

## EJERCICIOS DE APLICACIÓN. REGLA DE L'HÔPITAL.

1.- Calcula los siguientes límites:

a)  $\lim_{x \rightarrow 0} \frac{\operatorname{sen}^2 3x}{3x^2};$

b)  $\lim_{x \rightarrow 0} \frac{x^3 - x}{\sqrt{x+9} - 3}$

c)  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{\operatorname{Ln} x}$

d)  $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x}$

e)  $\lim_{x \rightarrow 0} \frac{e^{-x} - e^x}{3 \operatorname{sen} x}$

f)  $\lim_{x \rightarrow 7} \frac{x^2 - 9x + 14}{\operatorname{Ln}(x^2 - 6x - 6)}$

g)  $\lim_{x \rightarrow +\infty} \frac{\operatorname{Ln} x}{x}$

h)  $\lim_{x \rightarrow +\infty} \frac{e^x}{\operatorname{Ln} x}$

i)  $\lim_{x \rightarrow 0} \left( \frac{1}{4x} - \frac{2}{xe^x} \right)$

j)  $\lim_{x \rightarrow +\infty} \frac{x + \sqrt{x+1}}{3x}$

k)  $\lim_{x \rightarrow 0} x \operatorname{Ln} x$

l)  $\lim_{x \rightarrow 0} (\operatorname{tg} x)^x$

m)  $\lim_{x \rightarrow +\infty} \left( 1 + \frac{3}{x} \right)^{4x-5}$

n)  $\lim_{x \rightarrow \infty} \sqrt{x^2 + 2}$

ñ)  $\lim_{x \rightarrow +\infty} \left( \frac{x+3}{x-2} \right)^{3x+1}$

o)  $\lim_{x \rightarrow 2} [(x-2) \operatorname{Ln}(x-2)]$

p)  $\lim_{x \rightarrow 0} (\cos x + \operatorname{sen} x)^{\frac{1}{x}}$

q)  $\lim_{x \rightarrow 0} \frac{e^x - x - \cos x}{\operatorname{sen}^2 x}$

r)  $\lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{\cos x - 1}$

s)  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \operatorname{sen} x}$

t)  $\lim_{x \rightarrow 0} \frac{x}{\operatorname{Ln}(1+x)}$

u)  $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}$

v)  $\lim_{x \rightarrow 0} \frac{\operatorname{Ln} x}{\cot g x}$

w)  $\lim_{x \rightarrow 0} (e^x - x)^{\frac{1}{x}}$

aa)  $\lim_{x \rightarrow 0} \frac{1+x-e^x}{\operatorname{sen}^2 x}$

ab)  $\lim_{x \rightarrow 3} \frac{\sqrt{x^2-5}-2}{x-3}$

ac)  $\lim_{x \rightarrow +\infty} \left( 1 - \frac{1}{x^2} \right)^x$

ad)  $\lim_{x \rightarrow 0} \frac{e^x \operatorname{sen} x - x}{2x^2 + x^4}$

af)  $\lim_{x \rightarrow 0} \frac{x - \operatorname{sen} x}{\operatorname{tg} x - \operatorname{sen} x}$

ag)  $\lim_{x \rightarrow 1} \left( \frac{1}{\operatorname{Ln} x} - \frac{1}{x-1} \right)$

ah)  $\lim_{x \rightarrow +\infty} (\operatorname{Ln} x)^{e^{-x}}$

ai)  $\lim_{x \rightarrow 0} (\cos 2x)^{\frac{1}{\operatorname{sen} x^2}}$

aj)  $\lim_{x \rightarrow 1/2} \frac{1 - \operatorname{tg} \left( \frac{\pi}{2} x \right)}{\operatorname{Ln} 4x^2}$

2.- Se considera la función  $f(x) = \frac{x^2}{e^x}$ . Se pide calcular  $\lim_{x \rightarrow -\infty} f(x)$ ,  $\lim_{x \rightarrow +\infty} f(x)$

