1. **a)** Operar en forma binómica y simplificar: $\dfrac{(2-3i)^2-(2+3i)(3-2i)}{3i^{17}-1}-\dfrac{4}{5i^{-25}}$

   b) Operar en forma polar y pasar el resultado a binómica (para pasar a polar, dibujar previamente los complejos; al pasar a binómica, justificar todos los cálculos trigonométricos. No vale usar calculadora): $\dfrac{(-2\sqrt{3}-2i)^4}{(-1+\sqrt{3}\,i)^3\,(2-2i)^2}$ *(1,75 puntos)*

2. **a)** Dada $f(x)=\dfrac{2x+4}{x-2}$ se pide: **i)** $\lim\limits_{x\to-\infty} f(x)$ y $\lim\limits_{x\to\infty} f(x)$ <u>analíticamente</u>. ¿Qué A.H. presenta? **ii)** $\lim\limits_{x\to 2} f(x)$ <u>analíticamente</u>. ¿Qué A.V. presenta? **iii)** Cortes con los ejes **iv)** Con la información anterior (<u>no vale tabla de valores</u>), representarla.

   b) Dada $f(x)=|x^2-4x-5|$, se pide: **i)** Definición analítica por ramas. **ii)** Gráfica. *(1,75 puntos)*

3. Dada la función que figura a continuación, se pide: **a)** Gráfica **b)** Dom(f) e Im(f) **c)** Cortes con los ejes **d)** Intervalos de crecimiento. M y m **e)** Continuidad **f)** Ecuación de las posibles asíntotas **g)** Hallar, <u>analíticamente</u>, $\lim\limits_{x\to -3} f(x)$, $\lim\limits_{x\to 0} f(x)$ y $\lim\limits_{x\to 2} f(x)$ *(1,75 puntos)*

$$f(x)=\begin{cases} x^2+8x+7 & \text{si } x<-3 \\ x-5 & \text{si } -3\le x<2 \\ \sqrt{x-2} & \text{si } x\ge 2 \end{cases}$$

4. **a)** Hallar, razonadamente, $\log_2 64$ **b)** Ídem: $\log_3 \dfrac{\sqrt{3}}{9}$ **c)** Ídem: $\mathrm{Ln}\dfrac{e}{\sqrt[3]{e^2}}$ **d)** ¿En qué base se cumple que $\log_a 12+\log_a 3=2$? *(1,25 puntos)*

5. Resolver: **a)** $2^{x-3}=3^{x+1}$ **b)** $9^x+2\cdot 3^{x+1}=27$ **c)** $\log x^2+\log x^3=5$ *(1,75 puntos)*

6. Calcular: **a)** $\lim\limits_{x\to-2}\dfrac{x^2-4}{x^3+3x^2-4}$ **b)** $\lim\limits_{x\to-\infty}\dfrac{x^2-4}{x^3+3x^2-4}$ **c)** $\lim\limits_{x\to-\infty}\left(1+\dfrac{1}{e^x}\right)$ *(1,75 puntos)*

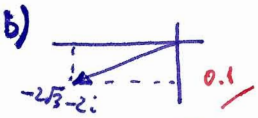
① a) $\dfrac{(2-3i)^2-(2+3i)(3-2i)}{3i^{17}-1}-\dfrac{4}{5i^{-25}}=\dfrac{4-12i+9i^2-(6-4i+9i-6i^2)}{3i^{17}-1}-\dfrac{4i^{25}}{5}=\dfrac{-5-12i-(12+5i)}{-1+3i}-\dfrac{4i}{5}=$

$=\dfrac{-17-17i}{-1+3i}-\dfrac{4i}{5}=\dfrac{(-17-17i)(-1-3i)}{(-1+3i)(-1-3i)}-\dfrac{4i}{5}=\dfrac{17+51i+17i+51i^2}{1-9i^2}-\dfrac{4i}{5}=\dfrac{-34+68i}{10}-\dfrac{4i}{5}=\dfrac{-34+68i-8i}{10}=$ 0.1 0.1 0.1 0.1

