

Resuelve las siguientes integrales:

$$\text{a) } \int \frac{2x-4}{(x-1)^2 (x+3)} dx$$

$$\text{b) } \int \frac{2x+3}{(x-2)(x+5)} dx$$

$$\text{c) } \int \frac{1}{(x-1)(x+3)^2} dx$$

$$\text{d) } \int \frac{3x-2}{x^2-4} dx$$

$$\text{a) } \int \frac{2x-4}{(x-1)^2 (x+3)} dx$$

Descomponemos en fracciones simples:

$$\frac{2x-4}{(x-1)^2 (x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$$

$$\frac{2x-4}{(x-1)^2 (x+3)} = \frac{A(x-1)(x+3) + B(x+3) + C(x-1)^2}{(x-1)^2 (x+3)}$$

$$2x-4 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$$

Hallamos A , B y C :

$$\left. \begin{array}{lll} x=1 & \rightarrow & -2 = 4B \quad \rightarrow \quad B = -1/2 \\ x=-3 & \rightarrow & -10 = 16C \quad \rightarrow \quad C = -5/8 \\ x=0 & \rightarrow & -4 = -3A + 3B + C \quad \rightarrow \quad A = 5/8 \end{array} \right\}$$

Por tanto:

$$\begin{aligned} \int \frac{2x-4}{(x-1)^2 (x+3)} dx &= \int \frac{5/8}{x-1} dx + \int \frac{-1/2}{(x-1)^2} dx + \int \frac{-5/8}{x+3} dx = \\ &= \frac{5}{8} \ln|x-1| + \frac{1}{2} \cdot \frac{1}{(x-1)} - \frac{5}{8} \ln|x+3| + k = \frac{5}{8} \ln \left| \frac{x-1}{x+3} \right| + \frac{1}{2x-2} + k \end{aligned}$$

$$\text{b) } \int \frac{2x+3}{(x-2)(x+5)} dx$$

Descomponemos en fracciones simples:

$$\frac{2x+3}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} = \frac{A(x+5) + B(x-2)}{(x-2)(x+5)}$$

$$2x+3 = A(x+5) + B(x-2)$$

Hallamos A y B :

$$\left. \begin{array}{lll} x=2 & \rightarrow & 7 = 7A \quad \rightarrow \quad A = 1 \\ x=-5 & \rightarrow & -7 = -7B \quad \rightarrow \quad B = 1 \end{array} \right\}$$

Por tanto:

$$\begin{aligned} \int \frac{2x+3}{(x-2)(x+5)} dx &= \int \frac{1}{x-2} dx + \int \frac{1}{x+5} dx = \\ &= \ln|x-2| + \ln|x+5| + k = \ln|(x-2)(x+5)| + k \end{aligned}$$

$$c) \int \frac{1}{(x-1)(x+3)^2} dx$$

Descomponemos en fracciones simples:

$$\frac{1}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$\frac{1}{(x-1)(x+3)^2} = \frac{A(x+3)^2 + B(x-1)(x+3) + C(x-1)}{(x-1)(x+3)^2}$$

$$1 = A(x+3)^2 + B(x-1)(x+3) + C(x-1)$$

Hallamos A , B y C :

$$\left. \begin{array}{lll} x=1 & \rightarrow & 1=16A \quad \rightarrow \quad A=1/16 \\ x=-3 & \rightarrow & 1=-4C \quad \rightarrow \quad C=-1/4 \\ x=0 & \rightarrow & 1=9A-3B-C \quad \rightarrow \quad B=-1/16 \end{array} \right\}$$

Por tanto:

$$\begin{aligned} \int \frac{1}{(x-1)(x+3)^2} dx &= \int \frac{1/16}{x-1} dx + \int \frac{-1/16}{x+3} dx + \int \frac{-1/4}{(x+3)^2} dx = \\ &= \frac{1}{16} \ln|x-1| - \frac{1}{16} \ln|x+3| + \frac{1}{4} \cdot \frac{1}{(x+3)} + k = \\ &= \frac{1}{16} \ln \left| \frac{x-1}{x+3} \right| + \frac{1}{4(x+3)} + k \end{aligned}$$

$$d) \int \frac{3x-2}{x^2-4} dx = \int \frac{3x-2}{(x-2)(x+2)} dx$$

Descomponemos en fracciones simples:

$$\frac{3x-2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2) + B(x-2)}{(x-2)(x+2)}$$

$$3x-2 = A(x+2) + B(x-2)$$

Hallamos A y B :

$$\left. \begin{array}{lll} x=2 & \rightarrow & 4=4A \quad \rightarrow \quad A=1 \\ x=-2 & \rightarrow & -8=-4B \quad \rightarrow \quad B=2 \end{array} \right\}$$

Por tanto:

$$\begin{aligned} \int \frac{3x-2}{x^2-4} dx &= \int \frac{1}{x-2} dx + \int \frac{2}{x+2} dx = \\ &= \ln|x-2| + 2 \ln|x+2| + k = \ln[|x-2|(x+2)^2] + k \end{aligned}$$