EJERCICIOS DE APLICACIONES. LA INTEGRAL INDEFINIDA.

1.- Calcula las siguientes integrales inmediatas.

a) $\int -4(3-2x^2)dx$; b) $\int 2x\sqrt{x^2-1}\ dx$ c) $\int x(1+2x^2)^3\ dx$

d) $\int x^2(2-2x^3)^3\ dx$ e) $\int \sqrt{7+2x}\ dx$ f) $\int (x^2+1)^2\ dx$

g) $\int \frac{2x}{x^2+3}dx$ h) $\int \frac{2}{x-3}dx$ i) $\int \frac{x}{(3x^2-2)^3}dx$

j) $\int \frac{x}{\sqrt[3]{2-x^2}}dx$ k) $\int (e^{-3x}+e^{x-2})dx$ l) $\int \frac{3^{5x-1}}{7}dx$

m) $\int \frac{x}{e^{x^2}}dx$ n) $\int sen(-x)\,dx$ ñ) $\int \frac{1}{\cos^2(x+1)}dx$

o) $\int \frac{x}{\cos^2(x^2-3)}dx$ p) $\int \frac{1}{\sqrt{1-25x^2}}dx$ q) $\int \frac{1}{\sqrt{1-(2x-3)^2}}dx$

r) $\int \frac{x}{1+9x^4}dx$

Resuelve las siguientes integrales (son por partes)

a) $\int Ln\,x\,dx$ b) $\int x\,Ln\,x\,dx$ c) $\int x^2 sen\,x\ dx$

d) $\int e^x(7+2x)\,dx$ e) $\int (4x^2-3x)\cos x\ dx$

Resuelve las siguientes integrales racionales:

a) $\int \frac{2}{x^2-1}dx$ b) $\int -\frac{3}{x^2+x-2}dx$ c) $\int \frac{2x+1}{x^4-5x^2+4}dx$

d) $\int \frac{7x-2}{x^3-2x^2-x+2}dx$ e) $\int \frac{3x-2}{(2-x)^2}dx$ f) $\int \frac{-2x^2+1}{x^3+6x^2+12x+8}dx$

g) $\int \frac{x-2}{x^4}dx$ h) $\int \frac{4x^2-2x}{(x+2)(x-3)^2}dx$ i) $\int \frac{-x^2+7x}{x^3-x^2-x+1}\ dx$

SOLUCIONES

1. a) $$\int -4(3-2x^2)\,dx = \int (-12+8x^2)\,dx = -12x + 8\frac{x^3}{3} + C$$

b) $$\int 2x\sqrt{x^2-1}\,dx = \int 2x(x^2-1)^{1/2}\,dx = \frac{(x^2-1)^{3/2}}{3/2} + C$$

c) $$\int x(1+2x^2)^3\,dx = \frac{1}{4}\int 4x(1+2x^2)^3\,dx = \frac{1}{4}\,\frac{(1+2x^2)^4}{4} + C =$$

$$= \frac{(1+2x^2)^4}{16} + C$$

d) $$\int x^2(2-2x^3)^3\,dx = -\frac{1}{6}\int -6x^2(2-2x^3)^3\,dx =$$

$$= -\frac{1}{6}\,\frac{(2-2x^3)^4}{4} + C = \frac{-(2-2x^3)^4}{24} + C$$

e) $$\int \sqrt{7+2x}\,dx = \int (7+2x)^{1/2}\,dx = \frac{1}{2}\int 2(7+2x)^{1/2}\,dx =$$

$$= \frac{1}{2}\,\frac{(7+2x)^{3/2}}{3/2} + C = \frac{(7+2x)^{3/2}}{3} + C$$

f) $$\int (x^2+1)^2\,dx = \int (x^4+2x^2+1)\,dx = \frac{x^5}{5} + \frac{2x^3}{3} + x + C$$

g) $$\int \frac{2x}{x^2+3}\,dx = \ln|x^2+3| + C$$

h) $$\int \frac{2}{x-3}\,dx = 2\int \frac{1}{x-3}\,dx = 2\ln|x-3| + C$$

i) $\int \frac{x}{(3x^2-2)^3}dx = \int x\,(3x^2-2)^{-3}dx = \frac{1}{6}\int 6x(3x^2-2)^{-3}dx =$

$= \frac{1}{6}\,\frac{(3x^2-2)^{-2}}{-2} + C = \frac{-1}{12(3x^2-2)^2} + C$

j) $\int \frac{x}{\sqrt[3]{2-x^2}}dx = \int x\,(2-x^2)^{-1/3}dx = -\frac{1}{2}\int -2x\,(2-x^2)^{-1/3}dx$