$=-\dfrac{34}{10}+\dfrac{60i}{10}=\boxed{-\dfrac{17}{5}+6i}$ 0.175

(0.875 cada apdo) $\boxed{1,75}$

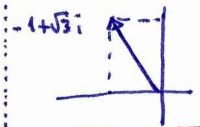
b)



0.1

$r=\sqrt{12+4}=\sqrt{16}=\boxed{4}$

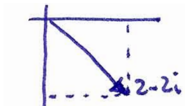
$\alpha=\operatorname{arctg}\dfrac{-2}{-2\sqrt3}=\operatorname{arctg}\dfrac{1}{\sqrt3}=\boxed{210^\circ}$



0.1

$r=\sqrt{1+3}=\sqrt4=\boxed{2}$

$\alpha=\operatorname{arctg}\dfrac{\sqrt3}{-1}=\operatorname{arctg}(-\sqrt3)=\boxed{120^\circ}$



0.1

$r=\sqrt{4+4}=\sqrt8=\boxed{2\sqrt2}$

$\alpha=\operatorname{arctg}\dfrac{-2}{2}=\operatorname{arctg}(-1)=\boxed{315^\circ}$

$\cos 210^\circ=\cos(180^\circ+30^\circ)=-\cos30^\circ=-\dfrac{\sqrt3}{2}$

$\operatorname{sen}210^\circ=\operatorname{sen}(180^\circ+30^\circ)=-\operatorname{sen}30^\circ=-\dfrac12$

$\dfrac{(-2\sqrt3-2i)^4}{(-1+\sqrt3 i)^3\cdot(2-2i)^2}=\dfrac{(4_{210^\circ})^4}{(2_{120^\circ})^3\cdot(2\sqrt2_{\,315^\circ})^2}=\dfrac{256_{840^\circ}}{8_{360^\circ}\cdot 8_{630^\circ}}=4_{-150^\circ}=\boxed{4_{210^\circ}}=4(\cos210^\circ+i\operatorname{sen}210^\circ)=4\left(-\dfrac{\sqrt3}{2}-\dfrac12 i\right)=\boxed{-2\sqrt3-2i}$ 0.1 0.1 0.1 0.175

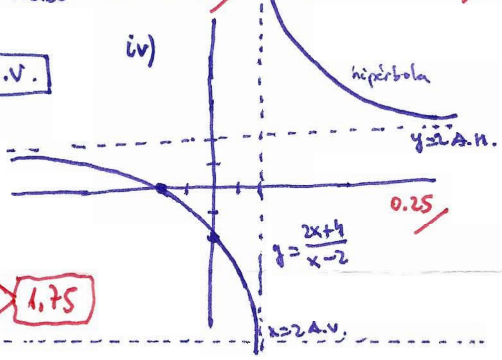
② a) $f(x)=\dfrac{2x+4}{x-2}$ (hipérbola) i) $\boxed{\lim\limits_{x\to-\infty}\dfrac{2x+4}{x-2}\overset{-\infty/-\infty}{\underset{\text{indet.}}{\sim}}\lim\limits_{x\to-\infty}\dfrac{2\cancel{x}}{\cancel{x}}=2}$ 0.1 ; $\boxed{\lim\limits_{x\to\infty}\dfrac{2x+4}{x-2}\overset{\infty/\infty}{\underset{\text{indet.}}{\sim}}\lim\limits_{x\to\infty}\dfrac{2\cancel{x}}{\cancel{x}}=2}$ 0.1 $\Rightarrow\boxed{y=2 \text{ A.H.}}$ 0.1

ii) $\lim\limits_{x\to2}\dfrac{2x+4}{x-2}=\dfrac{8}{0}=\begin{cases}\lim\limits_{x\to2^-}\dfrac{2x+4}{x-2}=\dfrac{8}{0^-}=-\infty\\ \lim\limits_{x\to2^+}\dfrac{2x+4}{x-2}=\dfrac{8}{0^+}=\infty\end{cases}$ 0.2 $\Rightarrow\boxed{\nexists\lim\limits_{x\to2}\dfrac{2x+4}{x-2}}\Rightarrow\boxed{x=2 \text{ A.V.}}$ 0.1

iii) corte eje x: $y=0\Rightarrow\dfrac{2x+4}{x-2}=0$; $2x+4=0$; $x=-2\to\boxed{(-2,0)}$ 0.1

corte eje y: $x=0\Rightarrow y=\dfrac{4}{-2}=-2\to\boxed{(0,-2)}$ 0.1

iv)



