

CALCULA LOS SIGUIENTES LÍMITES:

$$1) \lim_{x \rightarrow +\infty} \frac{2x^2 - 14x + 12}{x^2 - 10x + 4} = \frac{+\infty}{+\infty} (I) = \begin{cases} \lim_{x \rightarrow +\infty} \frac{\frac{2x^2}{x^2} - \frac{14x}{x^2} + \frac{12}{x^2}}{\frac{x^2}{x^2} - \frac{10x}{x^2} + \frac{4}{x^2}} = \lim_{x \rightarrow +\infty} \frac{2 - \frac{14}{x} + \frac{12}{x^2}}{1 - \frac{10}{x} + \frac{4}{x^2}} = \frac{2 - 0 + 0}{1 - 0 + 0} = \frac{2}{1} = 2 \\ \lim_{x \rightarrow +\infty} \frac{2x^2}{x^2} = \lim_{x \rightarrow +\infty} 2 = 2 \end{cases}$$

$$2) \lim_{x \rightarrow +\infty} \frac{(2x^2 - 3)(5x - 4)}{2x^3 - 2x^2 + 1} = \lim_{x \rightarrow +\infty} \frac{10x^3 - 8x^2 - 15x + 12}{2x^3 - 2x^2 + 1} = \frac{+\infty}{+\infty} (I) = \begin{cases} \lim_{x \rightarrow +\infty} \frac{10x^3 - 8x^2 - 15x + 12}{2x^3 - 2x^2 + 1} = \lim_{x \rightarrow +\infty} \frac{\frac{10x^3}{x^3} - \frac{8x^2}{x^3} - \frac{15x}{x^3} + \frac{12}{x^3}}{\frac{2x^3}{x^3} - \frac{2x^2}{x^3} + \frac{1}{x^3}} = \lim_{x \rightarrow +\infty} \frac{10 - \frac{8}{x} - \frac{15}{x^2} + \frac{12}{x^3}}{2 - \frac{2}{x} + \frac{1}{x^3}} = \frac{10 - 0 - 0 + 0}{2 - 0 + 0} = 5 \\ \lim_{x \rightarrow +\infty} \frac{10x^3 - 8x^2 - 15x + 12}{2x^3 - 2x^2 + 1} = \frac{+\infty}{+\infty} (I) = \lim_{x \rightarrow +\infty} \frac{10x^3}{2x^3} = \lim_{x \rightarrow +\infty} 5 = 5 \end{cases}$$

$$3) \lim_{x \rightarrow -1} \frac{x^3 + 1}{x^2 + 2x + 1} = \frac{0}{0} (I) = \lim_{x \rightarrow -1} \frac{(x+1)(x^2 - x + 1)}{(x+1)^2} = \lim_{x \rightarrow -1} \frac{x^2 - x + 1}{x+1} = \frac{3}{0} = \begin{cases} \lim_{x \rightarrow -1^-} \frac{x^2 - x + 1}{x+1} = \frac{3}{0^-} = -\infty \\ \lim_{x \rightarrow -1^+} \frac{x^2 - x + 1}{x+1} = \frac{3}{0^+} = +\infty \end{cases}$$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & +1 \\ -1 & & -1 & +1 & -1 \\ \hline & 1 & -1 & +1 & \boxed{0} \end{array} \Rightarrow x^3 + 1 = (x-1)(x^2 - x + 1)$$

$$x^2 + 2x + 1 = \underset{\substack{\text{identidad} \\ \text{notable}}}{(x+1)^2}$$

$$4) \lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 + 3x + 2} = \frac{0}{0} (I) = \lim_{x \rightarrow -1} \frac{(x+1)(x-1)}{(x+1)(x+2)} = \lim_{x \rightarrow -1} \frac{x-1}{x+2} = \frac{-2}{1} = -2$$

$$\begin{array}{c|ccc} & 1 & +3 & +2 \\ -1 & & -1 & -2 \\ \hline & 1 & +2 & \boxed{0} \end{array} \Rightarrow x^2 + 3x + 2 = (x+1)(x+2)$$

$$5) \lim_{x \rightarrow a} \frac{3x^2 - 2ax - a^2}{2x^2 - 3ax + a^2} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow a} \frac{(x-a)(3x+a)}{(x-a)(2x-a)} = \lim_{x \rightarrow a} \frac{3x+a}{2x-a} = \frac{3a+a}{2a-a} = \frac{4a}{a} = 4 \quad (a \neq 0)$$

$$\begin{array}{c|cc} & 3 & -2a & -a^2 \\ a & +3a & +a^2 & \\ \hline & 3 & +a & 0 \end{array} \Rightarrow 3x^2 - 2ax - a^2 = (x-a)(3x+a)$$

$$\begin{array}{c|cc} & 2 & -3a & +a^2 \\ a & +2a & -a^2 & \\ \hline & 2 & -a & 0 \end{array} \Rightarrow 2x^2 - 3ax + a^2 = (x-a)(2x-a)$$

