INTEGRACION POR PARTES

Existe una variedad de integrales que se pueden desarrollar, usando la relación: $\int udv = uv - \int vdu$.

El problema es elegir u y dv, por lo cual es útil la siguiente identificación:

I: Función trigonométrica inversa.
L: Función logarítmica.
A: Función algebraica.
T: Función trigonométrica.
E: Función exponencial.
Se usa de la manera siguiente:

EJERCICIOS DESARROLLADOS

4.1.-Encontrar: $\int x\cos xdx$

Solución.- I L A T E

$$\begin{matrix} \downarrow & \downarrow \\ x & \cos x \end{matrix}$$

$$\therefore \begin{matrix} u = x & dv = \cos xdx \\ du = dx & v = \operatorname{sen} x \end{matrix}$$

$$\therefore \int x\cos xdx = x\operatorname{sen} x - \int \operatorname{sen} xdx = x\operatorname{sen} x + \cos x + c$$

Respuesta: $\int x\cos xdx = x\operatorname{sen} x + \cos x + c$

4.2.-Encontrar: $\int x\sec^2 xdx$

Solución.- I L A T E

$$\begin{matrix} \downarrow & \downarrow \\ x & \sec^2 3x \end{matrix}$$

$$\therefore \begin{matrix} u = x & dv = \sec^2 3xdx \\ du = dx & v = \frac{1}{3}\tau g3x \end{matrix}$$

$$\therefore \int x\sec^2 xdx = \frac{1}{3}x\tau g3x - \frac{1}{3}\int \tau g3xdx = \frac{x\tau g3x}{3} - \frac{1}{9}\ell\eta\left|\sec 3x\right| + c$$

Respuesta: $\int x\sec^2 xdx = \frac{x\tau g3x}{3} - \frac{1}{9}\ell\eta\left|\sec 3x\right| + c$

4.3.-Encontrar: $\int x^2 \operatorname{sen} xdx$

Solución.- I L A T E

$$\begin{matrix} \downarrow & \downarrow \\ x^2 & \operatorname{sen} x \end{matrix}$$

$\therefore \quad \begin{matrix} u = x^2 & dv = \operatorname{sen} x dx \\ du = 2x dx & v = -\cos x \end{matrix}$

$\therefore \int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2\int x \cos x dx$, integrando por partes la segunda integral:

$\int x \cos x dx$; $\quad \begin{matrix} u = x & dv = \cos x dx \\ du = dx & v = \operatorname{sen} x \end{matrix}$

$\therefore \int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2\left[x \operatorname{sen} x - \int \operatorname{sen} x dx\right] = -x^2 \cos x + 2x \operatorname{sen} x + 2\cos x + c$

Respuesta: $\int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2x \operatorname{sen} x + 2\cos x + c$

4.4.-Encontrar: $\int (x^2 + 5x + 6)\cos 2x dx$

Solución.- I L A T E

$\downarrow \quad \searrow$

$x^2 + 5x + 6 \quad \cos 2x$

$\therefore \quad \begin{matrix} u = x^2 + 5x + 6 & dv = \cos 2x dx \\ du = (2x+5)dx & v = \frac{1}{2}\operatorname{sen} 2x \end{matrix}$

$\therefore \int (x^2 + 5x + 6)\cos 2x dx = \frac{(x^2 + 5x + 6)}{2}\operatorname{sen} 2x - \frac{1}{2}\int (2x+5)\operatorname{sen} 2x dx$

Integrando por partes la segunda integral:

I L A T E

$\swarrow \quad \searrow$

$2x + 5 \quad \operatorname{sen} 2x$

$\therefore \quad \begin{matrix} u = 2x + 5 & dv = \operatorname{sen} 2x dx \\ du = 2dx & v = -\frac{1}{2}\cos 2x \end{matrix}$

$\therefore \int (x^2 + 5x + 6)\cos 2x dx = \frac{1}{2}\operatorname{sen} 2x(x^2 + 5x + 6) - \frac{1}{2}\left[(2x+5)(-\tfrac{1}{2}\cos 2x) + \int \cos 2x dx\right]$

$= \frac{x^2 + 5x + 6}{2}\operatorname{sen} 2x + \frac{1}{4}\cos 2x(2x+5) - \frac{1}{2}\int \cos 2x dx$

$= \frac{x^2 + 5x + 6}{2}\operatorname{sen} 2x + \frac{2x+5}{4}\cos 2x - \frac{1}{4}\operatorname{sen} 2x + c$

Respuesta: $\int (x^2 + 5x + 6)\cos 2x dx = \frac{x^2 + 5x + 6}{2}\operatorname{sen} 2x + \frac{2x+5}{4}\cos 2x - \frac{1}{4}\operatorname{sen} 2x + c$

Nota.-Ya se habrá dado cuenta el lector, que la elección conveniente para el u y el dv, dependerá de la ubicación de los términos funcionales en la palabra ILATE. El de la izquierda corresponde al u, y el otro será el dv.

4.5.-Encontrar: $\int \ell\eta x dx$

Solución.- I L A T E

$\downarrow \quad \downarrow$

$\ell\eta x \quad 1$

$\therefore \quad u = \ell\eta x \qquad dv = 1dx$

$du = \dfrac{dx}{x} \qquad v = x$

$\therefore \int \ell\eta x dx = x\ell\eta x - \int dx = x\ell\eta x - x + c = x(\ell\eta x - 1) + c$

Respuesta: $\int \ell\eta x dx = x(\ell\eta x - 1) + c$

4.6.-Encontrar: $\int \ell\eta(a^2 + x^2)dx$

Solución.- I L A T E

$\downarrow \quad \searrow$

$\ell\eta(a^2 + x^2) \quad 1$

$\therefore \quad u = \ell\eta x \qquad dv = 1dx$

$du = \dfrac{dx}{x} \qquad v = x$

$\therefore \int \ell\eta(a^2 + x^2)dx = x\ell\eta(a^2 + x^2) - \int \dfrac{2x^2dx}{a^2 + x^2} = x\ell\eta(a^2 + x^2) - \int (2 - \dfrac{2a^2}{x^2 + a^2})dx$

$= x\ell\eta(a^2 + x^2) - 2\int dx + 2a^2 \int \dfrac{dx}{x^2 + a^2} = x\ell\eta(a^2 + x^2) - 2x + \dfrac{2a^2}{a} \operatorname{arc}\tau g \frac{x}{a} + c$

$= x\ell\eta(a^2 + x^2) - 2x + 2a \operatorname{arc}\tau g \frac{x}{a} + c$

Respuesta: $\int \ell\eta(a^2 + x^2)dx = x\ell\eta(a^2 + x^2) - 2x + 2a \operatorname{arc}\tau g \frac{x}{a} + c$

4.7.-Encontrar: $\int \ell\eta\left|x + \sqrt{x^2 - 1}\right|dx$

Solución.- I L A T E

$\downarrow \quad \searrow$

$\ell\eta\left|x + \sqrt{x^2 - 1}\right| \quad 1 \qquad dv = 1dx$

$v = x$

$u = \ell\eta\left|x + \sqrt{x^2 - 1}\right|$

$\therefore \quad du = \dfrac{1 + \dfrac{x}{\sqrt{x^2 - 1}}}{x + \sqrt{x^2 - 1}} d \Rightarrow du = \dfrac{\dfrac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}}{x + \sqrt{x^2 - 1}} dx \Rightarrow du = \dfrac{dx}{\sqrt{x^2 - 1}}$

$\therefore \int \ell\eta\left|x + \sqrt{x^2 - 1}\right|dx = x\ell\eta\left|x + \sqrt{x^2 - 1}\right| - \int \dfrac{xdx}{\sqrt{x^2 - 1}}$

Sea : $w = x^2 + 1, dw = 2xdx$.

Luego: $x\ell\eta\left|x + \sqrt{x^2 - 1}\right| - \dfrac{1}{2}\int (x^2 - 1)^{-1/2} 2xdx = x\ell\eta\left|x + \sqrt{x^2 - 1}\right| - \dfrac{1}{2}\int w^{-1/2}dw$

$= x\ell\eta\left|x + \sqrt{x^2 - 1}\right| - \dfrac{1}{2}\dfrac{w^{1/2}}{1/2} + c = x\ell\eta\left|x + \sqrt{x^2 - 1}\right| - w^{1/2} + c = x\ell\eta\left|x + \sqrt{x^2 - 1}\right| - \sqrt{x^2 - 1} + c$

Respuesta: $\int \ell\eta\left|x + \sqrt{x^2 - 1}\right|dx = x\ell\eta\left|x + \sqrt{x^2 - 1}\right| - \sqrt{x^2 - 1} + c$

4.8.-Encontrar: $\int \ell\eta^2 x dx$

Solución.- I L A T E

$\quad\downarrow\ \downarrow$

$\ell\eta^2 x \quad 1$

$$\therefore \begin{matrix} u = \ell\eta^2 x & dv = 1dx \\ du = 2\ell\eta x \frac{1}{x} dx & v = x \end{matrix}$$

$$\therefore \int \ell\eta^2 x dx = x\ell\eta^2 x - 2\int \ell\eta x \frac{1}{x} x dx = x\ell\eta^2 x - 2\int \ell\eta x dx$$

Por ejercicio 4.5, se tiene: $\int \ell\eta x dx = x(\ell\eta x - 1) + c$

Luego: $\int \ell\eta^2 x dx = x\ell\eta^2 x - 2\left[x(\ell\eta x - 1) + c\right] = x\ell\eta^2 x - 2x(\ell\eta x - 1) + c$

Respuesta: $\int \ell\eta^2 x dx = x\ell\eta^2 x - 2x(\ell\eta x - 1) + c$

4.9.-Encontrar: $\int \operatorname{arc}\tau g x dx$

Solución.- I L A T E

$\downarrow \quad \downarrow$

$\operatorname{arc}\tau g x \quad 1$

$$\therefore \begin{matrix} u = \operatorname{arc}\tau g x & dv = 1dx \\ du = \dfrac{dx}{1+x^2} & v = x \end{matrix}$$

$$\therefore \int \operatorname{arc}\tau g x dx = x \operatorname{arc}\tau g x - \int \frac{xdx}{1+x^2}$$

Sea: $w = 1 + x^2, dw = 2xdx$

Luego: $x \operatorname{arc}\tau g x - \frac{1}{2}\int \frac{2xdx}{1+x^2} = x \operatorname{arc}\tau g x - \frac{1}{2}\int \frac{dw}{w} = x \operatorname{arc}\tau g x - \frac{1}{2}\ell\eta|w| + c$

$= x \operatorname{arc}\tau g x - \frac{1}{2}\ell\eta\left|1 + x^2\right| + c$

Respuesta: $\int \operatorname{arc}\tau g x dx = x \operatorname{arc}\tau g x - \frac{1}{2}\ell\eta\left|1 + x^2\right| + c$

4.10.- $\int x^2 \operatorname{arc}\tau g x dx$

Solución.- I L A T E

$\downarrow \quad \downarrow$

$\operatorname{arc}\tau g x \quad x^2$

$$\therefore \begin{matrix} u = \operatorname{arc}\tau g x & dv = x^2 dx \\ du = \dfrac{dx}{1+x^2} & v = \dfrac{x^3}{3} \end{matrix}$$

$$\therefore \int x^2 \operatorname{arc}\tau g x dx = \frac{x^3}{3} \operatorname{arc}\tau g x - \frac{1}{3}\int \frac{x^2 dx}{1+x^2} = \frac{x^3}{3} \operatorname{arc}\tau g x - \frac{1}{3}\int (x - \frac{x}{x^2+1}) dx$$

$$= \frac{x^3}{3}\text{arc}\,\tau gx - \frac{1}{3}\int xdx - \frac{1}{3}\int \frac{x}{x^2+1}dx$$

Por ejercicio 4.9, se tiene: $\int \frac{xdx}{x^2+1} = \frac{1}{2}\ell\eta\left|x^2+1\right| + c$

Luego: $\frac{x^3}{3}\text{arc}\,\tau gx - \frac{1}{3}\int xdx + \frac{1}{6}\ell\eta\left|x^2+1\right| + c = \frac{x^3}{3}\text{arc}\,\tau gx - \frac{x^2}{6} + \frac{1}{6}\ell\eta\left|x^2+1\right| + c$

Respuesta: $\int x^2\,\text{arc}\,\tau gxdx = \frac{x^3}{3}\text{arc}\,\tau gx - \frac{x^2}{6} + \frac{1}{6}\ell\eta\left|x^2+1\right| + c$

4.11.-Encontrar: $\int \text{arc}\cos 2xdx$

Solución.- I L A T E

$\downarrow \quad \downarrow$

$\text{arc}\cos 2x \quad 1$

$$\therefore \quad u = \text{arc}\cos 2x \qquad dv = 1dx$$
$$du = -\frac{2dx}{\sqrt{1-4x^2}} \qquad v = x$$

$$\therefore \int \text{arc}\cos 2xdx = x\,\text{arc}\cos 2x + 2\int \frac{xdx}{\sqrt{1-4x^2}}$$