0.25

$\boxed{1{,}05}$ $\boxed{0{,}7}$ $\boxed{1{,}75}$

b) i) $f(x)=|x^2-4x-5|=\begin{cases}x^2-4x-5 & \text{si } x\in(-\infty,-1]\\ -x^2+4x+5 & \text{si } x\in(-1,5]\\ x^2-4x-5 & \text{si } x\in(5,\infty)\end{cases}$

raíces -1 y 5

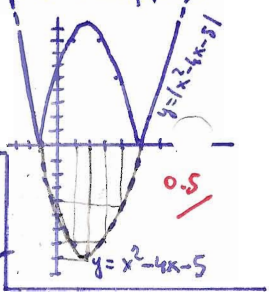
-1 5 / $+$ $-$ $+$

$f(x)=\begin{cases}x^2-4x-5 & \text{si } x\in(-\infty,-1]\cup(5,\infty)\\ -x^2+4x+5 & \text{si } x\in(-1,5]\end{cases}$ 0.2

ii) Para representar $|x^2-4x-5|$, representaremos la parábola $y=x^2-4x-5$, y a continuación reflejaremos su parte negativa en el semiplano positivo:

Vértice

x	…-3	-2	-1	0	1	2	3	4	5	6	7…
$y=x^2-4x-5$	…16	7	0	-5	-8	-9	-8	-5	0	7	16…



0.5

③ $f(x)=\begin{cases}x^2+8x+7 & \text{si } x<-3\\ x-5 & \text{si } -3\le x<2\\ \sqrt{x-2} & \text{si } x\ge2\end{cases}$

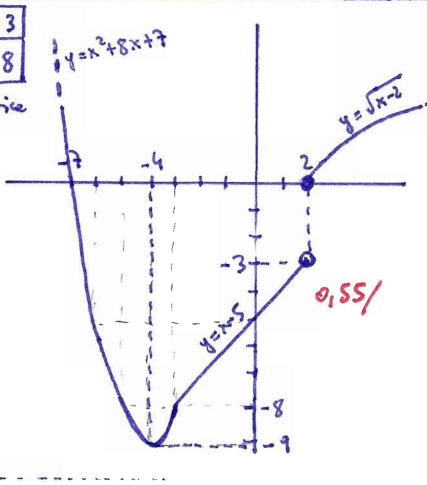
a)

x	…-9	-8	-7	-6	-5	-4	-3
$y=x^2+8x+7$	…16	7	0	-5	-8	-9	-8

Vértice

x	-3	2
$y=x-5$	-8	-3

x	2	3	4	5	6	7	8…∞
$y=\sqrt{x-2}$	0	1	1,41	1,73	2	2,24	2,45…∞



0,55

b) $\boxed{\operatorname{Dom}(f)=\mathbb{R}}$; $\boxed{\operatorname{Im}(f)=[-9,\infty)}$ 0.2

c) corte eje x: $\boxed{(-7,0)\text{ y }(2,0)}$; corte eje y: $\boxed{(0,-5)}$ 0.1

d) $f(x)\searrow\ \forall x\in(-\infty,-4)$; $f(x)\nearrow\ \forall x\in(-4,\infty)$ $\Rightarrow m(-4,-9)$ 0.2

e) $\boxed{f(x)\text{ discontinua en } x=2}$ 0.1

f) $\nexists$ asíntotas 0.1

$\boxed{1{,}75}$

g) $\lim\limits_{x\to-3^-}f(x)=\lim\limits_{x\to-3}(x^2+8x+7)=-8$; $\lim\limits_{x\to-3^+}f(x)=\lim\limits_{x\to-3}(x-5)=-8$ $\Rightarrow\boxed{\lim\limits_{x\to-3}f(x)=-8}$ 0.2 ; $\boxed{\lim\limits_{x\to0}f(x)=\lim\limits_{x\to0}(x-5)=-5}$ 0.1