3.- Determinése los valores de a y b para los cuales  $\lim_{x \rightarrow 0} \frac{ax^2 + bx + 1 - \cos x}{\operatorname{sen} x^2} = 1$

$$1. a) \lim_{x \rightarrow 0} \frac{\sin^2 3x}{3x^2} = \left[ \frac{0}{0} \right]_{L'H} = \lim_{x \rightarrow 0} \frac{\cancel{2} \sin 3x \cdot \cos 3x \cdot 3}{\cancel{6} x} =$$

$$= \left[ \frac{0}{0} \right]_{L'H} = \lim_{x \rightarrow 0} \frac{3 \cos 3x \cos 3x + 3 \sin 3x (-\sin 3x)}{1} = \frac{3}{1} = \underline{\underline{3}}$$

$$b) \lim_{x \rightarrow 0} \frac{x^3 - x}{\sqrt{x+9} - 3} = \left[ \frac{0}{0} \right]_{L'H} = \lim_{x \rightarrow 0} \frac{3x^2 - 1}{\frac{1}{2\sqrt{x+9}}} = \frac{-1}{\frac{1}{6}} = -6$$

$$c) \lim_{x \rightarrow 1} \frac{x^2 - 1}{\ln x} = \left[ \frac{0}{0} \right]_{L'H} = \lim_{x \rightarrow 1} \frac{2x}{\frac{1}{x}} = \frac{2}{1} = 1$$

$$d) \lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = \left[ \frac{0}{0} \right]_{L'H} = \lim_{x \rightarrow 0} \frac{(1 + \operatorname{tg}^2 x)}{1} = \frac{1}{1} = 1$$

$$e) \lim_{x \rightarrow 0} \frac{e^{-x} - e^x}{3 \cdot \sin x} = \left[ \frac{0}{0} \right] = \lim_{x \rightarrow 0} \frac{-e^{-x} - e^x}{3 \cos x} = \frac{-2}{3}$$

$$f) \lim_{x \rightarrow 7} \frac{x^2 - 9x + 14}{\ln(x^2 - 6x - 6)} = \left[ \frac{0}{0} \right] = \lim_{x \rightarrow 7} \frac{2x - 9}{\frac{2x - 6}{x^2 - 6x - 6}} =$$

$$= \lim_{x \rightarrow 7} \frac{(2x - 9)(x^2 - 6x - 6)}{(2x - 6)} = \frac{5 \cdot 1}{8} = \frac{5}{8}$$

$$g) \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \left[ \frac{\infty}{\infty} \right]_{L'H} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = \lim_{x \rightarrow \infty} \frac{1}{x} = \frac{1}{\infty} = 0$$

$$h) \lim_{x \rightarrow \infty} \frac{e^x}{\ln x} = \left[ \frac{\infty}{\infty} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{e^x}{\frac{1}{x}} = \lim_{x \rightarrow \infty} x e^x = \infty$$

$$i) \lim_{x \rightarrow 0} \left( \frac{1}{4x} - \frac{2}{x e^x} \right) = (\infty - \infty) = \lim_{x \rightarrow 0} \left( \frac{e^x - 8}{4x e^x} \right) =$$

$$= \frac{1 - 8}{0} = \frac{-7}{0} = -\infty.$$

$$j) \lim_{x \rightarrow \infty} \frac{x + \sqrt{x+1}}{3x} = \left[ \frac{\infty}{\infty} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{1 + \frac{1}{2\sqrt{x+1}}}{3} = \frac{1}{3}$$

$$k) \lim_{x \rightarrow 0} x \cdot \ln x = [0(-\infty)] = \lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{x}} = \left[ \frac{-\infty}{\infty} \right] \stackrel{\text{L'H}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{-1/x^2} = \lim_{x \rightarrow 0} -x = 0$$

$$l) \lim_{x \rightarrow 0} (\tan x)^x = [0^0] = e^0 = 1$$

$$\lim_{x \rightarrow 0} \ln (\tan x)^x = \lim_{x \rightarrow 0} x \cdot \ln (\tan x) = [0(-\infty)] =$$

$$\lim_{x \rightarrow 0} \frac{\ln(\tan x)}{\frac{1}{x}} = \left[ \frac{-\infty}{\infty} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\tan x} \cdot (1 + \tan^2 x)}{-1/x^2} =$$

$$= \lim_{x \rightarrow 0} \frac{x^2(1 + \tan^2 x)}{-\tan x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{2x(1 + \tan^2 x) + x^2 \cdot 2 \tan x (1 + \tan^2 x)}{-(1 + \tan^2 x)} = 0$$

$$m) \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^{4x-5} = [1^\infty] \underset{\uparrow}{=} e^{12}$$

$$\lim_{x \rightarrow \infty} \ln \left(1 + \frac{3}{x}\right)^{4x-5} = \lim_{x \rightarrow \infty} (4x-5) \cdot \ln \left(1 + \frac{3}{x}\right) = [\infty \cdot 0] =$$