$= -\frac{1}{2}\,\frac{(2-x^2)^{2/3}}{2/3} + C = -\frac{3}{4}\sqrt[3]{(2-x^2)^2} + C$

k) $\int (e^{-3x} + e^{x-2})dx = -\frac{1}{3}\int -3e^{-3x}dx + \int e^{x-2}dx =$

$= -\frac{1}{3}e^{-3x} + e^{x-2} + C$

l) $\int \frac{3^{5x-1}}{7}dx = \frac{1}{7}\int 3^{5x-1}dx = \frac{1}{7}\cdot\frac{1}{5}\int 5\cdot 3^{5x-1}dx =$

$= \frac{1}{35}\,\frac{3^{5x-1}}{\ln 3} + C$

m) $\int \frac{x}{e^{x^2}}dx = \int x\cdot e^{-x^2}dx = -\frac{1}{2}\int -2x\,e^{-x^2}dx =$

$= -\frac{1}{2}e^{-x^2} + C$

n) $\int \operatorname{sen}(-x)\,dx = -1\int -\operatorname{sen}(-x)\,dx = -(-\cos(-x)) + C =$

$= \cos(-x) + C$

ñ) $\int \frac{1}{\cos^2(x+1)}\,dx = \operatorname{tag}(x+1)+C$

o) $\int \frac{x}{\cos^2(x^2-3)}\,dx = \frac{1}{2}\int \frac{2x}{\cos^2(x^2-3)}\,dx = \frac{1}{2}\operatorname{tag}(x^2-3)+C$

p) $\int \frac{1}{\sqrt{1-25x^2}}\,dx = \int \frac{1}{\sqrt{1-(5x)^2}}\,dx = \frac{1}{5}\int \frac{5}{\sqrt{1-(5x)^2}}\,dx =$

$= \frac{1}{5}\operatorname{arcsen}(5x)+C$

q) $\int \frac{1}{\sqrt{1-(2x-3)^2}}\,dx = \frac{1}{2}\int \frac{2}{\sqrt{1-(2x-3)^2}}\,dx = \frac{1}{2}\operatorname{arcsen}(2x-3)+C$

r) $\int \frac{x}{1+9x^4}\,dx = \int \frac{x}{1+(3x^2)^2}\,dx = \frac{1}{6}\int \frac{6x}{1+(3x^2)^2}\,dx =$

$= \frac{1}{6}\operatorname{arctg}(3x^2)+C$

$$2.\ a)\int \ln x\,dx = \begin{cases} u=\ln x \to du=\frac{1}{x}dx \\ dv=dx \to v=x \end{cases} =$$

$$= x\cdot\ln x - \int x\cdot\frac{1}{x}dx = x\ln x - \int dx = x\ln x - x + C$$

$$b)\int x\cdot\ln x\cdot dx = \begin{cases} u=\ln x \to du=\frac{1}{x}dx \\ dv=x\,dx \Rightarrow v=\frac{x^2}{2} \end{cases} =$$

$$= \frac{x^2}{2}\cdot\ln x - \int\frac{x^2}{2}\frac{1}{x}dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx =$$

$$= \frac{x^2}{2}\cdot\ln x - \frac{1}{2}\frac{x^2}{2} + C = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$$

$$c)\int x^2\cdot \operatorname{sen} x\,dx = \begin{cases} u=x^2 \to du=2x\,dx \\ dv=\operatorname{sen} x\,dx \to v=-\cos x \end{cases}$$

$$= -x^2\cos x - \int -\cos x\cdot 2x\,dx = -x^2\cos x + 2\int x\cos x\,dx =$$

$$= \begin{cases} u=x \to du=dx \\ dv=\cos x\,dx \to v=\operatorname{sen} x \end{cases} = -x^2\cos x + 2\left[x\operatorname{sen} x - \int \operatorname{sen} x\,dx\right] =$$

$$= -x^2\cos x + 2x\operatorname{sen} x - 2(-\cos x) + C =$$

$$= -x^2\cos x + 2x\operatorname{sen} x + 2\cos x + C$$

d) $\int e^x(7+2x)\,dx = \begin{cases} u = 7+2x \to du = 2dx \\ dv = e^x dx \to v = e^x \end{cases}$

$= (7+2x)e^x - \int e^x \cdot 2dx = (7+2x)e^x - 2e^x + C$

e) $\int (4x^2-3x)\cos x\,dx = \begin{cases} u = 4x^2-3x \to du = (8x-3)dx \\ dv = \cos x\,dx \to v = \operatorname{sen} x \end{cases}$

$= (4x^2-3x)\operatorname{sen} x - \int \operatorname{sen} x(8x-3)\,dx = \begin{cases} u = 8x-3 \to du = 8dx \\ dv = \operatorname{sen} x\,dx \to v = -\cos x \end{cases}$

$= (4x^2-3x)\operatorname{sen} x - \left[(8x-3)(-\cos x) - \int -\cos x \cdot 8dx\right] =$