$$6) \lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^2} \right) = \infty - \infty \text{ (I)} = \lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{(1-x)(1+x)} \right) = \lim_{x \rightarrow 1} \frac{1+x-3}{(1-x)(1+x)} = \lim_{x \rightarrow 1} \frac{x-2}{(1-x)(1+x)} = \frac{-1}{0} =$$

$$\begin{cases} \lim_{x \rightarrow 1^-} \frac{x-2}{(1-x)(1+x)} = \frac{-1}{(0^+)(2)} = \frac{-1}{0^+} = -\infty \\ \lim_{x \rightarrow 1^+} \frac{x-2}{(1-x)(1+x)} = \frac{-1}{(0^-)(2)} = \frac{-1}{0^-} = +\infty \end{cases}$$

$$7) \lim_{x \rightarrow +\infty} \frac{\sqrt{x+4}}{x-2} = \frac{+\infty}{+\infty} \text{ (I)} = \begin{cases} \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{x}{x^2} + \frac{4}{x^2}}}{\frac{x}{x^2} - \frac{2}{x^2}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{1}{x} + \frac{4}{x^2}}}{1 - \frac{2}{x}} = \frac{\sqrt{0+0}}{1-0} = \frac{0}{1} = 0 \\ \lim_{x \rightarrow +\infty} \frac{\sqrt{x+4}}{x-2} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x}}{x} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{1}{2}}}{x} = \lim_{x \rightarrow +\infty} \frac{1}{x^{\frac{1}{2}}} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x}} = \frac{1}{+\infty} = 0 \end{cases}$$

$$8) \lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{\sqrt{x+7}-3} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 2} \frac{(\sqrt{x+2}-2)(\sqrt{x+2}+2)(\sqrt{x+7}+3)}{(\sqrt{x+7}-3)(\sqrt{x+7}+3)(\sqrt{x+2}+2)} = \lim_{x \rightarrow 2} \frac{[(\sqrt{x+2}-2)(\sqrt{x+2}+2)](\sqrt{x+7}+3)}{[(\sqrt{x+7}-3)(\sqrt{x+7}+3)](\sqrt{x+2}+2)} =$$

$$= \lim_{x \rightarrow 2} \frac{[(\sqrt{x+2})^2 - (2)^2](\sqrt{x+7}+3)}{[(\sqrt{x+7})^2 - (3)^2](\sqrt{x+2}+2)} = \lim_{x \rightarrow 2} \frac{(x+2-4)(\sqrt{x+7}+3)}{(x+7-9)(\sqrt{x+2}+2)} = \lim_{x \rightarrow 2} \frac{(x-2)(\sqrt{x+7}+3)}{(x-2)(\sqrt{x+2}+2)} =$$

$$= \lim_{x \rightarrow 2} \frac{(\sqrt{x+7}+3)}{(\sqrt{x+2}+2)} = \frac{\sqrt{2+7}+3}{\sqrt{2+2}+2} = \frac{3+3}{2+2} = \frac{6}{4} = \frac{3}{2}$$

$$9) \lim_{x \rightarrow 3} \frac{9-x^2}{\sqrt{3x}-3} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 3} \frac{(9-x^2)(\sqrt{3x}+3)}{(\sqrt{3x}-3)(\sqrt{3x}+3)} = \lim_{x \rightarrow 3} \frac{(9-x^2)(\sqrt{3x}+3)}{(\sqrt{3x})^2 - (3)^2} = \lim_{x \rightarrow 3} \frac{(9-x^2)(\sqrt{3x}+3)}{3x-9} \quad \text{factorizar polinomios}$$

$$= \lim_{x \rightarrow 3} \frac{(3-x)(3+x)(\sqrt{3x}+3)}{3(x-3)} = \lim_{x \rightarrow 3} \frac{-(x-3)(3+x)(\sqrt{3x}+3)}{3(x-3)} = \lim_{x \rightarrow 3} \frac{-(3+x)(\sqrt{3x}+3)}{3} = \frac{-6 \cdot 6}{3} = -12$$

$$\begin{aligned}
10) \quad \lim_{x \rightarrow +\infty} (\sqrt{x(x+1)} - x) &= \lim_{x \rightarrow +\infty} (\sqrt{x^2 + x} - x) = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + x} - x)(\sqrt{x^2 + x} + x)}{\sqrt{x^2 + x} + x} = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + x})^2 - (x)^2}{\sqrt{x^2 + x} + x} = \\
&= \lim_{x \rightarrow +\infty} \frac{x^2 + x - x^2}{\sqrt{x^2 + x} + x} = \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2 + x} + x} = \frac{+\infty}{+\infty} \text{ (I)} = \begin{cases} * \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2 + x} + x} = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{x}{x^2} + \frac{x}{x}}} = \\ = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} = \frac{1}{\sqrt{1+0} + 1} = \frac{1}{2} \\ * \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{x^2 + x}} = \lim_{x \rightarrow +\infty} \frac{x}{2x} = \lim_{x \rightarrow +\infty} \frac{1}{2} = \frac{1}{2} \end{cases}
\end{aligned}$$