Sea: $w = 1-4x^2, dw = -8xdx$

Luego: $x\,\text{arc}\cos 2x - \frac{2}{8}\int \frac{-8xdx}{\sqrt{1-4x^2}} = x\,\text{arc}\cos 2x - \frac{1}{4}\int w^{-\frac{1}{2}}dw = x\,\text{arc}\cos 2x - \frac{1}{4}\frac{w^{\frac{1}{2}}}{\frac{1}{2}} + c$

$$= x\,\text{arc}\cos 2x - \frac{1}{2}\sqrt{1-4x^2} + c$$

Respuesta: $\int \text{arc}\cos 2xdx = x\,\text{arc}\cos 2x - \frac{1}{2}\sqrt{1-4x^2} + c$

4.12.-Encontrar: $\int \frac{\text{arcs}\,e\,\text{n}\sqrt{x}}{\sqrt{x}}dx$

Solución.- I L A T E

$\downarrow \quad \searrow$

$\text{arcs}\,e\,\text{n}\sqrt{x} \quad 1$

$$\therefore \quad u = \text{arcs}\,e\,\text{n}\sqrt{x} \qquad dv = x^{-\frac{1}{2}}dx$$
$$du = \frac{1}{\sqrt{1-x}}\frac{dx}{\sqrt{x}} \qquad v = 2\sqrt{x}$$

$$\therefore \int \text{arcs}\,e\,\text{n}\sqrt{x}\,x^{-\frac{1}{2}}dx = 2\sqrt{x}\,\text{arcs}\,e\,\text{n}\sqrt{x} - \int \frac{dx}{\sqrt{1-x}}$$

Sea: $w = 1-x, dw = -dx$

Luego: $2\sqrt{x}\,\text{arcs}\,e\,\text{n}\sqrt{x} + \int \frac{-dx}{\sqrt{1-x}} = 2\sqrt{x}\,\text{arcs}\,e\,\text{n}\sqrt{x} + \int w^{-\frac{1}{2}}dw$

$$= 2\sqrt{x}\,\text{arcs}\,e\,\text{n}\sqrt{x} + 2w^{\frac{1}{2}} + c = 2\sqrt{x}\,\text{arcs}\,e\,\text{n}\sqrt{x} + 2\sqrt{1-x} + c$$

Respuesta: $\int \frac{\operatorname{arcsen}\sqrt{x}}{\sqrt{x}}dx = 2\sqrt{x}\operatorname{arcsen}\sqrt{x} + 2\sqrt{1-x} + c$

4.13.-Encontrar: $\int x \operatorname{arcsen} 2x^2 dx$

Solución.- I L A T E

$\downarrow \quad \searrow$

$\operatorname{arcsen} 2x^2 \quad x$

$u = \operatorname{arcsen} 2x^2 \qquad dv = xdx$

$\therefore \quad du = \frac{4xdx}{\sqrt{1-4x^4}} \qquad v = \frac{x^2}{2}$

$\therefore \int x \operatorname{arcsen} 2x^2 dx = \frac{x^2}{2}\operatorname{arcsen} 2x^2 - 2\int \frac{x^3 dx}{\sqrt{1-4x^4}}$

Sea: $w = 1-4x^4, dw = -16x^3 dx$

Luego: $\frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{2}{16}\int \frac{(-16x^3dx)}{\sqrt{1-4x^4}} = \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{8}\int w^{-1/2}dw$

$= \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{8}\frac{w^{1/2}}{1/2} + c = \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{4}w^{1/2} + c$

$= \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{4}\sqrt{1-4x^4} + c$

Respuesta: $\int x \operatorname{arcsen} 2x^2 dx = \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{4}\sqrt{1-4x^4} + c$

4.14.-Encontrar: $\int xe^{x/a}dx$

Sea: $w = \frac{x}{a}, dw = \frac{dx}{a}$

Luego: $\int xe^{x/a}dx = a^2\int \frac{x}{a}e^{x/a}\frac{dx}{a} = a^2\int we^w dw$, integrando por partes se tiene:

Solución.- I L A T E

$\downarrow \quad \downarrow$

$w \quad e^w$

$u = w \qquad dv = e^w dw$

$\therefore \quad du = dw \qquad v = e^w$

$\therefore a^2\int we^w dw = a^2\left(we^w - \int e^w dw\right) = a^2\left(we^w - e^w + c\right) = a^2\left(we^w - e^w\right) + c$

$= a^2\left(\frac{x}{a}e^{x/a} - e^{x/a}\right) + c = a^2 e^{x/a}(\frac{x}{a} - 1) + c$

Respuesta: $\int xe^{x/a}dx = a^2 e^{x/a}(\frac{x}{a} - 1) + c$

4.15.-Encontrar: $\int x^2 e^{-3x}dx$

Solución.- I L A T E

$$\begin{matrix} \downarrow & \downarrow \\ x^2 & e^{-3x} \end{matrix}$$

$$\therefore \quad \begin{matrix} u = x^2 & dv = e^{-3x}dx \\ du = 2xdx & v = -\frac{1}{3}e^{-3x} \end{matrix}$$

$\therefore \int x^2e^{-3x}dx = -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\int xe^{-3x}dx$, integrando por partes la segunda integral:

I L A T E

$$\begin{matrix} \downarrow & \downarrow \\ x & e^{-3x} \end{matrix}$$

$$\therefore \quad \begin{matrix} u = x & dv = e^{-3x}dx \\ du = dx & v = -\frac{1}{3}e^{-3x} \end{matrix}$$

$$\therefore \int x^2e^{-3x}dx = -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\left(-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}dx\right) = -\frac{x^2e^{-3x}}{3} - \frac{2}{9}xe^{-3x} + \frac{2}{9}\int e^{-3x}dx$$

$$= -\frac{x^2e^{-3x}}{3} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + c$$

Respuesta: $\int x^2e^{-3x}dx = \frac{-e^{-3x}}{3}\left(x^2 + \frac{2}{3}x + \frac{2}{9}\right) + c$

4.16.-Encontrar: $\int x^3e^{-x^2}dx$

Solución.- $\int x^3e^{-x^2}dx = \int x^2e^{-x^2}xdx$

Sea: $w = -x^2, dw = -2xdx$, además: $x^2 = -w$

Luego: $\int x^2e^{-x^2}xdx = -\frac{1}{2}\int x^2e^{-x^2}x(-2xdx) = -\frac{1}{2}\int -we^w dw = \frac{1}{2}\int we^w dw$, integrando por Partes se tiene:

I L A T E

$$\begin{matrix} \downarrow & \downarrow \\ w & e^w \end{matrix}$$

$$\therefore \quad \begin{matrix} u = w & dv = e^w dw \\ du = dw & v = e^w \end{matrix}$$

$$\therefore \frac{1}{2}\int we^w dw = \frac{1}{2}\left(we^w - \int e^w dw\right) = \frac{1}{2}we^w - \frac{1}{2}\int e^w dw = \frac{1}{2}we^w - \frac{1}{2}e^w + c$$

$$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + c = -\frac{1}{2}e^{-x^2}(x^2+1) + c$$

Respuesta: $\int x^3e^{-x^2}dx = -\frac{1}{2}e^{-x^2}(x^2+1) + c$

4.17.-Encontrar: $\int (x^2 - 2x + 5)e^{-x}dx$

Solución.- I L A T E

$$\begin{matrix} \downarrow & \downarrow \end{matrix}$$

$x^2-2x+5 \qquad e^{-x}$

$\therefore \quad u = x^2-2x+5 \qquad dv = e^{-x}dx$

$\quad du = (2x-2)dx \qquad v = -e^{-x}$

$\therefore \int (x^2-2x+5)e^{-x}dx = -e^{-x}(x^2-2x+5) + \int (2x-2)e^{-x}dx$ **,** integrando por partes la segunda integral:

I L A T E

$\downarrow \quad \downarrow$

$2x-2 \qquad e^{-x}$

$\therefore \quad u = 2x-2 \qquad dv = e^{-x}dx$

$\quad du = 2dx \qquad v = -e^{-x}$

$\therefore \int (x^2-2x+5)e^{-x}dx = -e^{-x}(x^2-2x+5) + \left[-e^{-x}(2x-2) + 2\int e^{-x}dx\right]$

$= -e^{-x}(x^2-2x+5) - e^{-x}(2x-2) + 2\int e^{-x}dx = -e^{-x}(x^2-2x+5) - e^{-x}(2x-2) - 2e^{-x} + c$

$= -e^{-x}(x^2 \cancel{-2x} + 5 \cancel{+2x} \cancel{-2+2}) + c = -e^{-x}(x^2+5) + c$

Respuesta: $\int (x^2-2x+5)e^{-x}dx = -e^{-x}(x^2+5) + c$

4.18.-Encontrar: $\int e^{ax}\cos bx dx$

Solución.- I L A T E

$\swarrow \quad \downarrow$

$\cos bx \qquad e^{ax}$

$\therefore \quad u = \cos bx \qquad dv = e^{ax}dx$

$\quad du = -b\,\mathrm{sen}\,bx dx \qquad v = \frac{1}{a}e^{ax}$

$\therefore \int e^{ax}\cos bx dx = \frac{e^{ax}\cos bx}{a} + \frac{b}{a}\int e^{ax}\,\mathrm{sen}\,bx dx$, Nótese que la segunda integral es semejante a la primera, salvo en la parte trigonométrica; integrando por partes la segunda integral:

I L A T E

$\swarrow \quad \downarrow$

$\mathrm{sen}\,bx \qquad e^{ax}$

$\therefore \quad u = \mathrm{sen}\,bx \qquad dv = e^{ax}dx$

$\quad du = b\cos bx dx \qquad v = \frac{1}{a}e^{ax}$

$\therefore = \frac{e^{ax}\cos bx}{a} + \frac{b}{a}\left(\frac{e^{ax}\,\mathrm{sen}\,bx}{a} - \frac{b}{a}\int e^{ax}\cos bx dx\right)$

$= \frac{e^{ax}\cos bx}{a} + \frac{be^{ax}\,\mathrm{sen}\,bx}{a^2} - \frac{b^2}{a^2}\int e^{ax}\cos bx dx$, Nótese que:

$\int e^{ax}\cos bx dx = \frac{e^{ax}\cos bx}{a} + \frac{be^{ax}\,\mathrm{sen}\,bx}{a^2} - \frac{b^2}{a^2}\int e^{ax}\cos bx dx$, la integral a encontrar aparece con coeficiente 1 en el primer miembro, y en el segundo con coeficiente:

$-\frac{b^2}{a^2}$. Transponiendo éste término al primer miembro y dividiendo por el nuevo coeficiente: $1+\frac{b^2}{a^2}=\frac{a^2+b^2}{a^2}$, se tiene:

$$\left(\frac{a^2+b^2}{a^2}\right)\int e^{ax}\cos bx dx=\frac{ae^{ax}\cos bx+be^{ax}\operatorname{sen}bx}{a^2}+c$$

$$\int e^{ax}\cos bx dx=\frac{\frac{ae^{ax}\cos bx+be^{ax}\operatorname{sen}bx}{\cancel{a^2}}}{\left(\frac{a^2+b^2}{\cancel{a^2}}\right)}+c=\frac{e^{ax}(a\cos bx+b\operatorname{sen}bx)}{a^2+b^2}+c$$

Respuesta: $\int e^{ax}\cos bx dx=\frac{e^{ax}(a\cos bx+b\operatorname{sen}bx)}{a^2+b^2}+c$

4.19.-Encontrar: $\int e^x\cos 2x dx$

Solución.- Este ejercicio es un caso particular del ejercicio anterior, donde: $a=1$ y $b=2$. Invitamos al lector, resolverlo por partes, aún cuando la respuesta es inmediata.

