EJERCICIOS DE LÍMITES CON L'HOPITAL

Calcular los límites planteados:

1) $\lim\limits_{x\to 0} \frac{e^{x} - e^{-x}}{\operatorname{sen} x}$

$$\lim_{x\to 0} \frac{e^{x} - e^{-x}}{\operatorname{sen} x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{e^{x} - e^{-x}}{\operatorname{sen} x} = \lim_{x\to 0} \frac{e^{x} + e^{-x}}{\cos x} = 2$$

2) $\lim\limits_{x\to\infty^+} \frac{x}{(\ln x)^3 + 2x}$

$$\lim_{x\to\infty^+} \frac{x}{(\ln x)^3 + 2x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty^+} \frac{x}{(\ln x)^3 + 2x} = \lim_{x\to\infty^+} \frac{1}{3(\ln x)^2 \frac{1}{x} + 2}$$

Si **comparamos infinitos** observamos que el numerador es un infinito de orden inferior al denominador, por tanto el límite es 0.

$$\lim_{x\to\infty^+} \frac{3(\ln x)^2}{x} = \frac{\infty}{\infty} \qquad \lim_{x\to\infty^+} \frac{3(\ln x)^2}{x} = 0$$

$$\lim_{x\to\infty^+} \frac{1}{3(\ln x)^2 \frac{1}{x} + 2} = \frac{1}{2}$$

3) $\lim\limits_{x\to 0}\left(\frac{1}{x}-\frac{1}{sen\,x}\right)$

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{sen\,x}\right)=\infty-\infty$$

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{sen\,x}\right)=\lim_{x\to 0}\frac{senx-x}{x\,senx}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{senx-x}{x\,senx}=\lim_{x\to 0}\frac{cos\,x-1}{senx+x\,cos\,x}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{cos\,x-1}{senx+x\,cos\,x}=\lim_{x\to 0}\frac{-senx}{cos\,x+cos\,x-x\,senx}=0$$

4) $\lim\limits_{x\to\frac{\pi}{4}}(tg\,x-1)\,sec\,2x$

$$\lim_{x\to\frac{\pi}{4}}(tg\,x-1)\,sec\,2x=0\cdot\infty$$

$$\lim_{x\to\frac{\pi}{4}}\frac{tg\,x-1}{cos\,2x}=\frac{0}{0}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{tg\,x-1}{cos\,2x}=\frac{1+tg^2x}{-2\,sen2x}=-1$$

5) $\lim\limits_{x \to 0} x^{\operatorname{tg} x}$

$$A = x^{\operatorname{tg} x} \qquad \ln A = \operatorname{tg} x \ln x \qquad A = e^{\operatorname{tg} x \ln x}$$

$$\lim_{x \to 0} x^{\operatorname{tg} x} = \lim_{x \to 0} e^{\operatorname{tg} x \ln x} = e^{\lim\limits_{x \to a}\left(e^{\operatorname{tg} x \ln x}\right)} = e^{\lim\limits_{x \to a} \frac{\ln x}{\operatorname{cotg} x}} = e^{\lim\limits_{x \to a}\left(-\frac{\frac{1}{x}}{\operatorname{cosec}^2 x}\right)} =$$

$$= e^{\lim\limits_{x \to a} \frac{-\operatorname{sen}^2 x}{x}} = e^{\lim\limits_{x \to a}\left(-2 \operatorname{sen} x \cos x\right)} = e^0 = 1$$

6) $\lim\limits_{x \to 0} \left(\operatorname{cotg} x\right)^{\operatorname{sen} x}$

$$A = \left(\operatorname{cotg} x\right)^{\operatorname{sen} x} \qquad \ln A = \operatorname{sen} x \ln\left(\operatorname{cotg} x\right)$$

$$A = e^{\operatorname{sen} x \ln\left(\operatorname{cotg} x\right)}$$

$$\lim_{x \to 0} \left(\operatorname{cotg} x\right)^{\operatorname{sen} x} = \lim_{x \to 0} e^{\operatorname{sen} x \ln\left(\operatorname{cotg} x\right)} = e^{\lim\limits_{x \to a} \frac{\ln\left(\operatorname{cotg} x\right)}{\operatorname{cosec} x}} =$$

$$e^{\lim\limits_{x \to a} \frac{\frac{-\operatorname{cosec}^2 x}{\operatorname{cotg} x}}{\frac{-\cos x}{\operatorname{sen}^2 x}}} = e^{\lim\limits_{x \to a} \frac{\operatorname{cosec}^2 x \operatorname{sen}^2 x}{\cos x \operatorname{cotg} x}} = e^{\lim\limits_{x \to a} \frac{1}{\cos x \frac{\cos x}{\operatorname{sen} x}}} = e^{\lim\limits_{x \to a} \frac{\operatorname{sen} x}{\cos^2 x}} = e^0 = 1$$

