sin(-3x) - 3\cos x \cdot \cos(3x)$
 d) $y' = 3\sin(-3x) \cdot \sin x + \cos(-3x) \cdot \cos x$

21. a) $y = x^2 \cos 4x$ b) $y = 2x^3 - \sin(3x)$
 c) $y = \sin^2(3x - 1)$ d) $y = \frac{\cos(5x)}{5x}$

Solución:

- a) $y' = 2x \cos 4x - 4x^2 \sin 4x$
 b) $y' = 6x^2 - 3\cos(3x)$
 c) $y' = 2\sin(3x - 1) \cdot 3\cos(3x - 1)$
 d) $y' = \frac{-5\sin(5x) \cdot 5x - 5\cos(5x)}{25x^2} = -\frac{5x\sin(5x) + \cos(5x)}{5x^2}$

22. a) $y = \frac{1}{\sin x}$ b) $y = \frac{1}{\cos x}$
 c) $y = \frac{\cos x}{\sin x}$ d) $y = \frac{1}{\sin x^2}$

Solución:

- a) $y' = \frac{-\cos x}{\sin^2 x}$
 b) $y' = \frac{\sin x}{\cos^2 x}$
 c) $y' = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} = -\frac{1}{\sin^2 x}$
 d) $y' = \frac{-2x \cos x^2}{\sin^2 x^2}$

23. El ejercicio anterior formulado de otra forma

- a) $y = \operatorname{cosec} x$ b) $y = \sec x$
 c) $y = \cotag x$ d) $y = \operatorname{cosec} x^2$

Solución:

- a) $y' = -(\operatorname{cosec} x)(\cotag x)$
 b) $y' = (\sec x)(\tag x)$
 c) $y' = -(\operatorname{cosec} x)^2$
 d) $y' = -2x(\operatorname{cosec} x^2)(\cotag x^2)$

24. a) $y' = 3e^{4x} \sin 5x$ b) $y = \cos(3e^x)$
 c) $y = e^{\cos 4x}$ d) $y = \cos(e^{\cos x})$

Solución:

a) $y' = 12e^{4x} \sin 5x + 15e^{4x} \cos 5x$
 b) $y' = -3e^{3x} \sin(3e^x)$
 c) $y' = -4 \sin(4x) \cdot e^{\cos 4x}$
 d) $y = -\sin x \cdot e^{\cos x} \cdot (-\sin(e^{\cos x})) = \sin x \cdot e^{\cos x} \cdot \sin(e^{\cos x})$

25. a) $y = \sin(\ln x)$ b) $y = \cos(\ln x)$
 c) $y = \cos \frac{1}{x}$ d) $y = \sqrt{\sin 2x}$

Solución:

a) $y' = \frac{1}{x} \cos(\ln x)$
 b) $y' = -\frac{1}{x} \sin(\ln x)$
 c) $y' = \frac{1}{x^2} \cdot \sin \frac{1}{x}$
 d) $y' = \frac{\cos 2x}{\sqrt{\sin 2x}}$

26. a) $y = \ln(\sin x)$ b) $y = \ln(\cos x)$
 c) $y = \ln(\tan x)$ d) $y = \ln(\sin x)^2$

Solución:

a) $y' = \frac{\cos x}{\sin x} = \cotag x$
 b) $y' = \frac{-\sin x}{\cos x} = -\tan x$
 c) $y = \ln(\tan x) = \ln(\sin x) - \ln(\cos x) \Rightarrow y' = \cotag x - \tan x$
 d) $y = \ln(\sin x)^2 = 2 \ln(\sin x) \Rightarrow y' = 2 \cotag x$

27. a) $y = \tag(x^2 - 1)$ b) $y = \tag(x - 1)^2$
 c) $y = 2 \tag(x - 1)$ d) $y = \tag^2(x - 1)$
 e) $y = (\tag(x - 1))^2$ f) $y = \tag \sqrt{x}$

Solución:

a) $y' = 2x(1 + \tag^2(x^2 - 1)) = \frac{2x}{\cos^2(x^2 - 1)}$
 b) $y' = 2(x - 1)(1 + \tag^2(x - 1)) = \frac{2(x - 1)}{\cos^2(x - 1)^2}$
 c) $y' = 2(1 + \tag^2(x - 1)) = \frac{2}{\cos^2(x - 1)}$

$$d) y' = 2\operatorname{tag}(x-1)(1 + \operatorname{tag}^2(x-1))$$

$$e) y = (\operatorname{tag}(x-1))^2 \text{ (es lo mismo que antes)}$$

$$f) y' = \frac{1}{2\sqrt{x}}(1 + \operatorname{tag}^2 \sqrt{x})$$

$$28. a) y = \arcsen 2x$$

$$b) y = \arcsen(2+x)$$

$$c) y = \arccos x^2$$

$$d) y = \arccos e^x$$

$$e) y = \operatorname{arctag}(3x+2)$$

$$f) y = \operatorname{arctag}(x^2)$$

Solución:

$$a) y' = \frac{2}{\sqrt{1-4x^2}}$$

$$b) y' = \frac{1}{\sqrt{1-(2+x)^2}}$$

$$c) y' = -\frac{2x}{\sqrt{1-(x^2)^2}}$$

$$d) y' = -\frac{e^x}{\sqrt{1-(e^x)^2}}$$

$$e) y' = \frac{3}{1+(3x+2)^2}$$

$$f) y' = \frac{2x}{1+x^4}$$