

ECUACIONES IRRAACIONALES

$$2) x-1 + \sqrt{x-2} = 13$$

$$\sqrt{x-2} = 13 - x + 1 \Rightarrow \sqrt{x-2} = 14 - x \Rightarrow$$

$$\Rightarrow (\sqrt{x-2})^2 = (14-x)^2 \Rightarrow x-2 = 196 + x^2 - 28x$$

$$\Rightarrow x^2 - 29x + 198 = 0 \Rightarrow$$

$$x = \frac{29 \pm \sqrt{29^2 - 4 \cdot 198}}{2} = \frac{29 \pm \sqrt{841 - 792}}{2} =$$

$$= \frac{29 \pm \sqrt{49}}{2} = \frac{29 \pm 7}{2} \begin{cases} 18 \\ 11 \end{cases}$$

veamos si algunas

de estas "candidatas" son realmente solución de la ecuación original:

$$x=11 \Rightarrow 11-1 + \sqrt{11-2} = 13 \Rightarrow 10 + 3 = 13 \Rightarrow \text{Sí}$$

$$\boxed{x=11 \text{ sí es solución}}$$

$$x=18 \Rightarrow 18-1 + \sqrt{18-2} = 13 \Rightarrow 17 + 4 = 13 \Rightarrow$$

$$\text{luego } \boxed{x=18 \text{ no es solución}} \quad 21 = 13 \quad \text{✗}$$

$$6) \sqrt{x-5} = \sqrt{2x-6} - 2 \Rightarrow$$

$$(\sqrt{x-5})^2 = (\sqrt{2x-6} - 2)^2 \Rightarrow$$

$$\Rightarrow x-5 = 2x-6 + 4 - 4\sqrt{2x-6} \Rightarrow$$

$$\Rightarrow x-5 = 2x-2 - 4\sqrt{2x-6} \Rightarrow$$

$$\Rightarrow x-5 - 2x + 2 = -4\sqrt{2x-6} \Rightarrow$$

$$-x-3 = -4\sqrt{2x-6}$$

$$(-x-3)^2 = (-4\sqrt{2x-6})^2$$

$$x^2 + 9 + 6x = 16(2x-6)$$

$$x^2 + 9 + 6x = 32x - 96$$

$$x^2 - 26x + 105 = 0$$

$$x = \frac{26 \pm \sqrt{(-26)^2 - 4 \cdot 105}}{2} = \frac{26 \pm \sqrt{676 - 420}}{2} =$$

$$= \frac{26 \pm \sqrt{256}}{2} = \frac{26 \pm 16}{2}$$

$$\begin{array}{l} 42/2 = 21 \\ 40/2 = 20 \end{array}$$

"candidatos" a solución son 5 y 21 veamos:

$$x=5 \quad \sqrt{5-5} = \sqrt{2 \cdot 5 - 6} - 2 \Rightarrow 0 = 2 - 2$$

$$\boxed{x=5 \text{ si es solución}} \quad 0=0 \Rightarrow \text{Sí}$$

$$x=21 \quad \sqrt{21-5} = \sqrt{2 \cdot 21 - 6} - 2$$

$$4 = \sqrt{36} - 2 \Rightarrow 4 = 4 \Rightarrow \text{Sí}$$

$$11) 2\sqrt{2x-7} - \sqrt{2x-15} = \sqrt{6x-23}$$

$$(2\sqrt{2x-7} - \sqrt{2x-15})^2 = (\sqrt{6x-23})^2$$

$$4(2x-7) + (2x-15) - 4\sqrt{(2x-7)(2x-15)} = 6x-23$$

$$8x - 28 + 2x - 15 - 4\sqrt{(2x-7)(2x-15)} = 6x - 23$$

$$10x - 43 - 4\sqrt{(2x-7)(2x-15)} = 6x - 23$$

$$-4\sqrt{(2x-7)(2x-15)} = 6x - 23 + 43 - 10x$$

$$-4\sqrt{(2x-7)(2x-15)} = -4x + 20$$

$$-\cancel{4}\sqrt{(2x-7)(2x-15)} = \cancel{4}(-x+5)$$

$$-\sqrt{(2x-7)(2x-15)} = -x+5$$

$$\left[-\sqrt{(2x-7)(2x-15)}\right]^2 = (-x+5)^2$$

$$(2x-7)(2x-15) = x^2 + 25 - 10x$$

$$\underline{4x^2} - \underline{30x} - \underline{14x} + \underline{105} = \underline{x^2} + 25 - \underline{10x}$$