$$\begin{aligned}
11) \quad \lim_{x \rightarrow 4} \left(\frac{x+6}{x^2-16} - \frac{x+1}{x^2-4x} \right) &= \infty - \infty \text{ (I)} = \lim_{x \rightarrow 4} \left(\frac{x+6}{(x-4)(x+4)} - \frac{x+1}{x(x-4)} \right) = \lim_{x \rightarrow 4} \left(\frac{x(x+6) - (x+4)(x+1)}{x(x-4)(x+4)} \right) = \\
&= \lim_{x \rightarrow 4} \left(\frac{x^2 + 6x - x^2 - x - 4x - 4}{x(x-4)(x+4)} \right) = \lim_{x \rightarrow 4} \frac{x-4}{x(x-4)(x+4)} = \lim_{x \rightarrow 4} \frac{1}{x(x+4)} = \frac{1}{32}
\end{aligned}$$

$$12) \quad \lim_{x \rightarrow 0} \left(\frac{x^2+3}{x^3} - \frac{1}{x} \right) = \infty - \infty \text{ (I)} = \lim_{x \rightarrow 0} \left(\frac{x^2+3-x^2}{x^3} \right) = \lim_{x \rightarrow 0} \left(\frac{3}{x^3} \right) = \frac{3}{0} = \begin{cases} \lim_{x \rightarrow 0^-} \frac{3}{x^3} = \frac{3}{0^-} = -\infty \\ \lim_{x \rightarrow 0^+} \frac{3}{x^3} = \frac{3}{0^+} = +\infty \end{cases}$$

$$\begin{aligned}
13) \quad \lim_{x \rightarrow 1} \left(\frac{2}{(x-1)^2} - \frac{1}{x(x-1)} \right) &= \infty - \infty \text{ (I)} = \lim_{x \rightarrow 1} \left(\frac{2x - (x-1)}{x(x-1)^2} \right) = \lim_{x \rightarrow 1} \frac{2x - x + 1}{x(x-1)^2} = \lim_{x \rightarrow 1} \frac{x+1}{x(x-1)^2} = \frac{2}{0} = \\
&\begin{cases} \lim_{x \rightarrow 1^-} \frac{x+1}{x(x-1)^2} = \frac{2}{1 \cdot (0^-)^2} = \frac{2}{0^+} = +\infty \\ \lim_{x \rightarrow 1^+} \frac{x+1}{x(x-1)^2} = \frac{2}{1 \cdot (0^+)^2} = \frac{2}{0^+} = +\infty \end{cases}
\end{aligned}$$

$$\begin{aligned}
14) \quad \lim_{x \rightarrow +\infty} \left(\sqrt{\frac{x}{1+2x}} \right)^{\frac{x}{x+1}} &= \lim_{x \rightarrow +\infty} \left(\left(\frac{x}{1+2x} \right)^{\frac{1}{2}} \right)^{\frac{x}{x+1}} = \lim_{x \rightarrow +\infty} \left(\frac{x}{1+2x} \right)^{\frac{x}{2x+2}} = \left[\lim_{x \rightarrow +\infty} \left(\frac{x}{1+2x} \right) \right]^{\lim_{x \rightarrow +\infty} \left(\frac{x}{2x+2} \right)} = \\
&= \left(\frac{1}{2} \right)^{\frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}
\end{aligned}$$

$$\circ \lim_{x \rightarrow +\infty} \left(\frac{x}{1+2x} \right) = \frac{+\infty}{+\infty} = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x}}{\frac{1}{x} + \frac{2x}{x}} = \lim_{x \rightarrow +\infty} \frac{1}{\frac{1}{x} + 2} = \frac{1}{0+2} = \frac{1}{2}$$

$$\circ \lim_{x \rightarrow +\infty} \left(\frac{x}{2x+2} \right) = \frac{1}{+\infty} = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x}}{\frac{2x}{x} + \frac{2}{x}} = \lim_{x \rightarrow +\infty} \frac{1}{2 + \frac{2}{x}} = \frac{1}{2+0} = \frac{1}{2}$$

$$15) \quad \lim_{x \rightarrow 2} \left(\frac{3}{x+1} \right)^{\frac{x^2+2x+5}{x}} = 1^{\frac{13}{2}} = 1$$

$$16) \lim_{x \rightarrow +\infty} \left(\frac{x+1}{x-1} \right)^x = 1^\infty = e^{\lim_{x \rightarrow +\infty} \left(\frac{x+1}{x-1} - 1 \right) \cdot x} = e^2 \quad (*)$$

$$(*) \lim_{x \rightarrow +\infty} \left(\frac{x+1}{x-1} - 1 \right) \cdot x = \lim_{x \rightarrow +\infty} \left(\frac{x+1-x+1}{x-1} \right) \cdot x = \lim_{x \rightarrow +\infty} \left(\frac{2}{x-1} \right) \cdot x = \lim_{x \rightarrow +\infty} \frac{2x}{x-1} = \frac{+\infty}{+\infty} \text{ (I)} = \begin{cases} \circ \lim_{x \rightarrow +\infty} \frac{2x}{x-1} = \lim_{x \rightarrow +\infty} \frac{\frac{2x}{x}}{\frac{x-1}{x}} = \lim_{x \rightarrow +\infty} \frac{2}{1-\frac{1}{x}} = \frac{2}{1-0} = 2 \\ \circ \lim_{x \rightarrow +\infty} \frac{2x}{x} = \lim_{x \rightarrow +\infty} 2 = 2 \end{cases}$$