$$= \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{3}{x}\right)}{\frac{1}{4x-5}} = \left[\frac{0}{0}\right] \underset{L'H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{\left(1 + \frac{3}{x}\right)} \cdot \left(-\frac{3}{x^2}\right)}{\frac{-4}{(4x-5)^2}} =$$

$$= \lim_{x \rightarrow \infty} \frac{3(4x-5)^2}{4\left(1 + \frac{3}{x}\right)x^2} = \lim_{x \rightarrow \infty} \frac{3(16x^2 - 40x + 25)}{4\left(\frac{x+3}{x}\right)x^2} =$$

$$= \lim_{x \rightarrow \infty} \frac{48x^2 - 120x + 75}{4x^2 + 12x} = \frac{48}{4} = 12 \quad \uparrow$$

$$n) \lim_{x \rightarrow \infty} \sqrt{x^2+2} = \lim_{x \rightarrow \infty} (x^2+2)^{1/x} = [\infty^0] \underset{\uparrow}{=} e^0 = 1$$

$$\lim_{x \rightarrow \infty} \ln (x^2+2)^{1/x} = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln(x^2+2) = [0 \cdot \infty] =$$

$$= \lim_{x \rightarrow \infty} \frac{\ln(x^2+2)}{x} = \left[\frac{\infty}{\infty}\right] \underset{L'H}{=} \lim_{x \rightarrow \infty} \frac{\frac{2x}{x^2+2}}{1} =$$

$$= \lim_{x \rightarrow \infty} \frac{2x}{x^2+2} = 0 \quad \uparrow$$

$$\text{ii) } \lim_{x \rightarrow \infty} \left( \frac{x+3}{x-2} \right)^{3x+1} = [1^\infty] = e^{15}$$

$$\lim_{x \rightarrow \infty} \ln \left( \frac{x+3}{x-2} \right)^{3x+1} = \lim_{x \rightarrow \infty} (3x+1) \ln \left( \frac{x+3}{x-2} \right) = [\infty \cdot 0] =$$

$$\lim_{x \rightarrow \infty} \frac{\ln \left( \frac{x+3}{x-2} \right)}{\frac{1}{3x+1}} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x+3} \cdot \frac{(x-2) - (x+3)}{(x-2)^2}}{\frac{-3}{(3x+1)^2}} =$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{+5}{(x+3)(x-2)}}{\frac{-3}{(3x+1)^2}} = \lim_{x \rightarrow \infty} \frac{5(3x+1)^2}{3(x+3)(x-2)} =$$

$$= \lim_{x \rightarrow \infty} \frac{45x^2 + \dots}{3x^2 + \dots} = \frac{45}{3} = 15 \uparrow$$

$$\text{0) } \lim_{x \rightarrow 2} [(x-2) \ln(x-2)] = [0(-\infty)] = \lim_{x \rightarrow 2} \frac{\ln(x-2)}{\frac{1}{x-2}} = \left[ \frac{-\infty}{\infty} \right]:$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 2} \frac{\frac{1}{x-2}}{\frac{-1}{(x-2)^2}} = \lim_{x \rightarrow 2} -(x-2) = 0$$

$$p) \lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x} = [1^\infty] = e^1$$

$$\lim_{x \rightarrow 0} \ln(\cos x + \sin x)^{1/x} = \lim_{x \rightarrow 0} \frac{1}{x} \ln(\cos x + \sin x) = [\infty \cdot 0] =$$

$$= \lim_{x \rightarrow 0} \frac{\ln(\cos x + \sin x)}{x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{-\sin x + \cos x}{\cos x + \sin x}$$

$$= \lim_{x \rightarrow 0} \frac{-\sin x + \cos x}{\cos x + \sin x} = \frac{1}{1} = 1$$

$$q) \lim_{x \rightarrow 0} \frac{e^x - x - \cos x}{\sin^2 x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{e^x - 1 + \sin x}{2 \sin x \cdot \cos x}$$

$$= \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{e^x + \cos x}{2 \cos^2 x - 2 \sin^2 x} = \frac{1+1}{2-0} = \frac{2}{2} = 1$$

$$r) \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{2x}{\sin x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{2}{\cos x} = \frac{2}{1} = 2$$

$$s) \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{0 + \sin x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{\cos x} = \frac{2}{1} = 2$$