$$3x^2 - 34x + 80 = 0$$

$$x = \frac{34 \pm \sqrt{(-34)^2 - 4 \cdot 3 \cdot 80}}{6} = \frac{34 \pm \sqrt{1156 - 960}}{6}$$

$$= \frac{34 \pm \sqrt{196}}{6} = \frac{34 \pm 14}{6}$$

$\begin{cases} 8 \rightarrow \text{si} \\ \frac{20}{6} = \frac{10}{3} \rightarrow \text{no} \end{cases}$

¡comprobemos!

ECUACIONES IRRACIONALES

$$7) \sqrt{3x+1} + \sqrt{10-x} = 5$$

$$\sqrt{3x+1} = 5 - \sqrt{10-x}$$

$$(\sqrt{3x+1})^2 = (5 - \sqrt{10-x})^2$$

$$3x+1 = 25 + 10-x - 10\sqrt{10-x}$$

$$3x+1 = 35-x - 10\sqrt{10-x}$$

$$3x+x+1-35 = -10\sqrt{10-x}$$

$$4x-34 = -10\sqrt{10-x}$$

$$\cancel{2}(2x-17) = -\cancel{2} \cdot 5\sqrt{10-x}$$

$$2x-17 = -5\sqrt{10-x}$$

$$(2x-17)^2 = 25(10-x)$$

$$4x^2 + 289 - 68x = 250 - 25x$$

$$4x^2 + 289 - \underline{68x} - 250 + \underline{25x} = 0$$

$$4x^2 - 43x + 39 = 0$$

$$x = \frac{43 \pm \sqrt{(-43)^2 - 16 \cdot 39}}{8} = \frac{43 \pm \sqrt{1849 - 624}}{8} =$$
$$= \frac{43 \pm \sqrt{1225}}{8} = \frac{43 \pm 35}{8} = \begin{cases} \frac{43+35}{8} = 39 \\ \frac{43-35}{8} = 1 \end{cases}$$

"candidates" $x = 39$ y $x = 1$ falta

$$\underline{x=1}$$

ver si verifican^{o no} la
ecuac original.

$$10) \sqrt{x^2-1} + 3 = 2\sqrt{x^2+4x+5} - x$$

$$\sqrt{x^2-1} + 3 + x = 2\sqrt{x^2+4x+5}$$

$$(\sqrt{x^2-1} + 3+x)^2 = (2\sqrt{x^2+4x+5})^2$$

$$x^2-1 + (3+x)^2 + 2(3+x)\sqrt{x^2-1} = 4(x^2+4x+5)$$

$$\underline{x^2-1} + \underline{9} + \underline{x^2} + \underline{6x} + 2(3+x)\sqrt{x^2-1} = 4x^2 + 16x + 20$$

$$2x^2 + 8 + 6x + 2(3+x)\sqrt{x^2-1} = 4x^2 + 16x + 20$$

$$2(3+x)\sqrt{x^2-1} = \underline{4x^2} + \underline{16x} + \underline{20} - \underline{6x} - \underline{8} - \underline{2x^2}$$

$$2(3+x)\sqrt{x^2-1} = 2x^2 + 10x + 12$$

$$\cancel{2}(3+x)\sqrt{x^2-1} = \cancel{2}(x^2 + 5x + 6)$$

$$(3+x)\sqrt{x^2-1} = (x^2 + 5x + 6) \rightarrow \text{¡¡dijato!!}$$

$$[(3+x)\sqrt{x^2-1}] = (\underline{x+3})(x+2) \quad \begin{array}{l} \text{las raíces} \\ \text{son } -3 \text{ y } -2 \end{array}$$

luego: 1:) si $x+3 \neq 0$ podemos simplificar por $x+3$ los dos miembros.