$$17) \lim_{x \rightarrow 1} \left(\frac{x^2 + 2x + 1}{x + 3} \right)^{\frac{x^3 + 2x^2 + 5}{x^2 + 4x - 5}} = 1^\infty \text{ (I)} = e^{\lim_{x \rightarrow 1} \left(\frac{x^2 + 2x + 1}{x + 3} - 1 \right) \left(\frac{x^3 + 2x^2 + 5}{x^2 + 4x - 5} \right)} = e^1 = e \quad (*)$$

$$(*) \lim_{x \rightarrow 1} \left(\frac{x^2 + 2x + 1}{x + 3} - 1 \right) \cdot \left(\frac{x^3 + 2x^2 + 5}{x^2 + 4x - 5} \right) = \lim_{x \rightarrow 1} \left(\frac{x^2 + 2x + 1 - x - 3}{x + 3} \right) \cdot \left(\frac{x^3 + 2x^2 + 5}{x^2 + 4x - 5} \right) =$$

$$= \lim_{x \rightarrow 1} \left(\frac{x^2 + x - 2}{x + 3} \right) \cdot \left(\frac{x^3 + 2x^2 + 5}{x^2 + 4x - 5} \right) \underset{\text{factorizar polinomios}}{=} = \lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{(x+3)} \cdot \frac{x^3 + 2x^2 + 5}{(x-1)(x+5)} = \lim_{x \rightarrow 1} \frac{(x+2)(x^3 + 2x^2 + 5)}{(x+3)(x+5)} = \frac{3 \cdot 8}{4 \cdot 6} = 1$$

$$18) \lim_{x \rightarrow +\infty} \left(1 - \frac{1}{x} \right)^{x^2} = 1^\infty \text{ (I)} = e^{\lim_{x \rightarrow +\infty} \left(1 - \frac{1}{x} - 1 \right) \cdot x^2} = e^{-\infty} = 0 \quad (*)$$

$$(*) \lim_{x \rightarrow +\infty} \left(1 - \frac{1}{x} - 1 \right) \cdot x^2 = \lim_{x \rightarrow +\infty} \left(-\frac{1}{x} \right) \cdot x^2 = \lim_{x \rightarrow +\infty} \left(-\frac{x^2}{x} \right) = \lim_{x \rightarrow +\infty} (-x) = -\infty$$

$$19) \lim_{x \rightarrow +\infty} (\sqrt{x^3 - x} - \sqrt{x^3 - 2x^2}) = \infty - \infty \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^3 - x} - \sqrt{x^3 - 2x^2})(\sqrt{x^3 - x} + \sqrt{x^3 - 2x^2})}{(\sqrt{x^3 - x} + \sqrt{x^3 - 2x^2})} =$$

$$= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^3 - x})^2 - (\sqrt{x^3 - 2x^2})^2}{(\sqrt{x^3 - x} + \sqrt{x^3 - 2x^2})} = \lim_{x \rightarrow +\infty} \frac{x^3 - x - (x^3 - 2x^2)}{(\sqrt{x^3 - x} + \sqrt{x^3 - 2x^2})} = \lim_{x \rightarrow +\infty} \frac{x^3 - x - x^3 + 2x^2}{(\sqrt{x^3 - x} + \sqrt{x^3 - 2x^2})} =$$

$$= \lim_{x \rightarrow +\infty} \frac{2x^2 - x}{(\sqrt{x^3 - x} + \sqrt{x^3 - 2x^2})} = \frac{+\infty}{+\infty} \text{ (I)} =$$

$$= \begin{cases} \lim_{x \rightarrow +\infty} \frac{\frac{2x^2}{x^2} - \frac{x}{x^2}}{\sqrt{\frac{x^3}{x^4} - \frac{x}{x^4}} + \sqrt{\frac{x^3}{x^4} - \frac{2x^2}{x^4}}} = \lim_{x \rightarrow +\infty} \frac{2 - \frac{1}{x}}{\sqrt{\frac{1}{x} - \frac{1}{x^3}} + \sqrt{\frac{1}{x} - \frac{2}{x^2}}} = \frac{2-0}{\sqrt{0-0} + \sqrt{0+0}} = \frac{2}{0} = +\infty \\ \lim_{x \rightarrow +\infty} \frac{2x^2}{\sqrt{x^3} + \sqrt{x^3}} = \lim_{x \rightarrow +\infty} \frac{2x^2}{2\sqrt{x^3}} = \lim_{x \rightarrow +\infty} \frac{x^2}{\sqrt{x^3}} = \lim_{x \rightarrow +\infty} \frac{x^2}{x^{\frac{3}{2}}} = \lim_{x \rightarrow +\infty} x^{\frac{1}{2}} = \lim_{x \rightarrow +\infty} \sqrt{x} = +\infty \end{cases}$$