$\lim\limits_{x\to2^-}f(x)=\lim\limits_{x\to2}(x-5)=-3$; $\lim\limits_{x\to2^+}f(x)=\lim\limits_{x\to2}\sqrt{x-2}=0$ $\Rightarrow\boxed{\nexists\lim\limits_{x\to2}f(x)}$ 0.2

④ a) $\boxed{\log_2 64 = 6}$ pq. $2^6 = 64$ 0.3125/

b) $\boxed{\log_3 \frac{\sqrt{3}}{9} = \log_3 \sqrt{3} - \log_3 9 = \frac{1}{2} \cancel{\log_3} 3 - \cancel{\log_3} 9 = \frac{1}{2} - 2 = -\frac{3}{2}}$ 0.3125/

$1{,}25$

c) $\boxed{Ln \frac{e}{\sqrt[3]{e^2}} = Lne - Ln\sqrt[3]{e^2} = Lne - \frac{1}{3} Lne^2 = \cancel{Ln}e - \frac{2}{3} \cancel{Ln}e = 1 - \frac{2}{3} = \frac{1}{3}}$ 0.3125/

d) $\log_a 12 + \log_a 3 = 2$; $\log_a(12 \cdot 3) = 2$; $\log_a 36 = 2 \Rightarrow a^2 = 36$; $\boxed{a = 6}$ (se descarta $a = -6$) 0.3125/

---

⑤ a) $2^{x-3} = 3^{x+1}$; $\log 2^{x-3} = \log 3^{x+1}$; $(x-3)\log 2 = (x+1)\log 3$; $x\log 2 - 3\log 2 = x\log 3 + \log 3$

$x\log 2 - x\log 3 = \log 3 + 3\log 2$; $x(\log 2 - \log 3) = \log 3 + 3\log 2$; $\boxed{x = \frac{\log 3 + 3\log 2}{\log 2 - \log 3}} \simeq \boxed{-7{,}8380\ldots}$ 0.6/

b) $9^x + 2 \cdot 3^{x+1} = 27$; $(3^2)^x + 2 \cdot 3^x \cdot 3 = 27$; $(3^x)^2 + 6 \cdot 3^x - 27 = 0$

$1{,}75$

cambio evar. $3^x = t \Rightarrow \boxed{t^2 + 6t - 27 = 0}$ 0.2/

$t = 3 = 3^x \Rightarrow \boxed{x = 1}$ 0.4/

$t = -9 = 3^x \Rightarrow \nexists$ soluc.

c) $\log x^2 + \log x^3 = 5$; $\log(x^2 \cdot x^3) = \log 100000 \Rightarrow x^5 = 100000 \Rightarrow \boxed{x = \sqrt[5]{100000} = 10}$ 0.55/

---

⑥ a) $\lim_{x\to -2} \frac{x^2-4}{x^3+3x^2-4} \overset{0/0}{\underset{\text{indet.}}{=}} \lim_{x\to -2} \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x^2+x-2)} = \frac{-4}{0} = \begin{cases} \lim_{x\to -2^-} \frac{x-2}{(x+2)(x-1)} = \frac{-4}{0^- \cdot (-3)} = \frac{4}{0^-} = -\infty \\ \lim_{x\to -2^+} \frac{x-2}{(x+2)(x-1)} = \frac{-4}{0^+ \cdot (-3)} = \frac{4}{0^+} = \infty \end{cases} \Rightarrow \boxed{\nexists \lim_{x\to -2} f(x)}$

raíces 1 y −2 0.25/ 0.5/

	1	3	0	−4
−2		−2	−2	4
	1	1	−2	$\boxed{0}$

b) $\boxed{\lim_{x\to -\infty} \frac{x^2-4}{x^3+3x^2-4}} \overset{\infty/\infty}{\underset{\text{indet.}}{\sim}} \lim_{x\to -\infty} \frac{x^2}{x^3} = \lim_{x\to -\infty} \frac{1}{x} = \boxed{0}$ 0.5/

$1{,}75$

c) $\boxed{\lim_{x\to -\infty} \left(1 + \frac{1}{e^x}\right)} = 1 + \frac{1}{e^{-\infty}} = 1 + e^{\infty} = 1 + \infty = \boxed{\infty}$ 0.5/