$$t) \lim_{x \rightarrow 0} \frac{x}{\ln(1+x)} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{1}{\frac{1}{1+x}} = 1$$

$$u) \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{2x}{\sin x} = \left[ \frac{0}{0} \right] =$$

$$= \lim_{x \rightarrow 0} \frac{2}{\cos x} = \frac{2}{1} = 2.$$

$$v) \lim_{x \rightarrow 0} \frac{\ln x}{\cot x} = \left[ \frac{-\infty}{\infty} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{1/x}{-1/\sin^2 x} = \lim_{x \rightarrow 0} \frac{-\sin^2 x}{x} = \left[ \frac{0}{0} \right]$$

$$= \lim_{x \rightarrow 0} \frac{-2\sin x \cos x}{1} = \frac{0}{1} = 0$$

$$w) \lim_{x \rightarrow 0} (e^x - x)^{1/x} = \left[ 1^\infty \right] = e^0 = 1$$

↑

$$\lim_{x \rightarrow 0} \ln (e^x - x)^{1/x} = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \ln(e^x - x) = \left[ \infty \cdot 0 \right] =$$

$$= \lim_{x \rightarrow 0} \frac{\ln(e^x - x)}{x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{e^x - 1}{e^x - x}}{1} =$$

$$= \lim_{x \rightarrow 0} \frac{e^x - 1}{e^x - x} = \frac{1 - 1}{1 - 0} = \frac{0}{1} = 0 \uparrow$$

$$\begin{aligned}
 \text{aa) } \lim_{x \rightarrow 0} \frac{1+x-e^x}{\sin^2 x} &= \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{1-e^x}{2 \sin x \cos x} = \left[ \frac{0}{0} \right] \\
 &\stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{-e^x}{2 \cos^2 x - 2 \sin^2 x} = \frac{-1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{ab) } \lim_{x \rightarrow 3} \frac{\sqrt{x^2-5}-2}{x-3} &= \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 3} \frac{\frac{1}{2} \frac{2x}{\sqrt{x^2-5}}}{1} = \\
 &= \lim_{x \rightarrow 3} \frac{x}{\sqrt{x^2-5}} = \frac{3}{2}
 \end{aligned}$$

$$\text{ac) } \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x^2}\right)^x = [1^\infty] = e^0 = 1$$

$$\lim_{x \rightarrow +\infty} \ln \left(1 + \frac{1}{x^2}\right)^x = \lim_{x \rightarrow +\infty} x \ln \left(1 + \frac{1}{x^2}\right) = [\infty \cdot 0] =$$

$$= \lim_{x \rightarrow +\infty} \frac{\ln \left(1 + \frac{1}{x^2}\right)}{\frac{1}{x}} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow +\infty} \frac{\frac{1}{\left(1 + \frac{1}{x^2}\right)} \cdot \frac{2}{x^3}}{-\frac{1}{x^2}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{-2x^2}{\left(\frac{x^2-1}{x^2}\right)x^3} = \lim_{x \rightarrow +\infty} \frac{-2x}{x^2-1} = 0 \uparrow$$

$$\text{ad) } \lim_{x \rightarrow 0} \frac{e^x \sin x - x}{2x^2 + x^4} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{e^x \cos x + e^x \sin x - 1}{4x + 4x^3} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{e^x \cos x - e^x \sin x + e^x \sin x + e^x \cos x}{4 + 12x^2} = \lim_{x \rightarrow 0} \frac{2e^x \cos x}{4 + 12x^2} = \frac{2}{4} = \frac{1}{2}$$

$$af) \lim_{x \rightarrow 0} \frac{x - \sin x}{\sqrt[3]{x} - \sin x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{1 - \cos x}{(1 + \sqrt[3]{x}) - \cos x} = \left[ \frac{0}{0} \right]$$

$$= \lim_{x \rightarrow 0} \frac{0 + \sin x}{2\sqrt[3]{x}(1 + \sqrt[3]{x}) + \sin x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=}$$

$$= \lim_{x \rightarrow 0} \frac{\cos x}{2(1 + \sqrt[3]{x})^2 + 4\sqrt[3]{x}(1 + \sqrt[3]{x}) + \cos x} = \frac{1}{2 + 1} = \frac{1}{3}$$

$$ag) \lim_{x \rightarrow 1} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right) = (\infty - \infty) =$$