Respuesta: $\int e^x\cos 2x dx=\frac{e^x(\cos 2x+2\operatorname{sen}2x)}{5}+c$

4.20.-Encontrar: $\int e^{ax}\operatorname{sen}bx dx$

Solución.- I L A T E

$$\swarrow \quad \downarrow$$

$$\operatorname{sen}bx \qquad e^{ax}$$

$$\therefore \quad \begin{array}{ll} u=\operatorname{sen}bx & dv=e^{ax}dx \\ du=b\cos bx dx & v=\frac{1}{a}e^{ax} \end{array}$$

$\therefore \int e^{ax}\operatorname{sen}bx dx=\frac{e^{ax}\operatorname{sen}bx}{a}-\frac{b}{a}\int e^{ax}\cos bx dx$, integrando por partes la segunda integral:

I L A T E

$$\swarrow \quad \downarrow$$

$$\cos bx \qquad e^{ax}$$

$$\therefore \quad \begin{array}{ll} u=\cos bx & dv=e^{ax}dx \\ du=-b\operatorname{sen}bx dx & v=\frac{1}{a}e^{ax} \end{array}$$

$$\therefore \int e^{ax}\operatorname{sen}bx dx=\frac{e^{ax}\operatorname{sen}bx}{a}-\frac{b}{a}\left(\frac{e^{ax}\cos bx}{a}+\frac{b}{a}\int e^{ax}\operatorname{sen}bx dx\right)$$

$$=\frac{e^{ax}\operatorname{sen}bx}{a}-\frac{be^{ax}\cos bx}{a^2}-\frac{b^2}{a^2}\int e^{ax}\operatorname{sen}bx dx,$$

Como habrá notado el lector, la integral a encontrar aparece con coeficiente 1 en el primer miembro, y en el segundo con coeficiente: $-\frac{b^2}{a^2}$. Transponiendo éste término al primer miembro y dividiendo por el nuevo coeficiente: $1+\frac{b^2}{a^2}=\frac{a^2+b^2}{a^2}$, se tiene:

$$\left(\frac{a^2+b^2}{a^2}\right)\int e^{ax}\operatorname{sen} bx dx=\frac{ae^{ax}\operatorname{sen} bx-be^{ax}\cos bx}{a^2}+c$$

$$\int e^{ax}\operatorname{sen} bx dx=\frac{\frac{ae^{ax}\operatorname{sen} bx-be^{ax}\cos bx}{\cancel{a^2}}}{\left(\frac{a^2+b^2}{\cancel{a^2}}\right)}+c=\int e^{ax}\operatorname{sen} bx dx=\frac{e^{ax}(a\operatorname{sen} bx-b\cos bx)}{a^2+b^2}+c$$

Respuesta: $\int e^{ax}\operatorname{sen} bx dx=\frac{e^{ax}(a\operatorname{sen} bx-b\cos bx)}{a^2+b^2}+c$

4.21.-Encontrar: $\int x\sqrt{1+x}dx$

Solución.- Cuando el integrando, está formado por el producto de funciones algebraicas, es necesario tomar como dv, la parte más fácil integrable y u como la parte más fácil derivable. Sin embargo, la opción de "más fácil" quedará a criterio del lector.

$$\therefore \quad \begin{array}{ll} u=x & dv=(1+x)^{1/2}dx \\ du=dx & v=\frac{2}{3}(1+x)^{3/2} \end{array}$$

$$\therefore \int x\sqrt{1+x}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{2}{3}\int(1+x)^{3/2}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{2}{3}\frac{(1+x)^{5/2}}{5/2}+c$$

$$=\frac{2}{3}x(1+x)^{3/2}-\frac{4(1+x)^{5/2}}{15}+c$$

Respuesta: $\int x\sqrt{1+x}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{4(1+x)^{5/2}}{15}+c$

4.22.-Encontrar: $\int\frac{x^2dx}{\sqrt{1+x}}$

Solución.- $\int\frac{x^2dx}{\sqrt{1+x}}=\int x^2(1+x)^{-1/2}dx$

$$\therefore \quad \begin{array}{ll} u=x^2 & dv=(1+x)^{-1/2}dx \\ du=2xdx & v=2(1+x)^{1/2} \end{array}$$

$\therefore \int\frac{x^2dx}{\sqrt{1+x}}=2x^2\sqrt{1+x}-4\int x\sqrt{1+x}dx$, integrando por partes la segunda integral:

$$\therefore \quad \begin{array}{ll} u = x & dv = (1+x)^{1/2}dx \\ du = dx & v = \frac{2}{3}(1+x)^{3/2} \end{array}$$

$$\int \frac{x^2dx}{\sqrt{1+x}} = 2x^2\sqrt{1+x} - 4\left[\frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\int (1+x)^{3/2}dx\right]$$

$$= 2x^2\sqrt{1+x} - \frac{8}{3}x(1+x)^{3/2} + \frac{8}{3}\frac{(1+x)^{5/2}}{5/2} + c = 2x^2\sqrt{1+x} - \frac{8}{3}x(1+x)^{3/2} + \frac{16}{15}(1+x)^{5/2} + c$$

Respuesta: $\int \frac{x^2dx}{\sqrt{1+x}} = 2x^2\sqrt{1+x} - \frac{8}{3}x(1+x)^{3/2} + \frac{16}{15}(1+x)^{5/2} + c$

4.23.-Encontrar: $\int \frac{xdx}{e^x}$

Solución.- $\int \frac{xdx}{e^x} = \int xe^{-x}dx$

I L A T E

$\quad\downarrow \quad \downarrow$

$\quad x \quad e^{-x}$

$$\therefore \quad \begin{array}{ll} u = x & dv = e^{-x}dx \\ du = dx & v = -e^{-x} \end{array}$$

$$\therefore \int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + c = e^{-x}(-x-1) + c = -e^{-x}(x+1) + c$$

Respuesta: $\int \frac{xdx}{e^x} = -e^{-x}(x+1) + c$

4.24.-Encontrar: $\int x^2\ell\eta\left|\sqrt{1-x}\right|dx$

Solución.- $\therefore$

$$u = \ell\eta\left|\sqrt{1-x}\right| \qquad dv = x^2dx$$

$$du = \frac{1}{\left|\sqrt{1-x}\right|}\frac{1}{2}(1-x)^{-1/2}(-1)dx \Rightarrow du = \frac{-dx}{2(1-x)} \qquad v = \frac{x^3}{3}$$

$$\therefore \int x^2\ell\eta\left|\sqrt{1-x}\right|dx = \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| + \frac{1}{6}\int\frac{x^3}{1-x}dx = \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\int\left(x^2 + x + 1 - \frac{1}{1-x}\right)dx$$

$$= \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\frac{x^3}{3} - \frac{1}{6}\frac{x^2}{2} - \frac{1}{6}x - \frac{1}{6}\ell\eta\left|1-x\right| + c$$

$$= \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\ell\eta\left|1-x\right| - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c$$

Respuesta: $\int x^2\ell\eta\left|\sqrt{1-x}\right|dx = \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\ell\eta\left|1-x\right| - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c$

4.25.-Encontrar: $\int x\,\mathrm{sen}^2 x\,dx$

Solución.-

$\therefore \quad \begin{array}{l} u = x \\ du = dx \end{array} \qquad \begin{array}{l} dv = \operatorname{sen}^2 x dx \\ v = \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x \end{array} \qquad \left(v = \int \frac{1-\cos 2x}{2} dx \right)$

$\therefore \int x \operatorname{sen}^2 x dx = \frac{1}{2}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{2}\int x dx + \frac{1}{4}\int \operatorname{sen} 2x dx$

$= \frac{1}{2}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{4}x^2 - \frac{1}{8}\cos 2x + c = \frac{1}{4}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{8}\cos 2x + c$

Respuesta: $\int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2x}{4} - \frac{\cos 2x}{8} + c$

Otra solución.-

$\int x \operatorname{sen}^2 x dx = \int x \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int x dx - \frac{1}{2}\int x \cos 2x dx = \frac{1}{2}\frac{x^2}{2} - \frac{1}{2}\int x \cos 2x dx$

$= \frac{x^2}{4} - \frac{1}{2}\int x \cos 2x dx$ **;** integrando por partes, la segunda integral:

$\therefore \quad \begin{array}{l} u = x \\ du = dx \end{array} \qquad \begin{array}{l} dv = \cos 2x dx \\ v = \frac{1}{2}\operatorname{sen} 2x \end{array}$

$\int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{1}{2}\left(\frac{x}{2}\operatorname{sen} 2x - \frac{1}{2}\int \operatorname{sen} 2x dx \right) = \frac{x^2}{4} - \frac{x}{4}\operatorname{sen} 2x + \frac{1}{4}\int \operatorname{sen} 2x dx$

$= \frac{x^2}{4} - \frac{x}{4}\operatorname{sen} 2x + \frac{1}{4}(-\frac{1}{2}\cos 2x) + c = \frac{x^2}{4} - \frac{x}{4}\operatorname{sen} 2x - \frac{\cos 2x}{8} + c$

Respuesta: $\int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2x}{4} - \frac{\cos 2x}{8} + c$

4.26.-Encontrar: $\int x(3x+1)^7 dx$

Solución.-

$\therefore \quad \begin{array}{l} u = x \\ du = dx \end{array} \qquad \begin{array}{l} dv = (3x+1)^7 dx \\ v = \frac{1}{24}(3x+1)^8 \end{array} \qquad \left(v = \int (3x+1)^7 dx \right)$

$\therefore \int x(3x+1)^7 dx = \frac{x}{24}(3x+1)^8 - \frac{1}{24}\int (3x+1)^8 dx = \frac{x}{24}(3x+1)^8 - \frac{1}{24}\frac{1}{3}\frac{(3x+1)^9}{9} + c$

$= \frac{x}{24}(3x+1)^8 - \frac{(3x+1)^9}{648} + c$

Respuesta: $\int x(3x+1)^7 dx = \frac{x}{24}(3x+1)^8 - \frac{(3x+1)^9}{648} + c$

EJERCICIOS PROPUESTOS

Usando esencialmente el mecanismo presentado, encontrar las integrales siguientes:

4.27.- $\int x(2x+5)^{10}dx$

4.28.- $\int \text{arcs}\,e\,\text{n}\,xdx$

4.29.- $\int x\,\text{s}\,e\,\text{n}\,xdx$

4.30.- $\int x\cos 3xdx$

4.31.- $\int x2^{-x}dx$

4.32.- $\int x^2e^{3x}dx$

4.33.- $\int x^3e^{-x/3}dx$

4.34.- $\int x\,\text{s}\,e\,\text{n}\,x\cos xdx$

4.35.- $\int x^2\ell\eta xdx$

4.36.- $\int \frac{\ell\eta x}{x^3}dx$

4.37.- $\int \frac{\ell\eta x}{\sqrt{x}}dx$

4.38.- $\int x\,\text{arc}\,\tau gxdx$

4.39.- $\int x\,\text{arcs}\,e\,\text{n}\,xdx$

4.40.- $\int \frac{xdx}{\text{s}\,e\,\text{n}^2 x}$

4.41.- $\int e^x\,\text{s}\,e\,\text{n}\,xdx$

4.42.- $\int 3^x\cos xdx$

4.43.- $\int \text{s}\,e\,\text{n}(\ell\eta x)dx$

4.44.- $\int (x^2-2x+3)\ell\eta xdx$

4.45.- $\int x\ell\eta\left|\frac{1-x}{1+x}\right|dx$

4.46.- $\int \frac{\ell\eta^2 x}{x^2}dx$

4.47.- $\int x^2\,\text{arc}\,\tau g3xdx$

4.48.- $\int x(\text{arc}\,\tau gx)^2dx$

4.49.- $\int (\text{arcs}\,e\,\text{n}\,x)^2dx$

4.50.- $\int \frac{\text{arcs}\,e\,\text{n}\,x}{x^2}dx$

4.51.- $\int \frac{\text{arcs}\,e\,\text{n}\sqrt{x}}{\sqrt{1-x}}dx$

4.52.- $\int \frac{\text{s}\,e\,\text{n}^2 x}{e^x}dx$

4.53.- $\int \tau g^2x\sec^3 xdx$

4.54.- $\int x^3\ell\eta^2xdx$

4.55.- $\int x\ell\eta(9+x^2)dx$

4.56.- $\int \text{arcs}\,e\,\text{n}\sqrt{x}dx$

4.57.- $\int x\,\text{arc}\,\tau g(2x+3)dx$

4.58.- $\int e^{\sqrt{x}}dx$

4.59.- $\int \cos^2(\ell\eta x)dx$

4.60.- $\int \frac{\ell\eta(\ell\eta x)}{x}dx$

4.61.- $\int \ell\eta|x+1|dx$

4.62.- $\int x^2e^xdx$

4.63.- $\int \cos^n xdx$

4.64.- $\int \text{s}\,e\,\text{n}^n xdx$

4.65.- $\int x^m(\ell\eta x)^ndx$

4.66.- $\int x^3(\ell\eta x)^2dx$

4.67.- $\int x^ne^xdx$

4.68.- $\int x^3e^xdx$

4.69.- $\int \sec^n xdx$

4.70.- $\int \sec^3 xdx$

4.71.- $\int x\ell\eta xdx$

4.72.- $\int x^n\ell\eta|ax|dx, n\neq -1$

4.73.- $\int \text{arcs}\,e\,\text{n}\,axdx$

4.74.- $\int x\,\text{s}\,e\,\text{n}\,axdx$

4.75.- $\int x^2\cos axdx$

4.76.- $\int x\sec^2 axdx$

4.77.- $\int \cos(\ell\eta x)dx$

4.78.- $\int \ell\eta(9+x^2)dx$

4.79.- $\int x\cos(2x+1)dx$

4.80.- $\int x\,\text{arc}\sec xdx$

4.81.- $\int \text{arc}\sec\sqrt{x}dx$

4.82.- $\int \sqrt{a^2-x^2}dx$

4.83.- $\int \ell\eta|1-x|dx$

4.84.- $\int \ell\eta(x^2+1)dx$

4.85.- $\int \text{arc}\,\tau g\sqrt{x}dx$

4.86.- $\int \frac{x\,\text{arcs}\,e\,\text{n}\,x}{\sqrt{1-x^2}}dx$

4.87.- $\int x\,\text{arc}\,\tau g\sqrt{x^2-1}dx$

4.88.- $\int \frac{x\,\text{arc}\,\tau gx}{(x^2+1)^2}dx$

4.89.- $\int \text{arcs}\,e\,\text{n}\,x\frac{xdx}{\sqrt{(1-x^2)^3}}$

4.90.- $\int x^2\sqrt{1-x}dx$

RESPUESTAS

4.27.- $\int x(2x+5)^{10}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = (2x+5)^{10}dx \\ du = dx & v = \dfrac{(2x+5)^{11}}{22} \end{array}$$