$$(\cancel{x+3})\sqrt{x^2-1} = (\cancel{x+3})(x+2)$$

$$\sqrt{x^2-1} = x+2$$

$$(\sqrt{x^2-1})^2 = (x+2)^2$$

$$\cancel{x^2}-1 = \cancel{x^2} + 4 + 4x$$

$$4x + 5 = 0 \Rightarrow \boxed{x = -5/4}$$

comprobar si verifica la ecuación inicial

2:) si $x+3=0$ queda luego para $\boxed{x = -3}$ otra solución

$$(\underline{x+3})\sqrt{x^2-1} = (\underline{x+3})(x+2)$$

$$11) 2\sqrt{2x-7} - \sqrt{2x-15} = \sqrt{6x-23}$$

$$(2\sqrt{2x-7} - \sqrt{2x-15})^2 = (\sqrt{6x-23})^2$$

$$4(2x-7) + 2x-15 - 4\sqrt{2x-7}\sqrt{2x-15} = 6x-23$$

$$8x-28+2x-15-4\sqrt{(2x-7)(2x-15)} = 6x-23$$

$$10x-43-4\sqrt{(2x-7)(2x-15)} = 6x-23$$

$$-4\sqrt{(2x-7)(2x-15)} = 6x-23-10x+43$$

$$-4\sqrt{(2x-7)(2x-15)} = -4x+20$$

$$\cancel{-4}\sqrt{(2x-7)(2x-15)} = \cancel{4}(-x+5)$$

$$-\sqrt{(2x-7)(2x-15)} = (-x+5)$$

$$\left[-\sqrt{(2x-7)(2x-15)}\right]^2 = (-x+5)^2$$

$$(2x-7)(2x-15) = x^2 + 25 - 10x$$

$$\underline{4x^2} - \underline{30x} - \underline{14x} + \underline{105} = \underline{x^2} + \underline{25} - \underline{10x}$$

$$3x^2 - 34x + 80 = 0 \Rightarrow x = \frac{34 \pm \sqrt{(-34)^2 - 12 \cdot 80}}{6}$$

$$= \frac{34 \pm \sqrt{1156 - 960}}{6} = \frac{34 \pm \sqrt{196}}{6} = \frac{34 \pm 14}{6} =$$

$$= \left\{ \begin{array}{l} \frac{34+14}{6} = \frac{48}{6} = 8 \\ \frac{34-14}{6} = \frac{20}{6} = \frac{10}{3} \end{array} \right\} \left\{ \begin{array}{l} x=8 \\ x=\frac{10}{3} \end{array} \right\} \text{ "candidatos" } \\ \text{ver si verifican} \\ \text{o no la ecuaci3n}$$

$$13) \sqrt{4x^2 + 4x - 1} = \sqrt{x^2 - 1} + x$$

$$(\sqrt{4x^2 + 4x - 1})^2 = (\sqrt{x^2 - 1} + x)^2$$

$$\cancel{4x^2} + 4x - \cancel{1} = \cancel{x^2} - \cancel{1} + \cancel{x^2} + 2x\sqrt{x^2 - 1}$$

$$2x^2 + 4x = 2x\sqrt{x^2 - 1}$$

$$\underline{x}(2x + 4) = 2\underline{x}\sqrt{x^2 - 1}$$

1.) si $x=0$ queda $\underset{0}{x}(2x+4) = 2\underbrace{x}_0\sqrt{x^2-1}$
 $0 = 0$

$x=0$ es una posible solución, veamos si verifica la ecuación original:

$$\sqrt{0^2 + 0 - 1} = \sqrt{0^2 - 1} + 0$$

$$\sqrt{-1} = \sqrt{-1} \quad \sqrt{-1} \text{ no es un n.º real}$$

luego $x=0$ no es solución

2.) si $x \neq 0$ podemos simplificar y queda:

$$\cancel{x}(2x+4) = 2\cancel{x}\sqrt{x^2-1}$$

$$\cancel{x}(x+2) = 2\sqrt{x^2-1}$$

$$(x+2)^2 = (\sqrt{x^2-1})^2$$

$$\cancel{x^2} + 4 + 4x = \cancel{x^2} - 1 \Rightarrow 4x = -5 \Rightarrow \boxed{x = -5/4}$$

¿es una posible solución? veamos:

$$\sqrt{4(-\frac{5}{4})^2 + 4(-\frac{5}{4}) - 1} = \sqrt{(-\frac{5}{4})^2 - 1} + (-\frac{5}{4})$$

$$\sqrt{\frac{25}{4} - 5 - 1} = \sqrt{\frac{25}{16} - 1 - \frac{5}{4}} \Rightarrow \sqrt{\frac{25}{4} - 6} = \sqrt{\frac{9}{16}} - \frac{5}{4}$$