$$\lim_{x \rightarrow 1} \frac{x-1 - \ln x}{\ln x(x-1)} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 1} \frac{1 - \frac{1}{x}}{\frac{x-1}{x} + \ln x} = \left[ \frac{0}{0} \right] =$$

$$= \lim_{x \rightarrow 1} \frac{x-1}{x-1+x \cdot \ln x} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 1} \frac{1}{1 + \ln x + 1} = \frac{1}{2}$$

$$ah) \lim_{x \rightarrow \infty} (\ln x)^{e^{-x}} = [\infty^0] = e^{\lim_{x \rightarrow \infty} \ln(\ln x) \cdot (-e^{-x})} = 1$$

$$\lim_{x \rightarrow \infty} \ln(\ln x)^{e^{-x}} = \lim_{x \rightarrow \infty} e^{-x} \cdot \ln(\ln x) = [0 \cdot \infty] =$$

$$\lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{e^x} = \left[ \frac{\infty}{\infty} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{e^x} =$$

$$= \lim_{x \rightarrow \infty} \frac{1}{x \cdot \ln x \cdot e^x} = \frac{1}{\infty} = 0 \quad \uparrow$$

$$a_i) \lim_{x \rightarrow 0} (\cos 2x)^{\frac{1}{\sin x^2}} = [1^\infty] = e^{-2}$$

$$\lim_{x \rightarrow 0} \ln (\cos 2x)^{\frac{1}{\sin x^2}} = \lim_{x \rightarrow 0} \frac{1}{\sin x^2} \cdot \ln (\cos 2x) =$$

$$= \lim_{x \rightarrow 0} \frac{\ln (\cos 2x)}{\sin x^2} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{-2 \sin 2x}{\cos 2x}}{2x \cdot \cos x^2} =$$

$$= \lim_{x \rightarrow 0} \frac{-\sin 2x}{x \cos 2x \cdot \cos x^2} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{-2 \cos 2x}{\cos 2x \cdot \cos x^2 + x(-2 \sin x) \cos x^2}$$

$$\frac{-2 \cos 2x \sin x^2}{-2x^2 \cos 2x \sin x^2} = \frac{-2}{1} = -2 \uparrow$$

$$a_j) \lim_{x \rightarrow \frac{1}{2}} \frac{1 - \operatorname{Arctg} \left( \frac{\pi}{2} x \right)}{\ln 4x^2} = \left[ \frac{0}{0} \right] \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \frac{1}{2}} \frac{-\frac{\pi}{2} \left( 1 + \frac{1}{9} \left( \frac{\pi}{2} x \right)^2 \right)}{\frac{8x}{4x^2}} =$$

$$= \lim_{x \rightarrow \frac{1}{2}} -\frac{\pi}{4} x \left( 1 + \frac{1}{9} \left( \frac{\pi}{2} x \right)^2 \right) = -\frac{\pi}{8} \cdot 2 = -\frac{\pi}{4}$$

————— 0 —————

$$2.- \lim_{x \rightarrow \infty} \frac{x^2}{e^x} = \left[ \frac{\infty}{0} \right] = \lim_{x \rightarrow \infty} x^2 \cdot \frac{1}{e^x} =$$

$$= \infty \cdot \frac{1}{e^{-\infty}} = \infty \cdot \frac{1}{0} = \infty \cdot \infty = \infty$$

$$\lim_{x \rightarrow \infty} \frac{x^2}{e^x} = \left[ \frac{\infty}{\infty} \right] \underset{L'H}{=} \lim_{x \rightarrow \infty} \frac{2x}{e^x} = \left[ \frac{\infty}{\infty} \right] \underset{L'H}{=}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$$


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$$3.- \lim_{x \rightarrow 0} \frac{ax^2 + bx + 1 - \cos x}{\sin x^2} = \left[ \frac{0}{0} \right] \underset{L'H}{=}$$

$$\lim_{x \rightarrow 0} \frac{2ax + b + \sin x}{\cos x^2 \cdot 2x} = \frac{b}{0} \Rightarrow \text{if } b = 0 \left[ \frac{0}{0} \right] \underset{L'H}{=}$$

$$\lim_{x \rightarrow 0} \frac{2a + \cos x}{-\sin x^2 \cdot 4x^2 + 2\cos x^2} = \frac{2a + 1}{2} = 1$$

$$2a + 1 = 2$$

$$\boxed{a = \frac{1}{2}}$$


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