$$\int x(2x+5)^{10}dx = \frac{x}{22}(2x+5)^{11} - \frac{1}{22}\int (2x+5)^{11}dx = \frac{x}{22}(2x+5)^{11} - \frac{1}{44}(2x+5)^{12} + c$$

$$= \frac{x}{22}(2x+5)^{11} - \frac{1}{528}(2x+5)^{12} + c$$

4.28.- $\int \text{arcs}e\text{n}\, xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \text{arcs}e\text{n}\, x & dv = dx \\ du = \dfrac{dx}{\sqrt{1-x^2}} & v = x \end{array}$$ Además: $w = 1 - x^2, dw = -2xdx$

$$\int \text{arcs}e\text{n}\, xdx = x\,\text{arcs}e\text{n}\, x - \int \frac{xdx}{\sqrt{1-x^2}} = x\,\text{arcs}e\text{n}\, x + \frac{1}{2}\int \frac{dw}{w^{1/2}} = x\,\text{arcs}e\text{n}\, x + \sqrt{1-x^2} + c$$

4.29.- $\int x\,\text{s}e\text{n}\, xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \text{s}e\text{n}\, xdx \\ du = dx & v = -\cos x \end{array}$$

$$\int x\,\text{s}e\text{n}\, xdx = -x\cos x + \int \cos xdx = -x\cos x + \text{s}e\text{n}\, x + c$$

4.30.- $\int x\cos 3xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \cos 3xdx \\ du = dx & v = \dfrac{1}{3}\text{s}e\text{n}\, 3x \end{array}$$

$$\int x\cos 3xdx = \frac{x}{3}\text{s}e\text{n}\, 3x - \int \frac{1}{3}\text{s}e\text{n}\, 3xdx = \frac{x}{3}\text{s}e\text{n}\, 3x + \frac{\cos 3x}{9} + c$$

4.31.- $\int x2^{-x}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = 2^{-x}dx \\ du = dx & v = -\dfrac{2^{-x}}{\ell\eta 2} \end{array}$$

$$\int x2^{-x}dx = -\frac{x2^{-x}}{\ell\eta 2} + \frac{1}{\ell\eta 2}\int 2^{-x}dx = -\frac{x2^{-x}}{\ell\eta 2} + \frac{1}{\ell\eta 2}\left(\frac{-2^{-x}}{\ell\eta 2}\right) + c = -\frac{x}{2^x\ell\eta 2} - \frac{1}{2^{-x}\ell\eta^2 2} + c$$

4.32.- $\int x^2 e^{3x}dx$

Solución.-

$\therefore \quad \begin{array}{ll} u = x^2 & dv = e^{3x}dx \\ du = 2xdx & v = \frac{1}{3}e^{3x} \end{array}$

$\int x^2 e^{3x}dx = \frac{x^2}{3}e^{3x} - \frac{2}{3}\int xe^{3x}dx$, integral la cual se desarrolla nuevamente por partes, esto es: $\therefore \quad \begin{array}{ll} u = x & dv = e^{3x}dx \\ du = dx & v = \frac{1}{3}e^{3x} \end{array}$

$$= \frac{x^2}{3}e^{3x} - \frac{2}{3}\left(\frac{x}{3}e^{3x} - \frac{1}{3}\int e^{3x}dx\right) = \frac{x^2}{3}e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\int e^{3x}dx = \frac{x^2}{3}e^{3x} - \frac{2x}{9}e^{3x} + \frac{2}{27}e^{3x} + c$$

4.33.- $\int x^3 e^{-x/3}dx$

Solución.-

$\therefore \quad \begin{array}{ll} u = x^3 & dv = e^{-x/3}dx \\ du = 3x^2dx & v = -3e^{-x/3} \end{array}$

$\int x^3 e^{-x/3}dx = -3x^3e^{-x/3} + 9\int x^2e^{-x/3}dx$, integral la cual se desarrolla nuevamente por partes, esto es: $\therefore \quad \begin{array}{ll} u = x^2 & dv = e^{-x/3}dx \\ du = 2xdx & v = -3e^{-x/3} \end{array}$

$$= -3x^3e^{-x/3} + 9\left(-3x^2e^{-x/3} + 6\int xe^{-x/3}dx\right) = -3x^3e^{-x/3} - 27x^2e^{-x/3} + 54\int xe^{-x/3}dx$$

, la nueva integral se desarrolla por partes, esto es:

$\therefore \quad \begin{array}{ll} u = x & dv = e^{-x/3}dx \\ du = dx & v = -3e^{-x/3} \end{array}$

$$= -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} + 54\left(-3xe^{-x/3} + 3\int e^{-x/3}dx\right) = -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} - \frac{162x}{e^{x/3}} + 162(-3e^{-x/3}) + c$$

$$= -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} - \frac{162x}{e^{x/3}} - \frac{486}{e^{x/3}} + c$$

4.34.- $\int x\,\mathrm{sen}\,x\cos x dx$

Solución.-

$\therefore \quad \begin{array}{ll} u = x & dv = \mathrm{sen}\,2xdx \\ du = dx & v = -\frac{\cos 2x}{2} \end{array}$

$$\int x\,\mathrm{sen}\,x\cos xdx = \frac{1}{2}\int x\,\mathrm{sen}\,2xdx = \frac{1}{2}\left(-\frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2xdx\right)$$

$$= -\frac{x}{4}\cos 2x + \frac{1}{4}\int \cos 2xdx = -\frac{x}{4}\cos 2x + \frac{1}{8}\mathrm{sen}\,2x + c$$

4.35.- $\int x^2 \ell\eta x dx$

Solución.-

$$u = \ell\eta x \qquad dv = x^2dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = \frac{x^3}{3}$$

$$\int x^2\ell\eta x dx = \frac{x^3\ell\eta x}{3} - \frac{1}{3}\int x^2dx = \frac{x^3\ell\eta x}{3} - \frac{x^3}{9} + c$$

4.36.- $\int \frac{\ell\eta x}{x^3}dx$

Solución.-

$$u = \ell\eta x \qquad dv = x^{-3}dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = -\frac{1}{2x^2}$$

$$\int \frac{\ell\eta x}{x^3}dx = \int x^{-3}\ell\eta x dx = -\frac{\ell\eta x}{2x^2} + \frac{1}{2}\int x^{-3}dx = -\frac{\ell\eta x}{2x^2} - \frac{1}{4x^2} + c$$

4.37.- $\int \frac{\ell\eta x}{\sqrt{x}}dx$

Solución.-

$$u = \ell\eta x \qquad dv = x^{-1/2}dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = 2\sqrt{x}$$

$$\int \frac{\ell\eta x}{\sqrt{x}}dx = \int x^{-1/2}\ell\eta x dx = 2\sqrt{x}\ell\eta x - 2\int x^{-1/2}dx = 2\sqrt{x}\ell\eta x - 4\sqrt{x} + c$$

4.38.- $\int x \operatorname{arc}\tau g x dx$

Solución.-

$$u = \operatorname{arc}\tau g x \qquad dv = xdx$$

$$\therefore \quad du = \frac{dx}{1+x^2} \qquad v = \frac{x^2}{2}$$

$$\int x \operatorname{arc}\tau g x dx = \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}\int \frac{x^2dx}{1+x^2} = \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}\int dx + \frac{1}{2}\int\frac{dx}{1+x^2} = \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}x + \frac{\operatorname{arc}\tau g x}{2} + c$$

4.39.- $\int x \operatorname{arcs}e\operatorname{n} x dx$

Solución.-

$$u = \operatorname{arcs}e\operatorname{n} x \qquad dv = xdx$$

$$\therefore \quad du = \frac{dx}{\sqrt{1+x^2}} \qquad v = \frac{x^2}{2}$$

$\int x \operatorname{arcs}e\operatorname{n} x dx = \frac{x^2}{2}\operatorname{arcs}e\operatorname{n} x - \frac{1}{2}\int\frac{x^2dx}{\sqrt{1+x^2}}$, integral para la cual se sugiere la sustitución siguiente: $\therefore$ $x = \operatorname{s}e\operatorname{n}\theta$, $dx = \cos\theta d\theta$

$$=\frac{x^2}{2}\arcsin x-\frac{1}{2}\int\frac{\operatorname{sen}^2\theta\cancel{\cos\theta}d\theta}{\cancel{\cos\theta}}$$

$$=\frac{x^2}{2}\arcsin x-\frac{1}{2}\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta=\frac{x^2}{2}\arcsin x-\frac{1}{4}\int d\theta+\frac{1}{4}\int\cos 2\theta d\theta$$

$$=\frac{x^2}{2}\arcsin x-\frac{1}{4}\theta+\frac{1}{8}\operatorname{sen}2\theta+c=\frac{x^2}{2}\arcsin x-\frac{1}{4}\arcsin x+\frac{2\operatorname{sen}\theta\cos\theta}{8}+c$$

Como: $\operatorname{sen}\theta=x,\cos\theta=\sqrt{1-x^2}$; luego:

$$=\frac{x^2}{2}\arcsin x-\frac{1}{4}\arcsin x+\frac{1}{4}x\sqrt{1-x^2}+c$$

4.40.- $\int\frac{xdx}{\operatorname{sen}^2 x}$

Solución.-

$$\therefore\quad u=x \qquad dv=\operatorname{cosec}^2 xdx$$
$$du=dx \qquad v=-\operatorname{co}\tau gx$$

$$\int\frac{xdx}{\operatorname{sen}^2 x}=\int x\operatorname{cosec}^2 xdx=-x\operatorname{co}\tau gx+\int\operatorname{co}\tau gxdx=-x\operatorname{co}\tau gx+\ell\eta\left|\operatorname{sen}x\right|+c$$

4.41.- $\int e^x\operatorname{sen}xdx$

Solución.-

$$\therefore\quad u=\operatorname{sen}x \qquad dv=e^x dx$$
$$du=\cos xdx \qquad v=e^x$$

$\int e^x\operatorname{sen}xdx=e^x\operatorname{sen}x-\int e^x\cos xdx$, integral la cual se desarrolla por partes, esto es:

$$\therefore\quad u=\cos x \qquad dv=e^x dx$$
$$du=-\operatorname{sen}xdx \qquad v=e^x$$

$$=e^x\operatorname{sen}x-\left(e^x\cos x+\int e^x\operatorname{sen}xdx\right)=e^x\operatorname{sen}x-e^x\cos x-\int e^x\operatorname{sen}xdx$$

Luego se tiene: $\int e^x\operatorname{sen}xdx=e^x\operatorname{sen}x-e^x\cos x-\int e^x\operatorname{sen}xdx$, de donde es inmediato:

$$2\int e^x\operatorname{sen}xdx=e^x(\operatorname{sen}x-\cos x)+c$$

$$\int e^x\operatorname{sen}xdx=\frac{e^x}{2}(\operatorname{sen}x-\cos x)+c$$

4.42.- $\int 3^x\cos xdx$

Solución.-

$$\therefore\quad u=\cos x \qquad dv=3^x dx$$
$$du=-\operatorname{sen}xdx \qquad v=\frac{3^x}{\ell\eta 3}$$

$\int 3^x \cos x dx = \cos x \frac{3^x}{\ell\eta 3} + \frac{1}{\ell\eta 3}\int 3^x \operatorname{sen} x dx$, integral la cual se desarrolla por partes, esto es: $\therefore$

$$u = \operatorname{sen} x \qquad dv = 3^x dx$$
$$du = \cos x dx \qquad v = \frac{3^x}{\ell\eta 3}$$

$$= \cos x \frac{3^x}{\ell\eta 3} + \frac{1}{\ell\eta 3}\left(\frac{3^x}{\ell\eta 3}\operatorname{sen} x - \frac{1}{\ell\eta 3}\int 3^x \cos x dx\right)$$