7) $\lim_{x\to 2}\left(\frac{x}{2}\right)^{\frac{1}{x-2}}$

$$\lim_{x\to 2}\left(\frac{x}{2}\right)^{\frac{1}{x-2}} = 1^{\infty} \qquad A = \left(\frac{x}{2}\right)^{\frac{1}{x-2}}$$

$$\ln A = \frac{1}{x-2}\ln\left(\frac{x}{2}\right) \qquad A = e^{\frac{1}{x-2}\ln\left(\frac{x}{2}\right)}$$

$$\lim_{x\to 2}\left(\frac{x}{2}\right)^{\frac{1}{x-2}} = e^{\lim_{x\to 2}\left[\frac{1}{x-2}\ln\left(\frac{x}{2}\right)\right]} = e^{\lim_{x\to 2}\left[\frac{\ln\left(\frac{x}{2}\right)}{x-2}\right]} =$$

$$e^{\lim_{x\to 2}\frac{\frac{1}{2}}{\frac{x}{2}}} = e^{\frac{1}{2}} = \sqrt{e}$$

8) $\lim_{x\to 1}\frac{\ln x}{x-1}$

$$\lim_{x\to 1}\frac{\ln x}{x-1} = \frac{0}{0}$$

$$\lim_{x\to 1}\frac{\ln x}{x-1} = \lim_{x\to 1}\frac{1}{x} = 1$$

9) $\lim_{x\to 0}\frac{\text{sen}3x}{x-\frac{3}{2}\text{sen}2x}$

$$\lim_{x\to 0}\frac{\text{sen}3x}{x-\frac{3}{2}\text{sen}2x} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\text{sen}3x}{x-\frac{3}{2}\text{sen}2x} = \lim_{x\to 0}\frac{3\cos 3x}{1-3\cos 2x} = -\frac{3}{2}$$

10) $\lim_{x\to 0}\left(arc\ sen\,x\ \cot g\,x\right)$

$$\lim_{x\to 0}\left(arc\ sen\,x\ \cot g\,x\right) = 0\cdot\infty$$

$$\lim_{x\to 0}\left(arc\ sen\,x\ \cot g\,x\right) = \lim_{x\to 0}\frac{\cos x\, arc\ sen\,x}{sen\,x} =$$

$$= \lim_{x\to 0}\frac{-sen\,x\, arc\ sen\,x + \dfrac{\cos x}{\sqrt{1-x^2}}}{\cos x} = 1$$

11) $\lim_{x\to 0}\left[\frac{1}{2}\frac{\text{sen}\,x}{\text{tg}\,x}\left(1+\text{tg}\,2x\right)^{\frac{4}{x}}\right]$

$$\lim_{x\to 0}\left[\frac{1}{2}\frac{\text{sen}\,x}{\frac{\text{sen}x}{\cos x}}\left(1+\text{tg}\,2x\right)^{\frac{4}{x}}\right] = \frac{1}{2}\lim_{x\to 0}\left(\cos x\right)\lim_{x\to 0}\left(1+\text{tg}\,2x\right)^{\frac{4}{x}} =$$

$$= \frac{1}{2}\lim_{x\to 0}\left(1+\text{tg}\,2x\right)^{\frac{4}{x}} = 1^{\infty}$$

$$A = \left(1+\text{tg}\,2x\right)^{\frac{4}{x}} \qquad \ln A = \frac{4}{x}\ln\left(1+\text{tg}\,2x\right) \qquad A = e^{\frac{4}{x}\ln(1+\text{tg}\,2x)}$$

$$\frac{1}{2}\lim_{x\to 0}\left(1+\text{tg}\,2x\right)^{\frac{4}{x}} = \frac{1}{2}e^{\lim_{x\to 0}\frac{4\ln(1+\text{tg}\,2x)}{x}} = \frac{1}{2}e^{\lim_{x\to 0}\frac{4\cdot 2\left(1+\text{tg}^2\,2x\right)}{\ln(1+\text{tg}\,2x)}} = \frac{1}{2}e^{8}$$