$$20) \lim_{x \rightarrow +\infty} \frac{3^x}{2^x} = \lim_{x \rightarrow +\infty} \left(\frac{3}{2} \right)^x = \left(\frac{3}{2} \right)^{+\infty} = +\infty$$

$$21) \lim_{x \rightarrow +\infty} \frac{2^x}{3^x} = \lim_{x \rightarrow +\infty} \left(\frac{2}{3} \right)^x = \left(\frac{2}{3} \right)^{+\infty} = 0$$

$$22) \lim_{x \rightarrow +\infty} \left(\frac{2x+5}{2x} \right)^{\frac{3x^2-1}{x+2}} = 1^\infty (I) = e^{\lim_{x \rightarrow +\infty} \left(\frac{2x+5}{2x} - 1 \right) \cdot \frac{3x^2-1}{x+2}} = e^{\frac{15}{2}} \quad (*)$$

$$(*) \lim_{x \rightarrow +\infty} \left(\frac{2x+5}{2x} - 1 \right) \cdot \frac{3x^2-1}{x+2} = \lim_{x \rightarrow +\infty} \left(\frac{2x+5-2x}{2x} \right) \cdot \frac{3x^2-1}{x+2} = \lim_{x \rightarrow +\infty} \left(\frac{5}{2x} \right) \cdot \frac{3x^2-1}{x+2} = \lim_{x \rightarrow +\infty} \frac{15x^2-5}{2x^2+4x} = \frac{+\infty}{+\infty} =$$

$$= \begin{cases} \lim_{x \rightarrow +\infty} \frac{15x^2-5}{2x^2+4x} = \lim_{x \rightarrow +\infty} \frac{\frac{15x^2}{x^2} - \frac{5}{x^2}}{\frac{2x^2}{x^2} + \frac{4x}{x^2}} = \lim_{x \rightarrow +\infty} \frac{15 - \frac{5}{x^2}}{2 + \frac{4}{x}} = \frac{15-0}{2+0} = \frac{15}{2} \\ \lim_{x \rightarrow +\infty} \frac{15x^2-5}{2x^2+4x} = \lim_{x \rightarrow +\infty} \frac{15x^2}{2x^2} = \lim_{x \rightarrow +\infty} \frac{15}{2} = \frac{15}{2} \end{cases}$$

$$23) \lim_{x \rightarrow +\infty} \frac{\sqrt{x+a} + \sqrt{x+b}}{\sqrt{x}} = \frac{+\infty}{+\infty} (I) = \begin{cases} \circ \lim_{x \rightarrow +\infty} \frac{\sqrt{x+a} + \sqrt{x+b}}{\sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{x}{x} + \frac{a}{x}} + \sqrt{\frac{x}{x} + \frac{b}{x}}}{\sqrt{\frac{x}{x}}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{1+\frac{a}{x}} + \sqrt{1+\frac{b}{x}}}{\sqrt{1}} = \\ = \frac{\sqrt{1+0} + \sqrt{1+0}}{\sqrt{1}} = \frac{1+1}{1} = 2 \\ \circ \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + \sqrt{x}}{\sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{2\sqrt{x}}{\sqrt{x}} = \lim_{x \rightarrow +\infty} 2 = 2 \end{cases}$$

$$24) \lim_{x \rightarrow +\infty} \left(\sqrt[x-3]{\frac{3x+1}{3x+5}} \right)^{x^2-2x} = \lim_{x \rightarrow +\infty} \left(\left(\frac{3x+1}{3x+5} \right)^{\frac{1}{x-3}} \right)^{x^2-2x} = \lim_{x \rightarrow +\infty} \left(\frac{3x+1}{3x+5} \right)^{\frac{x^2-2x}{x-3}} = 1^\infty (I) = e^{\lim_{x \rightarrow +\infty} \left(\frac{3x+1}{3x+5} - 1 \right) \cdot \frac{x^2-2x}{x-3}} = e^{\frac{-4}{3}} \quad (*)$$

$$(*) \lim_{x \rightarrow +\infty} \left(\frac{3x+1}{3x+5} - 1 \right) \cdot \frac{x^2-2x}{x-3} = \lim_{x \rightarrow +\infty} \left(\frac{3x+1-3x-5}{3x+5} \right) \cdot \frac{x^2-2x}{x-3} = \lim_{x \rightarrow +\infty} \left(\frac{-4}{3x+5} \right) \cdot \frac{x^2-2x}{x-3} =$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{-4x^2+8x}{3x^2-4x-15} \right) = \frac{-\infty}{+\infty} = \begin{cases} \lim_{x \rightarrow +\infty} \frac{-4x^2+8x}{3x^2-4x-15} = \lim_{x \rightarrow +\infty} \frac{\frac{-4x^2}{x^2} + \frac{8x}{x^2}}{\frac{3x^2}{x^2} - \frac{4x}{x^2} - \frac{15}{x^2}} = \lim_{x \rightarrow +\infty} \frac{-4 + \frac{8}{x}}{3 - \frac{4}{x} - \frac{15}{x^2}} = \frac{-4+0}{3-0-0} = -\frac{4}{3} \\ \lim_{x \rightarrow +\infty} \frac{-4x^2}{3x^2} = \lim_{x \rightarrow +\infty} \frac{-4}{3} = -\frac{4}{3} \end{cases}$$