$= \cos x \frac{3^x}{\ell\eta 3} + \frac{3^x \operatorname{sen} x}{\ell\eta^2 3} - \frac{1}{\ell\eta^2 3}\int 3^x \cos x dx$,luego:

$= \int 3^x \cos x dx = \frac{3^x}{\ell\eta}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3}\right) - \frac{1}{\ell\eta^2 3}\int 3^x \cos x dx$, de donde es inmediato:

$$= (1 + \frac{1}{\ell\eta^2 3})\int 3^x \cos x dx = \frac{3^x}{\ell\eta 3}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3}\right) + c$$

$$= (\frac{\ell\eta^2 3 + 1}{\ell\eta^2 3})\int 3^x \cos x dx = \frac{3^x}{\ell\eta 3}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3}\right) + c$$

$$= \int 3^x \cos x dx = \frac{3^x \ell\eta 3}{\ell\eta^2 3 + 1}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3}\right) + c$$

4.43.- $\int \operatorname{sen}(\ell\eta x)dx$

Solución.-

$$\therefore \quad u = \operatorname{sen}(\ell\eta x) \qquad dv = dx$$
$$du = \frac{\cos(\ell\eta x)}{x}dx \qquad v = x$$

$\int \operatorname{sen}(\ell\eta x)dx = x\operatorname{sen}(\ell\eta x) - \int \cos(\ell\eta x)dx$, integral la cual se desarrolla por partes, esto es:

$$\therefore \quad u = \cos(\ell\eta x) \qquad dv = dx$$
$$du = \frac{-\operatorname{sen}(\ell\eta x)}{x}dx \qquad v = x$$

$$= x\operatorname{sen}(\ell\eta x) - \left[x\cos(\ell\eta x) + \int \operatorname{sen}(\ell\eta x)dx\right] = x\operatorname{sen}(\ell\eta x) - x\cos(\ell\eta x) - \int \operatorname{sen}(\ell\eta x)dx$$

Se tiene por tanto:

$\int \operatorname{sen}(\ell\eta x)dx = x[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)] - \int \operatorname{sen}(\ell\eta x)dx$, de donde es inmediato:

$$2\int \operatorname{sen}(\ell\eta x)dx = x[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)] + c \quad \int \operatorname{sen}(\ell\eta x)dx = \frac{x}{2}[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)] + c$$

4.44.- $\int (x^2 - 2x + 3)\ell\eta x dx$

Solución.-

$$u = \ell\eta x \qquad dv = (x^2 - 2x + 3)dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = \frac{x^3}{3} - x^2 + 3x$$

$$\int (x^2 - 2x + 3)\ell\eta x dx = (\frac{x^3}{3} - x^2 + 3x)\ell\eta x - \int (\frac{x^2}{3} - x + 3)dx$$

$$= (\frac{x^3}{3} - x^2 + 3x)\ell\eta x - \int \frac{x^2}{3}dx - \int xdx + 3\int dx = (\frac{x^3}{3} - x^2 + 3x)\ell\eta x - \frac{x^3}{9} - \frac{x^2}{2} + 3x + c$$

4.45.- $\int x\ell\eta\left|\frac{1-x}{1+x}\right|dx$

Solución.-

$$u = \ell\eta\left|\frac{1-x}{1+x}\right| \qquad dv = xdx$$

$$\therefore \quad du = \frac{2dx}{x^2 - 1} \qquad v = \frac{x^2}{2}$$

$$\int x\ell\eta\left|\frac{1-x}{1+x}\right|dx = \frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right| - \int \frac{x^2 dx}{x^2 - 1} = \frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right| - \int (1 + \frac{1}{x^2 - 1})dx$$

$$= \frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right| - \int dx - \int \frac{dx}{x^2 - 1} = \frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right| - x - \frac{1}{2}\ell\eta\left|\frac{x-1}{x+1}\right| + c$$

4.46.- $\int \frac{\ell\eta^2 x}{x^2}dx$

Solución.-

$$u = \ell\eta^2 x \qquad dv = x^{-2}dx$$

$$\therefore \quad du = \frac{2\ell\eta x}{x}dx \qquad v = -\frac{1}{x}$$

$\int \frac{\ell\eta^2 x}{x^2}dx = -\frac{\ell\eta^2 x}{x} + 2\int \frac{\ell\eta x}{x^2}dx = -\frac{\ell\eta^2 x}{x} + 2\int x^{-2}\ell\eta x dx$, integral la cual se desarrolla por partes, esto es:

$$u = \ell\eta x \qquad dv = x^{-2}dx$$

$$\therefore \quad du = \frac{dx}{x} \qquad v = -\frac{1}{x}$$

$$= -\frac{\ell\eta^2 x}{x} + 2\left(-\frac{\ell\eta x}{x} + \int \frac{dx}{x^2}\right) = -\frac{\ell\eta^2 x}{x} - \frac{2\ell\eta x}{x} + 2\int \frac{dx}{x^2} = -\frac{\ell\eta^2 x}{x} - \frac{2\ell\eta x}{x} - \frac{2}{x} + c$$

4.47.- $\int x^2 \operatorname{arc}\tau g 3x dx$

Solución.-

$$u = \operatorname{arc}\tau g 3x \qquad dv = x^2 dx$$

$$\therefore \quad du = \frac{3dx}{1 + 9x^2} \qquad v = \frac{x^3}{3}$$

$$\int x^2 \operatorname{arc}\tau g3xdx = \frac{x^3}{3}\operatorname{arc}\tau g3x - \int \frac{x^3dx}{1+9x^2} = \frac{x^3}{3}\operatorname{arc}\tau g3x - \frac{1}{9}\int \frac{x^3dx}{\frac{1}{9}+x^2}$$

$$= \frac{x^3}{3}\operatorname{arc}\tau g3x - \frac{1}{9}\left[\int\left(x - \frac{\frac{1}{9}x}{x^2+\frac{1}{9}}\right)dx\right] = \frac{x^3}{3}\operatorname{arc}\tau g3x - \frac{1}{9}\frac{x^2}{2} + \frac{1}{81}\int \frac{xdx}{x^2+\frac{1}{9}}$$

$$= \frac{x^3}{3}\operatorname{arc}\tau g3x - \frac{x^2}{18} + \frac{1}{162}\ell\eta\left|x^2+\frac{1}{9}\right| + c$$

4.48.- $\int x(\operatorname{arc}\tau gx)^2 dx$

Solución.-

$$u = (\operatorname{arc}\tau gx)^2 \qquad dv = xdx$$

$$\therefore \quad du = \frac{2\operatorname{arc}\tau gxdx}{1+x^2} \qquad v = \frac{x^2}{2}$$

$\int x(\operatorname{arc}\tau gx)^2 dx = \frac{x^2}{2}(\operatorname{arc}\tau gx)^2 - \int (\operatorname{arc}\tau gx)\frac{x^2dx}{1+x^2}$, integral la cual se desarrolla por partes, esto es:

$$u = \operatorname{arc}\tau gx \qquad dv = \frac{x^2dx}{1+x^2}$$

$$\therefore \quad du = \frac{dx}{1+x^2} \qquad v = x - \operatorname{arc}\tau gx$$

$$= \frac{(x\operatorname{arc}\tau gx)^2}{2} - \left[(x-\operatorname{arc}\tau gx)\operatorname{arc}\tau gx - \int (x-\operatorname{arc}\tau gx)\frac{dx}{1+x^2}\right]$$

$$= \frac{(x\operatorname{arc}\tau gx)^2}{2} - x\operatorname{arc}\tau gx + (\operatorname{arc}\tau gx)^2 + \int \frac{xdx}{1+x^2} - \int \frac{\operatorname{arc}\tau gxdx}{1+x^2}$$

$$= \frac{(x\operatorname{arc}\tau gx)^2}{2} - x\operatorname{arc}\tau gx + (\operatorname{arc}\tau gx)^2 + \frac{1}{2}\ell\eta(1+x^2) - \frac{(\operatorname{arc}\tau gx)^2}{2} + c$$

4.49.- $\int (\operatorname{arcs}e\operatorname{n}x)^2 dx$

Solución.-

$$u = (\operatorname{arcs}e\operatorname{n}x)^2 \qquad dv = dx$$

$$\therefore \quad du = \frac{2\operatorname{arcs}e\operatorname{n}xdx}{\sqrt{1-x^2}} \qquad v = x$$

$\int (\operatorname{arcs}e\operatorname{n}x)^2 dx = x(\operatorname{arcs}e\operatorname{n}x)^2 - 2\int \operatorname{arcs}e\operatorname{n}x\frac{xdx}{\sqrt{1-x^2}}$, integral la cual se desarrolla por partes, esto es: $\therefore$

$$u = \operatorname{arcs}e\operatorname{n}x \qquad dv = \frac{xdx}{\sqrt{1-x^2}}$$

$$du = \frac{dx}{\sqrt{1-x^2}} \qquad v = -\sqrt{1-x^2}$$

$$= x(\operatorname{arcs}e\operatorname{n}x)^2 - 2\left[-\sqrt{1-x^2}\operatorname{arcs}e\operatorname{n}x + \int dx\right]$$

$$= x(\operatorname{arcs}e\operatorname{n}x)^2 + 2\sqrt{1-x^2}\operatorname{arcs}e\operatorname{n}x - 2x + c$$

4.50.- $\int \frac{\arcsin x}{x^2}dx$

Solución.-

$$\therefore \quad u = \arcsin x \qquad dv = x^{-2}dx$$
$$du = \frac{dx}{\sqrt{1-x^2}} \qquad v = -\frac{1}{x}$$

$$\int \frac{\arcsin x}{x^2}dx = \int x^{-2}\arcsin x dx = -\frac{\arcsin x}{x} + \int \frac{dx}{x\sqrt{1-x^2}}$$

$$= -\frac{\arcsin x}{x} + \ell\eta\left|\frac{x}{1+\sqrt{1-x^2}}\right| + c$$

4.51.- $\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}}dx$

Solución.-

$$\therefore \quad u = \arcsin\sqrt{x} \qquad dv = \frac{dx}{\sqrt{1-x}}$$
$$du = \frac{dx}{\sqrt{1-x}}\frac{1}{2\sqrt{x}} \qquad v = -2\sqrt{1-x}$$

$$\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}}dx = -2\sqrt{1-x}\arcsin\sqrt{x} + \int\frac{dx}{\sqrt{x}} = -2\sqrt{1-x}\arcsin\sqrt{x} + 2\sqrt{x} + c$$

4.52.- $\int \frac{\operatorname{sen}^2 x}{e^x}dx$

Solución.-

$$\therefore \quad u = \operatorname{sen}^2 x \qquad dv = e^{-x}dx$$
$$du = 2\operatorname{sen} x\cos x \qquad v = -e^{-x}$$

$$\int \frac{\operatorname{sen}^2 x}{e^x}dx = \int \operatorname{sen}^2 x e^{-x}dx = -e^{-x}\operatorname{sen}^2 x + 2\int \operatorname{sen} x\cos x e^{-x}dx$$

$= -e^{-x}\operatorname{sen}^2 x + \not{2}\int \frac{\operatorname{sen} 2x}{\not{2}}e^{-x}dx$, $*$ Integral la cual se desarrolla por partes, esto es:

$$\therefore \quad u = \operatorname{sen} 2x \qquad dv = e^{-x}dx$$
$$du = 2\cos 2x dx \qquad v = -e^{-x}$$

$= -e^{-x}\operatorname{sen}^2 x + 2\int \cos 2x e^{-x}dx$, Integral la cual se desarrolla por partes, esto es:

$$\therefore \quad u = \cos 2x \qquad dv = e^{-x}dx$$
$$du = -2\operatorname{sen} 2x dx \qquad v = -e^{-x}$$

$$\int \operatorname{sen} 2x e^{-x}dx = -e^{-x}\operatorname{sen} 2x + 2\left(-e^{-x}\cos 2x - 2\int \operatorname{sen} 2x e^{-x}dx\right)$$

$\int \operatorname{sen} 2x e^{-x}dx = -e^{-x}\operatorname{sen} 2x - 2e^{-x}\cos 2x - 4\int \operatorname{sen} 2x e^{-x}dx$, de donde:

$$5\int \operatorname{sen} 2x e^{-x}dx = -e^{-x}(\operatorname{sen} 2x + 2\cos 2x) + c$$

$\int \operatorname{sen} 2x e^{-x} dx = \frac{-e^{-x}}{5}(\operatorname{sen} 2x + 2\cos 2x) + c$, Sustituyendo en: $*$

$$\int \frac{\operatorname{sen}^2 x dx}{e^x} = -e^{-x}\operatorname{sen}^2 x - \frac{2e^{-x}}{5}(\operatorname{sen} 2x + 2\cos 2x) + c$$

