

1.-(2.5 puntos) Resolver la ecuación trigonométrica (sin calculadora)

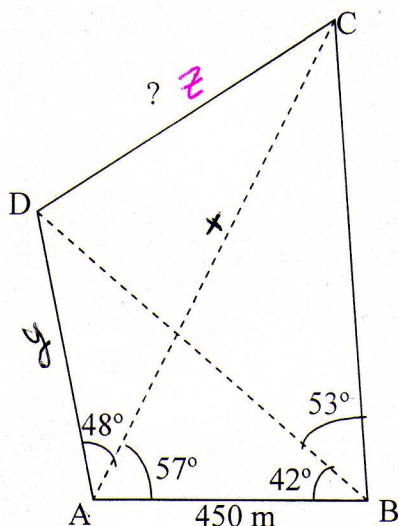
$$\cos(2x) + \sin(2x) \cdot \sin x = -\frac{3}{2} - 2\cos^3 x$$

2.-(2.5 puntos) Sabiendo que $\sin \frac{\alpha}{2} = -\frac{\sqrt{5}}{5}$, con $\pi < \frac{\alpha}{2} < \frac{3\pi}{2}$. (sin calculadora)

Halla: a) $\sin \alpha$

b) $\tan(\frac{\pi}{4} + \frac{\alpha}{4})$

3.-(2.5 puntos) Halla la distancia que hay entre dos barcos C y D, sabiendo que hemos medido la distancia que hay entre A y B y hemos obtenido 450 m, y que con el teodolito hemos obtenido que $\angle CAD = 48^\circ$, $\angle BAC = 57^\circ$, $\angle ABD = 42^\circ$ y $\angle CBD = 53^\circ$



4.-(2.5 puntos) Sabemos que en un triángulo ABC: $\hat{A} = 40^\circ$, $b = 40\text{cm}$, $c = 8\text{cm}$.

Determina:

- a) La longitud del lado a b) El ángulo $\hat{B}$ c) La longitud de la bisectriz trazada desde el ángulo $\hat{A}$ d) El área del triángulo ABC.

(nº1)

$$\cos 2x + \sin 2x \cdot \sin x = -\frac{3}{2} - 2\cos^3 x$$

$$\cos^2 x - \sin^2 x + 2\sin^2 x \cos x = -\frac{3}{2} - 2\cos^3 x \quad (\sin^2 x = 1 - \cos^2 x)$$

$$\cos^2 x - (1 - \cos^2 x) + 2(1 - \cos^2 x) \cos x = -\frac{3}{2} - 2\cos^3 x$$

$$\cos^2 x - 1 + \cos^2 x + 2\cos x - 2\cos^3 x = -\frac{3}{2} - 2\cos^3 x$$

$$2\cos^2 x + 2\cos x + \frac{1}{2} = 0$$

$$4\cos^2 x + 4\cos x + 1 = 0 \Rightarrow \cos x = \frac{-4 \pm \sqrt{16 - 16}}{8} \quad \cos x = -\frac{1}{2}$$

$$x = \arccos\left(-\frac{1}{2}\right) \begin{cases} x_1 = 180^\circ - 60^\circ + 360^\circ k = 120^\circ + 360^\circ k \\ x_2 = 180^\circ + 60^\circ + 360^\circ k = 240^\circ + 360^\circ k \end{cases} \quad x_1 = \frac{2\pi}{3} + 2k\pi \quad x_2 = \frac{4\pi}{3} + 2k\pi$$

№2 $\sin \frac{\alpha}{2} = -\frac{\sqrt{5}}{5}$

$\pi < \frac{\alpha}{2} < \frac{3\pi}{2}$

$\frac{\pi}{2} < \frac{\alpha}{4} < \frac{3\pi}{4}$

(a) $\sin \alpha = 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} = 2 \cdot \left(-\frac{\sqrt{5}}{5}\right) \cdot \left(-\frac{\sqrt{20}}{5}\right) = \frac{4}{5}$

Calculo: $\cos \frac{\alpha}{2} = 1 - \sin^2 \frac{\alpha}{2} = 1 - \left(-\frac{\sqrt{5}}{5}\right)^2 = 1 - \frac{5}{25} = \frac{20}{25}$
 $\cos \frac{\alpha}{2} = \pm \frac{\sqrt{20}}{5}$
 $\alpha/2 \in \text{III} \Rightarrow \cos \frac{\alpha}{2} = -\frac{\sqrt{20}}{5} = -\frac{2\sqrt{5}}{5}$