$$25) \lim_{x \rightarrow +\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} = \frac{+\infty}{+\infty} = \begin{cases} \lim_{x \rightarrow +\infty} \frac{2x^2-3x-4}{\sqrt{x^4+1}} = \lim_{x \rightarrow +\infty} \frac{\frac{2x^2}{x^2} - \frac{3x}{x^2} - \frac{4}{x^2}}{\sqrt{\frac{x^4}{x^4} + \frac{1}{x^4}}} = \lim_{x \rightarrow +\infty} \frac{2 - \frac{3}{x} - \frac{4}{x^2}}{\sqrt{1 + \frac{1}{x^4}}} = \frac{2-0-0}{\sqrt{1+0}} = \frac{2}{1} = 2 \\ \lim_{x \rightarrow +\infty} \frac{2x^2}{\sqrt{x^4}} = \lim_{x \rightarrow +\infty} \frac{2x^2}{x^2} = \lim_{x \rightarrow +\infty} 2 = 2 \end{cases}$$

$$26) \lim_{x \rightarrow +\infty} \frac{x^2}{10 + x\sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{x^2}{10 + \sqrt{x^3}} = \frac{+\infty}{+\infty} (I) = \begin{cases} \lim_{x \rightarrow +\infty} \frac{x^2}{10 + \sqrt{x^3}} = \lim_{x \rightarrow +\infty} \frac{\frac{x^2}{x^2}}{\frac{10}{x^2} + \sqrt{\frac{x^3}{x^4}}} = \lim_{x \rightarrow +\infty} \frac{1}{\frac{10}{x^2} + \sqrt{\frac{1}{x}}} = \frac{1}{0} = +\infty \\ \lim_{x \rightarrow +\infty} \frac{x^2}{\sqrt{x^3}} = \lim_{x \rightarrow +\infty} \frac{x^2}{x^{\frac{3}{2}}} = \lim_{x \rightarrow +\infty} x^{\frac{1}{2}} = \lim_{x \rightarrow +\infty} \sqrt{x} = +\infty \end{cases}$$

$$27) \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^2+1}}{x+1} = \frac{+\infty}{+\infty} (I) = \begin{cases} \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^2+1}}{x+1} = \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{\frac{x^2}{x^3} + \frac{1}{x^3}}}{\frac{x}{x} + \frac{1}{x}} = \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{\frac{1}{x} + \frac{1}{x^3}}}{1 + \frac{1}{x}} = \frac{\sqrt[3]{0+0}}{1+0} = \frac{0}{1} = 0 \\ \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^2}}{x} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{2}{3}}}{x} = \lim_{x \rightarrow +\infty} x^{-\frac{1}{3}} = \lim_{x \rightarrow +\infty} \frac{1}{x^{\frac{1}{3}}} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt[3]{x}} = \frac{1}{+\infty} = 0 \end{cases}$$

$$28) \lim_{x \rightarrow a} \frac{x^2 - (a+1)x + a}{x^3 - a^3} = \frac{0}{0} (I) = \lim_{x \rightarrow a} \frac{(x-a)(x-1)}{(x-a)(x^2 + ax + a^2)} = \lim_{x \rightarrow a} \frac{(x-1)}{(x^2 + ax + a^2)} = \frac{a-1}{a^2 + a^2 + a^2} = \frac{a-1}{3a^2}$$

$$\begin{array}{c|ccc} & 1 & -(a+1) & +a \\ a & & +a & -a \\ \hline & 1 & -1 & \boxed{0} \end{array} \Rightarrow x^2 - (a+1)x + a = (x-a)(x-1)$$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -a^3 \\ a & & +a & +a^2 & +a^3 \\ \hline & 1 & +a & +a^2 & \boxed{0} \end{array} \Rightarrow x^3 - a^3 = (x-a)(x^2 + ax + a^2)$$

$$29) \lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1} = \frac{0}{0} (I) = \lim_{x \rightarrow 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{(\sqrt{x})^2 - (1)^2}{(x-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{1}{(\sqrt{x}+1)} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

$$30) \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^2-2x+3}}{x-5} = \frac{+\infty}{+\infty} (I) = \begin{cases} \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^2-2x+3}}{x-5} = \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{\frac{x^2}{x^3} - \frac{2x}{x^3} + \frac{3}{x^3}}}{\frac{x}{x} - \frac{5}{x}} = \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{\frac{1}{x} - \frac{2}{x^2} + \frac{3}{x^3}}}{1 - \frac{5}{x}} = \frac{\sqrt[3]{0-0+0}}{1-0} = 0 \\ \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^2}}{x} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{2}{3}}}{x} = \lim_{x \rightarrow +\infty} \frac{1}{x^{\frac{1}{3}}} = \lim_{x \rightarrow +\infty} \frac{1}{\sqrt[3]{x}} = \frac{1}{+\infty} = 0 \end{cases}$$