4.53.- $\int \tau g^2 x \sec^3 x dx = \int (\sec^2 x - 1)\sec^3 x dx = \int \sec^5 x dx (*) - \int \sec^3 x dx (**)$

Solución.-

$*\int \sec^5 x dx$, Sea: $u = \sec^3 x$ $\quad dv = \sec^2 x dx$

$du = 3\sec^3 x \tau g x dx$ $\quad v = \tau g x$

$$\int \sec^5 x dx = \int \sec^3 x \sec^2 x dx = \sec^3 x \tau g x - 3\int \sec^3 x \tau g^2 x dx$$

$**\int \sec^3 x dx$, Sea: $u = \sec x$ $\quad dv = \sec^2 x dx$

$du = \sec x \tau g x dx$ $\quad v = \tau g x$

$$\int \sec^3 x dx = \int \sec x \sec^2 x dx = \sec x \tau g x - \int \sec x \tau g^2 x dx = \sec x \tau g x - \int \sec x (\sec x^2 - 1) dx$$

$= \sec x \tau g x - \int \sec^3 x dx + \int \sec x dx$, luego: $2\int \sec^3 x dx = \sec x \tau g x + \int \sec x dx$

Esto es: $\int \sec^3 x dx = \frac{1}{2}(\sec x \tau g x + \ell n|\sec x \tau g x|) + c$, ahora bien:

$\int \tau g^2 x \sec^3 x dx = \int \sec^5 x dx - \int \sec^3 x dx$, con ($*$ y $**$)

$$\int \tau g^2 x \sec^3 x dx = \sec^3 x \tau g x - 3\int \sec^3 x \tau g^2 x dx - \frac{1}{2}(\sec x \tau g x + \ell n|\sec x \tau g x|) + c$$

De lo anterior: $4\int \tau g^2 x \sec^3 x dx = \sec^3 x \tau g x - \frac{1}{2}(\sec x \tau g x + \ell n|\sec x \tau g x|) + c$

Esto es: $\int \tau g^2 x \sec^3 x dx = \frac{1}{4}\sec^3 x \tau g x - \frac{1}{8}(\sec x \tau g x + \ell n|\sec x \tau g x|) + c$

4.54.- $\int x^3 \ell \eta^2 x dx$

Solución.-

$u = \ell \eta^2 x$ $\quad dv = x^3 dx$

$\therefore$ $du = \frac{2\ell \eta x}{x} dx$ $\quad v = \frac{x^4}{4}$

$\int x^3 \ell \eta^2 x dx = \frac{x^4}{4}\ell \eta^2 x - \frac{1}{2}\int x^3 \ell \eta x dx$, integral la cual se desarrolla por partes, esto es:

$u = \ell \eta x$ $\quad dv = x^3 dx$

$du = \frac{dx}{x}$ $\quad v = \frac{x^4}{4}$

$$= \frac{x^4}{4}\ell \eta^2 x - \frac{1}{2}\left(\frac{x^4}{4}\ell \eta x - \frac{1}{4}\int x^3 dx\right) = \frac{x^4}{4}\ell \eta^2 x - \frac{1}{8}x^4 \ell \eta x + \frac{1}{8}\frac{x^4}{4} + c$$

$$= \frac{x^4}{4}\ell \eta^2 x - \frac{1}{8}x^4 \ell \eta x + \frac{x^4}{32} + c$$

4.55.- $\int x\ell\eta(9+x^2)dx$

Solución.-

$u=\ell\eta(9+x^2)$ $\qquad dv=xdx$

$\therefore$ $du=\dfrac{2xdx}{9+x^2}$ $\qquad v=\dfrac{x^2}{2}$

$$\int x\ell\eta(9+x^2)dx=\frac{x^2}{2}\ell\eta(9+x^2)-\int\frac{x^3}{9+x^2}dx=\frac{x^2}{2}\ell\eta(9+x^2)-\int\left(x-\frac{9x}{x^2+9}\right)dx$$

$$=\frac{x^2}{2}\ell\eta(9+x^2)-\int xdx+9\int\frac{xdx}{9+x^2}=\frac{x^2}{2}\ell\eta(9+x^2)-\frac{x^2}{2}+\frac{9}{2}\ell\eta(x^2+9)+c$$

$$=\frac{x^2}{2}\left[\ell\eta(9+x^2)-1\right]+\frac{9}{2}\ell\eta(x^2+9)+c$$

4.56.- $\int \text{arcsen}\sqrt{x}dx$

Solución.-

$u=\text{arcsen}\sqrt{x}dx$ $\qquad dv=dx$

$\therefore$ $du=\dfrac{dx}{\sqrt{1-x^2}}\dfrac{1}{2\sqrt{x}}$ $\qquad v=x$

$$\int \text{arcsen}\sqrt{x}dx=x\,\text{arcsen}\sqrt{x}-\int\frac{xdx}{\sqrt{1-x}}\frac{1}{2\sqrt{x}}=x\,\text{arcsen}\sqrt{x}-\frac{1}{2}\int\frac{\sqrt{x}dx}{\sqrt{1-x}}$$

Para la integral resultante, se recomienda la siguiente sustitución: $\sqrt{1-x}=t$, de donde: $x=1-t^2$, y $dx=-2tdt$ (ver capitulo 9)

$=x\,\text{arcsen}\sqrt{x}-\dfrac{1}{2}\dfrac{\sqrt{1-t^2}(-2\not{t}dt)dx}{\not{t}}=x\,\text{arcsen}\sqrt{x}+\sqrt{1-t^2}dt$, Se recomienda la sustitución: $t=\text{sen}\,\theta$, de donde: $\sqrt{1-t^2}=\cos\theta$, y $dt=\cos\theta d\theta$. Esto es:

$$=x\,\text{arcsen}\sqrt{x}+\int\cos^2\theta d\theta=x\,\text{arcsen}\sqrt{x}+\frac{1}{2}\int(1+\cos2\theta)d\theta$$

$$=x\,\text{arcsen}\sqrt{x}+\frac{1}{2}\theta+\frac{1}{4}\text{sen}\,2\theta+c=x\,\text{arcsen}\sqrt{x}+\frac{1}{2}\theta+\frac{1}{2}\text{sen}\,\theta\cos\theta+c$$

$$=x\,\text{arcsen}\sqrt{x}+\frac{\text{arcsen}\,t}{2}+\frac{t}{2}\sqrt{1-t^2}+c=x\,\text{arcsen}\sqrt{x}+\frac{\text{arcsen}\sqrt{1-x}}{2}+\frac{\sqrt{1-x}}{2}\sqrt{x}+c$$

4.57.- $\int x\,\text{arc}\tau g(2x+3)dx$

Solución.-

$u=\text{arc}\tau g(2x+3)$ $\qquad dv=xdx$

$\therefore$ $du=\dfrac{2dx}{1+(2x+3)^2}$ $\qquad v=\dfrac{x^2}{2}$

$$\int x\,\text{arc}\tau g(2x+3)dx=\frac{x^2}{2}\text{arc}\tau g(2x+3)-\int\frac{x^2dx}{1+4x^2+12x+9}$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \int \frac{x^2dx}{4x^2+12x+10} = \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \int \left(\frac{1}{4} - \frac{3x+5/2}{4x^2+12x+10} \right) dx$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \frac{1}{4}\int dx + \int \frac{3x+5/2}{4x^2+12x+10}dx$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \frac{1}{4}x + 3\int \frac{x+5/6}{4x^2+12x+10}dx$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \frac{1}{4}x + \frac{3}{8}\int \frac{8x+40/6}{4x^2+12x+10}dx$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \frac{1}{4}x + \frac{3}{8}\int \frac{8x+12-32/6}{4x^2+12x+10}dx$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \frac{1}{4}x + \frac{3}{8}\int \frac{(8x+12)dx}{4x^2+12x+10} - \frac{3}{8}\frac{32}{6}\int \frac{dx}{4x^2+12x+10}$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \frac{1}{4}x + \frac{3}{8}\ell\eta\left|4x^2+12x+10\right| - 2\int \frac{dx}{4x^2+12x+10}$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \frac{1}{4}x + \frac{3}{8}\ell\eta\left|4x^2+12x+10\right| - 2\int \frac{dx}{(2x+3)^2+1}$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \frac{1}{4}x + \frac{3}{8}\ell\eta\left|4x^2+12x+10\right| - \frac{2}{2}\int \frac{2dx}{(2x+3)^2+1}$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g(2x+3) - \frac{1}{4}x + \frac{3}{8}\ell\eta\left|4x^2+12x+10\right| - \operatorname{arc}\tau g(2x+3) + c$$

$$= \frac{1}{2}\left[(x^2-2)\operatorname{arc}\tau g(2x+3) - \frac{1}{2}x + \frac{3}{4}\ell\eta\left|4x^2+12x+10\right| \right] + c$$

4.58.- $\int e^{\sqrt{x}}dx$

Solución.-

$$\therefore \quad u = e^{\sqrt{x}} \qquad du = \frac{e^{\sqrt{x}}dx}{2\sqrt{x}} \qquad dv = dx \qquad v = x$$

$\int e^{\sqrt{x}}dx = xe^{\sqrt{x}} - \frac{1}{2}\int \frac{xe^{\sqrt{x}}dx}{2\sqrt{x}}$, Se recomienda la sustitución: $z = \sqrt{x}, dz = \frac{dx}{2\sqrt{x}}$

$= xe^{\sqrt{x}} - \frac{1}{2}\int z^2e^z dz$, Esta integral resultante, se desarrolla por partes:

$$\therefore \quad u = z^2 \qquad du = 2zdz \qquad dv = e^z dz \qquad v = e^z$$

$= xe^{\sqrt{x}} - \frac{1}{2}\left(z^2e^z - 2\int ze^z dz\right) = xe^{\sqrt{x}} - \frac{z^2e^z}{2} + \int ze^z dz$, integral que se desarrolla por partes:

$\therefore \quad u = z \quad dv = e^z dz$

$du = dz \quad v = e^z$

$$= xe^{\sqrt{x}} - \frac{z^2 e^z}{2} + ze^z - \int e^z dz = xe^{\sqrt{x}} - \frac{z^2 e^z}{2} + ze^z - e^z + c = xe^{\sqrt{x}} - \frac{xe^{\sqrt{x}}}{2} + \sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$$

$$= e^{\sqrt{x}}\left(\frac{x}{2} + \sqrt{x} - 1\right) + c$$

4.59.- $\int \cos^2(\ell\eta x)dx$

Solución.-

$u = \cos(2\ell\eta x) \quad dv = dx$

$\therefore \quad du = -\frac{[\mathrm{sen}(2\ell\eta x)]2dx}{x} \quad v = x$

$$\int \cos^2(\ell\eta x)dx = \int \frac{1+\cos(2\ell\eta x)}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2\ell\eta x)dx$$

$$= \frac{1}{2}x + \frac{1}{2}\left[x\cos(2\ell\eta x) + 2\int \mathrm{sen}(2\ell\eta x)dx\right] = \frac{x}{2} + \frac{x}{2}\cos(2\ell\eta x) + \int \mathrm{sen}(2\ell\eta x)dx *$$

Integral que se desarrolla por partes:

$u = \mathrm{sen}(2\ell\eta x) \quad dv = dx$

$\therefore \quad du = -\frac{[\cos(2\ell\eta x)]2dx}{x} \quad v = x$

$$* = \frac{x}{2} + \frac{x}{2}\cos(2\ell\eta x) + x\,\mathrm{sen}(2\ell\eta x) - 2\int \cos(2\ell\eta x)dx,$$

Dado que apareció nuevamente: $\int \cos(2\ell\eta x)dx$, igualamos: $*$

$$\cancel{\frac{x}{2}} + \frac{1}{2}\int \cos(2\ell\eta x)dx = \cancel{\frac{x}{2}} + \frac{x}{2}\cos(2\ell\eta x) + x\,\mathrm{sen}(2\ell\eta x) - 2\int \cos(2\ell\eta x)dx$$, de donde:

$$\frac{5}{2}\int \cos(2\ell\eta x)dx = \frac{x}{2}\cos(2\ell\eta x) + x\,\mathrm{sen}(2\ell\eta x) + c$$

$$\frac{1}{2}\int \cos(2\ell\eta x)dx = \frac{x}{10}\cos(2\ell\eta x) + \frac{x}{5}\mathrm{sen}(2\ell\eta x) + c$$, Por tanto:

$$\int \cos^2(\ell\eta x)dx = \frac{x}{2} + \frac{x}{10}\cos(2\ell\eta x) + \frac{x}{5}\mathrm{sen}(2\ell\eta x) + c$$

4.60.- $\int \frac{\ell\eta(\ell\eta x)}{x}dx$, Sustituyendo por: $w = \ell\eta x, dw = \frac{dx}{x}$, Se tiene:

Solución.-

$\int \frac{\ell\eta(\ell\eta x)}{x}dx = \int \ell\eta w dw$, Esta integral se desarrolla por partes:

$u = \ell\eta w \quad dv = dw$

$\therefore \quad du = \frac{dw}{w} \quad v = w$

$$= w\ell\eta w - \int dw = w\ell\eta w - w + c = w(\ell\eta w - 1) + c = \ell\eta x[\ell\eta(\ell\eta x) - 1] + c$$

4.61.- $\int \ell\eta\left|x+1\right|dx$

Solución.-

$$\therefore \quad u=\ell\eta\left|x+1\right| \qquad dv=dx$$
$$du=\frac{dx}{x+1} \qquad v=x$$

$$\int \ell\eta\left|x+1\right|dx = x\ell\eta\left|x+1\right| - \int\frac{xdx}{x+1} = x\ell\eta\left|x+1\right| - \int\left(1-\frac{1}{x+1}\right)dx$$
$$= x\ell\eta\left|x+1\right| - x + \ell\eta\left|x+1\right| + c$$

4.62.- $\int x^2e^xdx$

Solución.-

$$\therefore \quad u=x^2 \qquad dv=e^xdx$$
$$du=2xdx \qquad v=e^x$$

$$\int x^2e^xdx = x^2e^x - 2\int xe^xdx$$

Integral que se desarrolla nuevamente por partes:

$$\therefore \quad u=x \qquad dv=e^xdx$$
$$du=dx \qquad v=e^x$$

$$= x^2e^x - 2\left[xe^x - \int e^xdx\right] = x^2e^x - 2xe^x + 2e^x + c$$

4.63.- $\int \cos^n xdx = \int\cos^{n-1}x\cos xdx$

Solución.-

$$\therefore \quad u=\cos^{n-1}x \qquad dv=\cos xdx$$
$$du=(n-1)\cos^{n-2}x(-\operatorname{sen}x)dx \qquad v=\operatorname{sen}x$$

$$= \cos^{n-1}x\operatorname{sen}x + (n-1)\int\operatorname{sen}^2x\cos^{n-2}xdx$$
$$= \cos^{n-1}x\operatorname{sen}x + (n-1)\int(1-\cos^2x)\cos^{n-2}xdx$$

$= \cos^{n-1}x\operatorname{sen}x + (n-1)\int\cos^{n-2}xdx - (n-1)\int\cos^nxdx$, Se tiene:

$\int\cos^nxdx = \cos^{n-1}x\operatorname{sen}x + (n-1)\int\cos^{n-2}xdx - (n-1)\int\cos^nxdx$, Esto es:

$$n\int\cos^nxdx = \cos^{n-1}x\operatorname{sen}x + (n-1)\int\cos^{n-2}xdx$$
$$\int\cos^nxdx = \frac{\cos^{n-1}x\operatorname{sen}x}{n} + \frac{(n-1)}{n}\int\cos^{n-2}xdx$$

4.64.- $\int\operatorname{sen}^nxdx = \int\operatorname{sen}^{n-1}x\operatorname{sen}xdx$

Solución.-

$$\therefore \quad u=\operatorname{sen}^{n-1}x \qquad dv=\operatorname{sen}xdx$$
$$du=(n-1)\operatorname{sen}^{n-2}x(\cos x)dx \qquad v=-\cos x$$

$$= -\operatorname{sen}^{n-1}x\cos x + (n-1)\int\cos^2x\operatorname{sen}^{n-2}xdx$$
$$= -\operatorname{sen}^{n-1}x\cos x + (n-1)\int(1-\operatorname{sen}^2x)\operatorname{sen}^{n-2}xdx$$

$= -\operatorname{sen}^{n-1} x\cos x + (n-1)\int \operatorname{sen}^{n-2} xdx - (n-1)\int \operatorname{sen}^{n} xdx$, Se tiene:

$$\int \operatorname{sen}^{n} xdx = -\operatorname{sen}^{n-1} x\cos x + (n-1)\int \operatorname{sen}^{n-2} xdx - (n-1)\int \operatorname{sen}^{n} xdx$$

$$n\int \operatorname{sen}^{n} xdx = -\operatorname{sen}^{n-1} x\cos x + (n-1)\int \operatorname{sen}^{n-2} xdx$$

$$\int \operatorname{sen}^{n} xdx = \frac{-\operatorname{sen}^{n-1} x\cos x}{n} + \frac{(n-1)}{n}\int \operatorname{sen}^{n-2} xdx$$

4.65.- $\int x^m (\ell\eta x)^n dx = x^{m+1}(\ell\eta x)^n - n\int x^m (\ell\eta x)^{n-1} dx - m\int x^m (\ell\eta x)^n dx$

Solución.-

$\therefore$ $u = x^m (\ell\eta x)^n$ $\quad dv = dx$

$du = x^m n(\ell\eta x)^{n-1}\frac{dx}{x} + mx^{m-1}(\ell\eta x)^n dx$ $\quad v = x$

Se tiene: $(m+1)\int x^m (\ell\eta x)^n dx = x^{m+1}(\ell\eta x)^n - n\int x^m (\ell\eta x)^{n-1} dx$

$$\int x^m (\ell\eta x)^n dx = \frac{x^{m+1}(\ell\eta x)^n}{(m+1)} - \frac{n}{(m+1)}\int x^m (\ell\eta x)^{n-1} dx$$

4.66.- $\int x^3 (\ell\eta x)^2 dx$

Solución.-

Puede desarrollarse como caso particular del ejercicio anterior, haciendo: $m = 3, n = 2$

$$\int x^3(\ell\eta x)^2 dx = \frac{x^{3+1}(\ell\eta x)^2}{3+1} - \frac{2}{3+1}\int x^3(\ell\eta x)^{2-1} dx = \frac{x^4(\ell\eta x)^2}{4} - \frac{1}{2}\int x^3(\ell\eta x)dx *$$

Para la integral resultante: $\int x^3(\ell\eta x)dx *$

$\int x^3(\ell\eta x)dx = \frac{x^4(\ell\eta x)}{4} - \frac{1}{4}\int x^3 dx = \frac{x^4(\ell\eta x)}{4} - \frac{x^4}{16} + c$, introduciendo en: $*$

$$\int x^3(\ell\eta x)^2 dx = \frac{x^4(\ell\eta x)^2}{4} - \frac{x^4}{8}(\ell\eta x) + \frac{x^4}{32} + c$$

4.67.- $\int x^n e^x dx$

Solución.-

$\therefore$ $u = x^n$ $\quad dv = e^x dx$

$du = nx^{n-1}dx$ $\quad v = e^x$

$$\int x^n e^x dx = x^n e^x - n\int x^{n-1} e^x dx$$

4.68.- $\int x^3 e^x dx$

Solución.-

$\therefore$ $u = x^3$ $\quad dv = e^x dx$

$du = 3x^2 dx$ $\quad v = e^x$

Puede desarrollarse como el ejercicio anterior, haciendo: $n = 3$

$\int x^3 e^x dx = x^3 e^x - 3\int x^2 e^x dx *$, Además:

$*\int x^2e^xdx = x^2e^x - 2\int xe^xdx **$, Además: $\int xe^xdx = xe^x - \int e^xdx = xe^x - e^x + c$

Reemplazando en $**$ y luego en $*$:

$\int x^3e^xdx = x^3e^x - 3\left[x^2e^x - 2(xe^x - e^x)\right] + c$

$\int x^3e^xdx = e^x(x^3 - 3x^2 + 6x - 6) + c$

4.69.- $\int \sec^n xdx = \int \sec^{n-2} x\sec^2 xdx$

Solución.-

$$\therefore \quad u = \sec^{n-2} x \qquad dv = \sec^2 xdx$$
$$du = (n-2)\sec^{n-3} x\sec x\tau gxdx \qquad v = \tau gx$$

$= \sec^{n-2} x\tau gx - (n-2)\int \tau g^2 x\sec^{n-2} xdx = \sec^{n-2} x\tau gx - (n-2)\int (\sec^2 x - 1)\sec^{n-2} xdx$

$= \sec^{n-2} x\tau gx - (n-2)\int \sec^n xdx + (n-2)\int \sec^{n-2} xdx$, Se tiene:

$\int \sec^n xdx = \sec^{n-2} x\tau gx - (n-2)\int \sec^n xdx + (n-2)\int \sec^{n-2} xdx$

$(n-1)\int \sec^n xdx = \sec^{n-2} x\tau gx + (n-2)\int \sec^{n-2} xdx$

$$\int \sec^n xdx = \frac{\sec^{n-2} x\tau gx}{(n-1)} + \frac{(n-2)}{(n-1)}\int \sec^{n-2} xdx$$

4.70.- $\int \sec^3 xdx$

Solución.-

Puede desarrollarse como caso particular del ejercicio anterior, haciendo: $n = 3$

$$\int \sec^3 xdx = \frac{\sec^{3-2} x\tau gx}{3-1} + \frac{3-2}{3-1}\int \sec^{3-2} xdx = \frac{\sec x\tau gx}{2} + \frac{1}{2}\int \sec xdx$$

$$= \frac{\sec x\tau gx}{2} + \frac{1}{2}\ell\eta\left|\sec x\tau gx\right| + c$$

4.71.- $\int x\ell\eta xdx$

Solución.-

$$\therefore \quad u = \ell\eta x \qquad dv = xdx$$
$$du = \frac{dx}{x} \qquad v = \frac{x^2}{2}$$

$$\int x\ell\eta xdx = \frac{x^2}{2}\ell\eta x - \int \frac{xdx}{2} = \frac{x^2}{2}\ell\eta x - \frac{1}{4}x^2 + c$$

4.72.- $\int x^n\ell\eta\left|ax\right|dx, n \neq -1$

Solución.-

$$\therefore \quad u = \ell\eta\left|ax\right| \qquad dv = xdx$$
$$du = \frac{dx}{x} \qquad v = \frac{x^{n+1}}{n+1}$$

$$\int x^n\ell\eta\left|ax\right|dx = \frac{x^{n+1}}{n+1}\ell\eta\left|ax\right| - \frac{1}{n+1}\int x^ndx = \frac{x^{n+1}}{n+1}\ell\eta\left|ax\right| - \frac{x^{n+1}}{(n+1)^2} + c$$

4.73.- $\int \arcsin ax\,dx$

Solución.-

$$\therefore \quad u = \arcsin ax, \quad du = \frac{adx}{\sqrt{1-a^2x^2}} \qquad dv = dx, \quad v = x$$

$$\int \arcsin ax\,dx = x\arcsin ax - \int \frac{axdx}{\sqrt{1-a^2x^2}} = x\arcsin ax + \frac{1}{2a}\int \frac{(-2a^2x)dx}{\sqrt{1-a^2x^2}}$$

$$= x\arcsin ax + \frac{1}{2a}\frac{(1-a^2x^2)^{1/2}}{1/2} + c = x\arcsin ax + \frac{1}{a}\sqrt{1-a^2x^2} + c$$

4.74.- $\int x\,\mathrm{sen}\,ax\,dx$

Solución.-

$$\therefore \quad u = x, \quad du = dx \qquad dv = \mathrm{sen}\,ax\,dx, \quad v = -\frac{1}{a}\cos ax$$

$$\int x\,\mathrm{sen}\,ax\,dx = -\frac{x}{a}\cos ax + \frac{1}{a}\int \cos ax\,dx = -\frac{x}{a}\cos ax + \frac{1}{a^2}\mathrm{sen}\,ax + c$$

$$= \frac{1}{a^2}\mathrm{sen}\,ax - \frac{x}{a}\cos ax + c$$

4.75.- $\int x^2\cos ax\,dx$

Solución.-

$$\therefore \quad u = x^2, \quad du = 2xdx \qquad dv = \cos ax\,dx, \quad v = -\frac{1}{a}\mathrm{sen}\,ax$$

$\int x^2\cos ax\,dx = \frac{x^2}{a}\mathrm{sen}\,ax - \frac{2}{a}\int x\,\mathrm{sen}\,ax\,dx$, aprovechando el ejercicio anterior:

$$= \frac{x^2}{a}\mathrm{sen}\,ax - \frac{2}{a}\left(\frac{1}{a^2}\mathrm{sen}\,ax - \frac{x}{a}\cos ax\right) + c = \frac{x^2}{a}\mathrm{sen}\,ax - \frac{2}{a^3}\mathrm{sen}\,ax - \frac{2x}{a^2}\cos ax + c$$