12) $\lim_{x\to 0} \frac{\ln(1+x) - senx}{x\, senx}$

$$\lim_{x\to 0} \frac{\ln(1+x) - senx}{x\, senx} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\ln(1+x) - senx}{x\, senx} = \lim_{x\to 0} \frac{\frac{1}{1+x} - \cos x}{senx + x\cos x} =$$

$$= \lim_{x\to 0} \frac{-\frac{1}{(1+x)^2} + senx}{\cos x + \cos x - xsenx} = -\frac{1}{2}$$

13) $\lim_{x\to 0} \frac{1 + senx - e^x}{(arc\ tgx)^2}$

$$\lim_{x\to 0} \frac{1 + senx - e^x}{(arc\ tgx)^2} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{1 + senx - e^x}{(arc\ tgx)^2} = \lim_{x\to 0} \frac{\cos x - e^x}{\frac{2\, arc\ tgx}{1+x^2}} =$$

$$= \lim_{x\to 0} \frac{senx - e^x}{\frac{2 - 4x\, arc\ tgx}{(1+x^2)^2}} = -\frac{1}{2}$$

14) $\lim_{x\to 0}\left[\frac{1}{\ln(1+x)}-\frac{1}{x}\right]$

$$\lim_{x\to 0}\left[\frac{1}{\ln(1+x)}-\frac{1}{x}\right]=\infty-\infty$$

$$\lim_{x\to 0}\left[\frac{1}{\ln(1+x)}-\frac{1}{x}\right]=\lim_{x\to 0}\frac{x-\ln(1+x)}{x\ln(1+x)}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{x-\ln(1+x)}{x\ln(1+x)}=\lim_{x\to 0}\frac{1-\frac{1}{1+x}}{\ln(1+x)+\frac{x}{1+x}}=$$

$$\lim_{x\to 0}\frac{\frac{1+x-1}{1+x}}{\frac{(1+x)\ln(1+x)+x}{1+x}}=\lim_{x\to 0}\frac{x}{\ln(1+x)+x\ln(1+x)+x}=$$

$$=\lim_{x\to 0}\frac{1}{\frac{1}{1+x}+\ln(1+x)+\frac{x}{1+x}+1}=\frac{1}{2}$$

15) $\lim_{x \to 0} \left(\frac{1+tgx}{1+senx} \right)^{\frac{1}{senx}}$

$$\lim_{x \to 0} \left(\frac{1+tgx}{1+senx} \right)^{\frac{1}{senx}} = 1^{\infty}$$

$$A = \left(\frac{1+tgx}{1+senx} \right)^{\frac{1}{senx}} \qquad \ln A = \frac{1}{senx} \ln\left(\frac{1+tgx}{1+senx} \right)$$

Aplicando las **propiedades de los logaritmos** en el segundo miembro tenemos:

$$\ln A = \frac{1}{senx} \left[\ln(1+tgx) - \ln(1+senx) \right]$$

$$\ln A = \frac{\ln(1+tgx) - \ln(1+senx)}{senx}$$

$$A = e^{\frac{\ln(1+tgx) - \ln(1+senx)}{senx}}$$

$$\lim_{x \to 0} \left(\frac{1+tgx}{1+senx} \right)^{\frac{1}{senx}} = e^{\lim_{x \to a} \frac{\ln(1+tgx) - \ln(1+senx)}{senx}} =$$

$$= e^{\lim_{x \to a} \frac{\frac{1+tg^2 x}{1+tgx} - \frac{\cos x}{1+senx}}{\cos x}} = e^0 = 1$$

16) $\lim\limits_{x \to 0} x^{\operatorname{sen} x}$

$$\lim_{x \to 0} x^{\operatorname{sen} x} = 0^0$$

$$A = x^{\operatorname{sen} x} \qquad \ln A = \operatorname{sen} x \ln x \qquad A = e^{\operatorname{sen} x \ln x}$$

$$\lim_{x \to 0} x^{\operatorname{sen} x} = e^{\lim\limits_{x \to 0} (\operatorname{sen} x \ln x)} = e^{\lim\limits_{x \to 0} \frac{\ln x}{\frac{1}{\operatorname{sen} x}}} = e^{\lim\limits_{x \to 0} \frac{\frac{1}{x}}{\frac{-\cos x}{\operatorname{sen}^2 x}}} =$$

$$e^{\lim\limits_{x \to 0} \frac{-\operatorname{sen}^2 x}{x \cos x}} = e^{\lim\limits_{x \to 0} \frac{-2 \operatorname{sen} x \cos x}{\cos x - x \operatorname{sen} x}} = e^0 = 1$$