$$31) \lim_{x \rightarrow -\infty} \frac{\sqrt{3x+x^4}}{2+5x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{3(-x)+(-x)^4}}{2+5(-x)} = \lim_{x \rightarrow +\infty} \frac{\sqrt{-3x+x^4}}{2-5x} = \frac{+\infty}{-\infty} (I) =$$

$$\left\{ \begin{array}{l} \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{-3x}{x^4} + \frac{x^4}{x^4}}}{\frac{2}{x^2} - \frac{5x}{x^2}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{-3}{x^3} + 1}}{\frac{2}{x^2} - \frac{5}{x}} = \frac{\sqrt{0+1}}{0-0} = \frac{1}{0} = -\infty \\ \lim_{x \rightarrow +\infty} \frac{\sqrt{x^4}}{-5x} = \lim_{x \rightarrow +\infty} \frac{x^2}{-5x} = \lim_{x \rightarrow +\infty} \frac{x}{-5} = \frac{+\infty}{-5} = -\infty \end{array} \right.$$

$$32) \lim_{x \rightarrow +\infty} \frac{\sqrt{4x+x^2}}{2+5x^2} = \frac{+\infty}{+\infty} (I) = \left\{ \begin{array}{l} \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{4x}{x^4} + \frac{x^2}{x^4}}}{\frac{2}{x^2} + \frac{5x^2}{x^2}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{4}{x^3} + \frac{1}{x^2}}}{\frac{2}{x^2} + 5} = \frac{\sqrt{0+0}}{0+5} = \frac{0}{5} = 0 \\ \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2}}{5x^2} = \lim_{x \rightarrow +\infty} \frac{x}{5x^2} = \lim_{x \rightarrow +\infty} \frac{2}{5x} = \frac{2}{+\infty} = 0 \end{array} \right.$$

$$33) \lim_{x \rightarrow -\infty} \frac{5x+1}{\sqrt[3]{x^2-3x-1}} = \lim_{x \rightarrow +\infty} \frac{5(-x)+1}{\sqrt[3]{(-x)^2-3(-x)-1}} = \lim_{x \rightarrow +\infty} \frac{-5x+1}{\sqrt[3]{x^2+3x-1}} = \frac{-\infty}{+\infty} (I) =$$

$$\left\{ \begin{array}{l} \lim_{x \rightarrow +\infty} \frac{-5x+1}{\sqrt[3]{x^2+3x-1}} = \lim_{x \rightarrow +\infty} \frac{\frac{-5x}{x} + \frac{1}{x}}{\sqrt[3]{\frac{x^2}{x^3} + \frac{3x}{x^3} - \frac{1}{x^3}}} = \lim_{x \rightarrow +\infty} \frac{-5 + \frac{1}{x}}{\sqrt[3]{\frac{1}{x} + \frac{3}{x^2} - \frac{1}{x^3}}} = \frac{-5+0}{\sqrt[3]{0+0-0}} = \frac{-5}{0} = -\infty \\ \lim_{x \rightarrow +\infty} \frac{-5x+1}{\sqrt[3]{x^2+3x-1}} = \lim_{x \rightarrow +\infty} \frac{-5x}{\sqrt[3]{x^2}} = \lim_{x \rightarrow +\infty} \frac{-5x}{\frac{2}{x^3}} = \lim_{x \rightarrow +\infty} (-5x^{\frac{1}{3}}) = \lim_{x \rightarrow +\infty} (-5\sqrt[3]{x}) = -\infty \end{array} \right.$$

$$34) \lim_{x \rightarrow 7} \frac{2-\sqrt{x-3}}{x^2-49} = \frac{0}{0} (I) = \lim_{x \rightarrow 7} \frac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x^2-49)(2+\sqrt{x-3})} = \lim_{x \rightarrow 7} \frac{(2)^2 - (\sqrt{x-3})^2}{(x^2-49)(2+\sqrt{x-3})} = \lim_{x \rightarrow 7} \frac{4-(x-3)}{(x^2-49)(2+\sqrt{x-3})} =$$

$$= \lim_{x \rightarrow 7} \frac{4-x+3}{(x^2-49)(2+\sqrt{x-3})} = \lim_{x \rightarrow 7} \frac{7-x}{(x^2-49)(2+\sqrt{x-3})} = \lim_{x \rightarrow 7} \frac{-(x-7)}{(x-7)(x+7)(2+\sqrt{x-3})} =$$

$$= \lim_{x \rightarrow 7} \frac{-1}{(x+7)(2+\sqrt{x-3})} = \frac{-1}{14 \cdot (2+2)} = -\frac{1}{56}$$

$$35) \lim_{x \rightarrow +\infty} \frac{3^x}{10x^2+5x-2} = \frac{+\infty}{+\infty} (I) = +\infty$$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de x.