4.76.- $\int x\sec^2 ax\,dx$

Solución.-

$$\therefore \quad u = x, \quad du = dx \qquad dv = \sec^2 ax\,dx, \quad v = \frac{1}{a}\tau g ax$$

$$\int x\sec^2 ax\,dx = \frac{x}{a}\tau g ax - \frac{1}{a}\int \tau g ax\,dx = \frac{x}{a}\tau g ax - \frac{1}{a}\frac{1}{a}\ell\eta\left|\sec ax\right| + c$$

$$= \frac{x}{a}\tau g ax - \frac{1}{a^2}\ell\eta\left|\sec ax\right| + c$$

4.77.- $\int \cos(\ell\eta x)dx$

Solución.-

$$\therefore \quad u=\cos(\ell\eta x) \qquad dv = dx$$
$$du = -\frac{\operatorname{sen}(\ell\eta x)}{x}dx \qquad v = x$$

$\int \cos(\ell\eta x)dx = x\cos(\ell\eta x) + \int \operatorname{sen}(\ell\eta x)dx$, aprovechando el ejercicio:4.43

$\int \operatorname{sen}(\ell\eta x)dx = \frac{x}{2}\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] + c$, Luego:

$$= x\cos(\ell\eta x) + \frac{x}{2}\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] + c = x\cos(\ell\eta x) + \frac{x}{2}\operatorname{sen}(\ell\eta x) - \frac{x}{2}\cos(\ell\eta x) + c$$

$$= \frac{x}{2}\left[\cos(\ell\eta x) + \operatorname{sen}(\ell\eta x)\right] + c$$

4.78.- $\int \ell\eta(9+x^2)dx$

Solución.-

$$\therefore \quad u = \ell\eta(9+x^2) \qquad dv = dx$$
$$du = \frac{2xdx}{9+x^2} \qquad v = x$$

$$\int \ell\eta(9+x^2)dx = x\ell\eta(9+x^2) - 2\int \frac{x^2dx}{9+x^2} = x\ell\eta(9+x^2) - 2\int\left(1 - \frac{9}{9+x^2}\right)dx$$

$$= x\ell\eta(9+x^2) - 2\int dx + 18\int \frac{dx}{9+x^2} = x\ell\eta(9+x^2) - 2x + 6\operatorname{arc}\tau g\, {}^{x}\!/_{3} + c$$

4.79.- $\int x\cos(2x+1)dx$

Solución.-

$$\therefore \quad u = x \qquad dv = \cos(2x+1)dx$$
$$du = dx \qquad v = \frac{1}{2}\operatorname{sen}(2x+1)$$

$$\int x\cos(2x+1)dx = \frac{x}{2}\operatorname{sen}(2x+1) - \frac{1}{2}\int \operatorname{sen}(2x+1)dx$$

$$= \frac{x}{2}\operatorname{sen}(2x+1) + \frac{1}{4}\cos(2x+1) + c$$

4.80.- $\int x \operatorname{arc}\sec x dx$

Solución.-

$$\therefore \quad u = \operatorname{arc}\sec x \qquad dv = xdx$$
$$du = \frac{dx}{x\sqrt{x^2-1}} \qquad v = \frac{x^2}{2}$$

$$\int x\operatorname{arc}\sec x dx = \frac{x^2}{2}\operatorname{arc}\sec x - \frac{1}{2}\int \frac{xdx}{\sqrt{x^2-1}} = \frac{x^2}{2}\operatorname{arc}\sec x - \frac{1}{2}\sqrt{x^2-1} + c$$

4.81.- $\int \operatorname{arc}\sec\sqrt{x}dx$

Solución.-

$$\therefore \quad u = \operatorname{arcsec} x \qquad dv = dx$$
$$du = \frac{1}{2}\frac{dx}{x\sqrt{x-1}} \qquad v = x$$

$$\int \operatorname{arcsec}\sqrt{x}dx = x\operatorname{arcsec} x - \frac{1}{2}\int\frac{dx}{\sqrt{x-1}} = x\operatorname{arcsec} x - \sqrt{x-1} + c$$

4.82.- $\int\sqrt{a^2-x^2}dx = \int\frac{a^2-x^2}{\sqrt{a^2-x^2}}dx = a^2\int\frac{dx}{\sqrt{a^2-x^2}} - \int\frac{x^2dx}{\sqrt{a^2-x^2}}$

$= a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} - \int x\frac{xdx}{\sqrt{a^2-x^2}} *$, integral que se desarrolla por partes:

Solución.-

$$\therefore \quad u = x \qquad dv = \frac{xdx}{\sqrt{a^2-x^2}}$$
$$du = dx \qquad v = -\sqrt{a^2-x^2}$$

$* = a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} - \left(-x\sqrt{a^2-x^2} + \int\sqrt{a^2-x^2}dx\right)$, Se tiene que:

$\int\sqrt{a^2-x^2}dx = a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} + x\sqrt{a^2-x^2} - \int\sqrt{a^2-x^2}dx$, De donde:

$$2\int\sqrt{a^2-x^2}dx = a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} + x\sqrt{a^2-x^2} + c$$

$$\int\sqrt{a^2-x^2}dx = \frac{a^2}{2}\operatorname{arcs}e\operatorname{n}\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + c$$

4.83.- $\int \ell\eta\left|1-x\right|dx$

Solución.-

$$\therefore \quad u = \ell\eta\left|1-x\right| \qquad dv = dx$$
$$du = -\frac{dx}{1-x} \qquad v = x$$

$$\int \ell\eta\left|1-x\right|dx = x\ell\eta\left|1-x\right| - \int\frac{xdx}{x-1} = x\ell\eta\left|1-x\right| - \int\left(1+\frac{1}{x-1}\right)dx$$

$$= x\ell\eta\left|1-x\right| - \int dx - \int\frac{dx}{x-1} = x\ell\eta\left|1-x\right| - x - \ell\eta\left|x-1\right| + c$$

4.84.- $\int \ell\eta(x^2+1)dx$

Solución.-

$$\therefore \quad u = \ell\eta(x^2+1) \qquad dv = dx$$
$$du = \frac{2xdx}{x^2+1} \qquad v = x$$

$$\int \ell\eta(x^2+1)dx = x\ell\eta(x^2+1) - 2\int\frac{x^2dx}{x^2+1} = x\ell\eta(x^2+1) - 2\int\left(1-\frac{1}{x^2+1}\right)dx$$

$$= x\ell\eta(x^2+1) - 2x + 2\operatorname{arc}\tau gx + c$$

4.85.- $\int \text{arc}\,\tau g\sqrt{x}dx$

Solución.-

$$\therefore \quad u = \text{arc}\,\tau g\sqrt{x} \qquad dv = dx$$
$$du = \frac{dx}{1+x}\frac{1}{2\sqrt{x}} \qquad v = x$$

$\int \text{arc}\,\tau g\sqrt{x}dx = x\,\text{arc}\,\tau g\sqrt{x} - \frac{1}{2}\int\frac{\sqrt{x}dx}{1+x}*$ En la integral resultante, se recomienda la sustitución: $\sqrt{x} = t$, esto es $x = t^2, dx = 2tdt$

$$= x\,\text{arc}\,\tau g\sqrt{x} - \frac{1}{2}\int\frac{t\,2tdt}{1+t^2} = x\,\text{arc}\,\tau g\sqrt{x} - \int\frac{t^2dt}{1+t^2} = x\,\text{arc}\,\tau g\sqrt{x} - \int\left(1-\frac{1}{1+t^2}\right)dt$$

$$= x\,\text{arc}\,\tau g\sqrt{x} - \int dt + \int\frac{dt}{1+t^2} = x\,\text{arc}\,\tau g\sqrt{x} - t + \text{arc}\,\tau g t + c$$

$$= x\,\text{arc}\,\tau g\sqrt{x} - \sqrt{x} + \text{arc}\,\tau g\sqrt{x} + c$$

4.86.- $\int \frac{x\,\text{arcs}\,e\,\text{n}\,x}{\sqrt{1-x^2}}dx$

Solución.-

$$\therefore \quad u = \text{arcs}\,e\,\text{n}\,x \qquad dv = \frac{xdx}{\sqrt{1-x^2}}$$
$$du = \frac{dx}{\sqrt{1-x^2}} \qquad v = -\sqrt{1-x^2}$$

$$\int\frac{x\,\text{arcs}\,e\,\text{n}\,x}{\sqrt{1-x^2}}dx = -\sqrt{1-x^2}\,\text{arcs}\,e\,\text{n}\,x + \int dx = -\sqrt{1-x^2}\,\text{arcs}\,e\,\text{n}\,x + x + c$$

4.87.- $\int x\,\text{arc}\,\tau g\sqrt{x^2-1}dx$

Solución.-

$$\therefore \quad u = \text{arc}\,\tau g\sqrt{x^2-1} \qquad dv = xdx$$
$$du = \frac{dx}{x\sqrt{x^2-1}} \qquad v = \frac{x^2}{2}$$

$$\int x\,\text{arc}\,\tau g\sqrt{x^2-1}dx = \frac{x^2}{2}\text{arc}\,\tau g\sqrt{x^2-1} - \frac{1}{2}\int\frac{xdx}{\sqrt{x^2-1}} = \frac{x^2}{2}\text{arc}\,\tau g\sqrt{x^2-1} - \frac{1}{2}\sqrt{x^2-1} + c$$

4.88.- $\int \frac{x\,\text{arc}\,\tau gx}{(x^2+1)^2}dx$

Solución.-

$$\therefore \quad u = \text{arc}\,\tau gx \qquad dv = \frac{xdx}{(x^2+1)^2}$$
$$du = \frac{dx}{x^2+1} \qquad v = \frac{-1}{2(x^2+1)}$$

$\int\frac{x\,\text{arc}\,\tau gx}{(x^2+1)^2}dx = \frac{-\text{arc}\,\tau gx}{2(x^2+1)} + \frac{1}{2}\int\frac{dx}{(x^2+1)^2}*$, Se recomienda la siguiente sustitución:

$x = \tau g\theta$, de donde: $dx = \sec^2\theta d\theta$; $x^2+1 = \sec^2\theta$

$$* = \frac{-\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{2}\int \frac{\sec^2\theta d\theta}{\sec^4\theta} = -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{2}\int \cos^2\theta d\theta = -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{2}\int \frac{1+\cos 2\theta d\theta}{2}$$

$$= -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{4}\theta + \frac{1}{8}\operatorname{sen} 2\theta + c = -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{4}\operatorname{arc}\tau gx + \frac{1}{4}\operatorname{sen}\theta\cos\theta + c$$

$$= -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{4}\operatorname{arc}\tau gx + \frac{1}{4}\frac{x}{\sqrt{x^2+1}}\frac{1}{\sqrt{x^2+1}} + c$$

$$= -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{4}\operatorname{arc}\tau gx + \frac{x}{4(x^2+1)} + c$$

4.89.- $\int \operatorname{arcsen} x \frac{xdx}{\sqrt{(1-x^2)^3}}$

Solución.-

$$u = \operatorname{arcsen} x \qquad dv = \frac{xdx}{(1-x^2)^{3/2}}$$

$$\therefore \quad du = \frac{dx}{\sqrt{1-x^2}} \qquad v = \frac{1}{\sqrt{1-x^2}}$$

$$\int \operatorname{arcsen} x \frac{xdx}{\sqrt{(1-x^2)^3}} = \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}} - \int \frac{dx}{1-x^2} = \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}} + \frac{1}{2}\ell\eta\left|\frac{1-x}{1+x}\right| + c$$

4.90.- $\int x^2\sqrt{1-x}dx$

Solución.-

$$u = \sqrt{1-x} \qquad dv = x^2 dx$$

$$\therefore \quad du = -\frac{dx}{2\sqrt{1-x}} \qquad v = \frac{x^3}{3}$$

$\int x^2\sqrt{1-x}dx = \frac{x^3}{3}\sqrt{1-x} + \frac{1}{6}\int \frac{x^3dx}{\sqrt{1-x}} *$, Se recomienda usar la siguiente sustitución: $\sqrt{1-x} = t$, o sea: $x = 1-t^2$, De donde: $dx = -2tdt$

$$= \frac{x^3}{3}\sqrt{1-x} + \frac{1}{6}\int \frac{(1-t^2)^3(-2tdt)}{t} = \frac{x^3}{3}\sqrt{1-x} - \frac{1}{3}\int (1-t^2)^3 dt$$

$$= \frac{x^3}{3}\sqrt{1-x} - \frac{1}{3}\int (1-3t^2+3t^4-t^6)dt = \frac{x^3}{3}\sqrt{1-x} - \frac{1}{3}(t - t^3 + \frac{3t^5}{5} - \frac{t^7}{7}) + c$$

$$= \frac{x^3}{3}\sqrt{1-x} - \frac{1}{3}\left[\sqrt{1-x} - (1-x)\sqrt{1-x} + \frac{3}{5}(1-x)^2\sqrt{1-x} - \frac{3}{7}(1-x)^3\sqrt{1-x}\right] + c$$

$$= \frac{\sqrt{1-x}}{3}\left[x^3 - 1 - (1-x) + \frac{3}{5}(1-x)^2 - \frac{1}{7}(1-x)^3\right] + c$$