(b) $\tan\left(\frac{\pi}{4} + \frac{\alpha}{4}\right) = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \tan \frac{\alpha}{4}}{1 - \frac{1}{\sqrt{2}} \tan \frac{\alpha}{4}} = \frac{1 + \tan \frac{\alpha}{4}}{1 - \tan \frac{\alpha}{4}} = \frac{1 - 2\sqrt{5}}{1 + 2\sqrt{5}} = \frac{-1\sqrt{5}}{2\sqrt{5}} = \frac{(-1\sqrt{5})(3-\sqrt{5})}{2\sqrt{5}(3-\sqrt{5})} = \frac{-3+5}{2(3-\sqrt{5})} = \frac{2}{2(3-\sqrt{5})} = \frac{1}{3-\sqrt{5}}$

$\tan \frac{\alpha}{4} = \pm \sqrt{\frac{1 - \cos \alpha/2}{1 + \cos \alpha/2}} = -\sqrt{\frac{1 - \frac{2\sqrt{5}}{5}}{1 + \frac{2\sqrt{5}}{5}}} = -\sqrt{\frac{5 - 2\sqrt{5}}{5 + 2\sqrt{5}}} = -\sqrt{\frac{(5 - 2\sqrt{5})^2}{25 - 20}} = -\frac{5 - 2\sqrt{5}}{\sqrt{5}} = -2 + \sqrt{5}$

$\alpha/4 \in \text{II} \Rightarrow \tan \frac{\alpha}{4} = -2 + \sqrt{5}$

№3 $\triangle ACB$ - Calculo de AC (x) $\angle C = 180^\circ - 57^\circ - 95^\circ = 28^\circ$

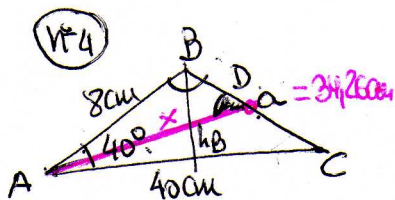
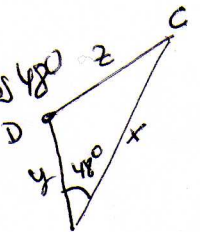
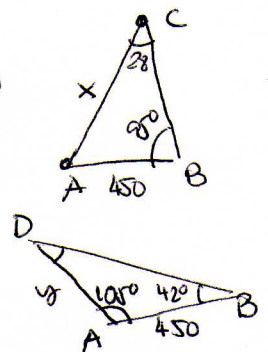
T. seno $\frac{\sin 90^\circ}{450} = \frac{\sin 28^\circ}{x} \Rightarrow x = \frac{450 \cdot \sin 90^\circ}{\sin 28^\circ} \approx 954,88 \text{ m}$

$\triangle ADB$ - Calculo de AD (y) $\angle D = 180^\circ - 105^\circ - 42^\circ = 33^\circ$

T. seno $\frac{450}{\sin 33^\circ} = \frac{y}{\sin 42^\circ} \Rightarrow y = \frac{450 \cdot \sin 42^\circ}{\sin 33^\circ} \approx 552,86 \text{ m}$

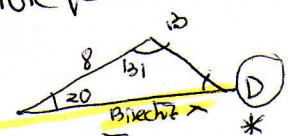
$\triangle ADC$ - Calculo DC (z)

T. coseno $z^2 = y^2 + x^2 - 2yx \cdot \cos 48^\circ = (552,86)^2 + (954,88)^2 - 2 \cdot 552,86 \cdot 954,88 \cdot \cos 48^\circ$
 $z^2 = 510961,88 \Rightarrow z = 714,81 \text{ m}$



(a) $a^2 = 8^2 + 40^2 - 2 \cdot 8 \cdot 40 \cdot \cos 40^\circ = 1173,73 \Rightarrow a = 34,26 \text{ cm}$
 (b) $\frac{40}{\sin B} = \frac{34,26}{\sin 40^\circ} \Rightarrow \sin B = \frac{40 \sin 40^\circ}{34,26} = 0,75$
 $B = \arcsin 0,75$
 $B = 48,63^\circ$
 $B = 131,37^\circ$

Imposible por el lado c = 8cm no puede tener por angulo opuesto $C = 91,37^\circ > B = 48,63^\circ$. Por la medidas de los lados el angulo B ha de ser el mayor. hence $B > C$



(c) $\frac{x}{\sin B} = \frac{8}{\sin 28,63^\circ}$

$x = \frac{8 \cdot \sin 131,37^\circ}{\sin 28,63^\circ} \approx 12,53 \text{ cm}$ longitud de la bisectriz.

(d) $A = \frac{40 \cdot \sin 40^\circ}{2} = 20 \cdot 0,64 = 12,8 \text{ cm}^2$

$\sin 40^\circ = \frac{h}{8} \Rightarrow h = 8 \sin 40^\circ = 5,14$