$$36) \lim_{x \rightarrow +\infty} \frac{\sqrt{x^5-3}}{10x^2-9} = \frac{+\infty}{+\infty} (I) = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^5}}{10x^2} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{5}{2}}}{10x^2} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{1}{2}}}{10} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x}}{10} = +\infty$$

$$37) \lim_{x \rightarrow -\infty} \frac{\sqrt{x^5 - 3}}{10x^2 - 9} = \lim_{x \rightarrow +\infty} \frac{\sqrt{(-x)^5 - 3}}{10(-x)^2 - 9} = \lim_{x \rightarrow +\infty} \frac{\sqrt{-x^5 - 3}}{10x^2 - 9} = \frac{-\infty}{+\infty} (I) = \lim_{x \rightarrow +\infty} \frac{\sqrt{-x^5}}{10x^2} = \lim_{x \rightarrow +\infty} \frac{-x^{\frac{5}{2}}}{10x^2} = \lim_{x \rightarrow +\infty} \frac{-x^{\frac{1}{2}}}{10} =$$

$$= \lim_{x \rightarrow +\infty} \frac{-\sqrt{x}}{10} = -\infty$$

$$38) \lim_{x \rightarrow +\infty} \frac{\log(x^3 + 1)}{10x^2 + 1} = \frac{+\infty}{+\infty} (I) = 0$$

Las potencias de x son infinitos de orden superior a cualquier función logarítmica.

$$39) \lim_{x \rightarrow +\infty} \frac{5^x}{\log(x^3 + 1)} = \frac{+\infty}{+\infty} (I) = +\infty$$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier función logarítmica.

$$40) \lim_{x \rightarrow +\infty} (2^x - \sqrt{x^5 - 1}) = \infty - \infty (I) = +\infty$$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de x .

$$41) \lim_{x \rightarrow +\infty} (10x^2 - \sqrt{x^5 - 1}) = \infty - \infty (I) = -\infty$$

Dadas dos potencias de x la de mayor exponente es un infinito de orden superior

$$42) \lim_{x \rightarrow +\infty} [\log(x^3) - 10x^2] = \infty - \infty (I) = -\infty$$

Las potencias de x son infinitos de orden superior a cualquier función logarítmica.

$$43) \lim_{x \rightarrow +\infty} (e^x - x^3) = \infty - \infty (I) = +\infty$$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de x .

$$44) \lim_{x \rightarrow +\infty} \frac{x^2 + 1}{e^x} = \frac{+\infty}{+\infty} (I) = 0$$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de x .

$$45) \lim_{x \rightarrow +\infty} (\sqrt{x^2 + x} - \sqrt{x + 7}) = \infty - \infty (I) = +\infty$$

Dadas dos potencias de x la de mayor exponente es un infinito de orden superior

$$46) \lim_{x \rightarrow +\infty} \frac{\ln(x^2 + 1)}{x} = \frac{+\infty}{+\infty} = 0$$

Las potencias de x son infinitos de orden superior a cualquier función logarítmica.

$$47) \lim_{x \rightarrow -\infty} (0,5^x + 1) = \lim_{x \rightarrow +\infty} (0,5^{-x} + 1) = 0,5^{-\infty} + 1 = +\infty + 1 = +\infty$$

$$48) \lim_{x \rightarrow -\infty} (2^{x+1} - 5) = 2^{-\infty} - 5 = 0 - 5 = -5$$

$$49) \lim_{x \rightarrow +\infty} \left(1, 2^x - \frac{3x^2}{x+1} \right) = \infty - \infty (I) = +\infty$$

Las funciones exponenciales de base mayor que 1 son infinitos de orden superior a cualquier potencia de x .

$$50) \lim_{x \leftarrow -\infty} \left(\frac{2x+7}{x} \right)^{1+3x} = \left[\lim_{x \leftarrow -\infty} \left(\frac{2x+7}{x} \right) \right]^{\lim_{x \leftarrow -\infty} (1+3x)} = 2^{-\infty} = 0$$

$$\circ \lim_{x \leftarrow -\infty} \left(\frac{2x+7}{x} \right) = \frac{-\infty}{-\infty} (I) = \lim_{x \leftarrow -\infty} \left(\frac{2x}{x} \right) = \lim_{x \leftarrow -\infty} 2 = 2$$

$$\circ \lim_{x \leftarrow -\infty} (1+3x) = -\infty$$

$$51) \lim_{x \rightarrow +\infty} \frac{x + \log x}{\log x} = \frac{+\infty}{+\infty} (I) = \lim_{x \rightarrow +\infty} \left(\frac{x}{\log x} + \frac{\log x}{\log x} \right) = \lim_{x \rightarrow +\infty} \left(\frac{x}{\log x} + 1 \right) = +\infty$$

Las potencias de x son infinitos de orden superior a cualquier función logarítmica.

$$52) \lim_{x \rightarrow +\infty} \frac{3 \cdot 2^x}{2^x + 1} = \frac{+\infty}{+\infty} (I) = \lim_{x \rightarrow +\infty} \frac{3 \cdot 2^x}{2^x} = \lim_{x \rightarrow +\infty} 3 = 3$$