$= (4x^2-3x)\operatorname{sen} x + (8x-3)\cos x - 8\int \cos x\,dx =$

$= (4x^2-3x)\operatorname{sen} x + (8x-3)\cos x - 8\operatorname{sen} x + C =$

$= (4x^2-3x-8)\operatorname{sen} x + (8x-3)\cos x + C$

INTEGRALES RACIONALES

a) $$I=\int \frac{2}{x^2-1}\,dx=\int \frac{2}{(x+1)(x-1)}\,dx=\int\left(\frac{A}{x+1}+\frac{B}{(x-1)}\right)dx$$

$$\frac{2}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}=\frac{A(x-1)+B(x+1)}{(x+1)(x-1)} \rightarrow$$

$$2=A(x-1)+B(x+1)$$

Si $x=1 \quad 2=B\cdot 2 \rightarrow B=1$

Si $x=-1 \quad 2=-2A \rightarrow A=-1$

$$\Rightarrow I=\int\left(\frac{-1}{x+1}+\frac{1}{x-1}\right)dx=-\ln|x+1|+\ln|x-1|+C=$$

$$=\ln\left|\frac{x-1}{x+1}\right|+C$$

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b) $$I=\int \frac{-3}{x^2+x-2}\,dx=\int\frac{-3}{(x-1)(x+2)}\,dx=\int\left(\frac{A}{x-1}+\frac{B}{x+2}\right)dx$$

$x^2+x-2=0$

$$x=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm 3}{2}\begin{cases}1\\-2\end{cases}$$

$$\frac{-3}{x^2+x-2}=\frac{A}{x-1}+\frac{B}{x+2}=\frac{A(x+2)+B(x-1)}{x^2+x-2}$$

$$-3=A(x+2)+B(x-1)$$

Si $x=-2 \rightarrow -3=-3B \rightarrow B=1$

Si $x=1 \rightarrow -3=3A \rightarrow A=-1$

$$\Rightarrow I=\int\left[\frac{-1}{x-1}+\frac{1}{x+2}\right]dx=-\ln|x-1|+\ln|x+2|+C=\ln\left|\frac{x+2}{x-1}\right|+C$$

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c) $\displaystyle\int \frac{2x+1}{x^4-5x^2+4}\,dx = \int\left[\frac{A}{(x+2)}+\frac{B}{(x-2)}+\frac{C}{(x+1)}+\frac{D}{(x-1)}\right]dx$

$x^4-5x^2+4=0$

$x^2=\frac{5\pm\sqrt{25-16}}{2}=\frac{5\pm3}{2}<^{4}_{1} \to x=\pm2,\ x=\pm1$

$$\frac{2x+1}{x^4-5x^2+4}=\frac{A(x-2)(x+1)(x-1)+B(x+2)(x+1)(x-1)+C(x+2)(x-2)(x-1)+D(x+2)(x-2)(x+1)}{x^4-5x^2+4}$$

Si $x=2 \to 5=12B \to B=5/12$

Si $x=-2 \to -3=-40A \to A=3/40$

Si $x=1 \to 3=-6D \to D=-1/2$

Si $x=-1 \to -1=6C \to C=-1/6$

$$\Rightarrow I=\int\left[\frac{3/40}{x+2}+\frac{5/12}{x-2}+\frac{-1/6}{x+1}+\frac{-1/2}{x-1}\right]dx=$$

$$=\frac{3}{40}\ln|x+2|+\frac{5}{12}\ln|x-2|-\frac{1}{6}\ln|x+1|-\frac{1}{2}\ln|x-1|+C.$$

d) $$\int \frac{7x-2}{x^3-2x^2-x+2}\,dx = \int\left[\frac{A}{x-2}+\frac{B}{x-1}+\frac{C}{x+1}\right]dx$$

$$\begin{array}{r|rrrr}
 & 1 & -2 & -1 & 2 \\
2 & & 2 & 0 & -2 \\
\hline
 & 1 & 0 & -1 & \boxed{0} \\
1 & & 1 & 1 & \\
\hline
 & 1 & 1 & \boxed{0} & \\
-1 & & -1 & & \\
\hline
 & 1 & \boxed{0} & &
\end{array}$$

$$7x-2 = A(x-1)(x+1)+B(x-2)(x+1)+C(x-2)(x-1)$$

Si $x=1 \to 5=-2B \to B=-5/2$

Si $x=-1 \to -9=6C \to C=-3/2$

Si $x=2 \to 12=3A \to A=4$

$$I=\int\left[\frac{4}{x-2}+\frac{-5/2}{x-1}+\frac{-3/2}{x+1}\right]dx =$$

$$= 4\cdot\ln|x-2| - \frac{5}{2}\ln|x-1| - \frac{3}{2}\ln|x+1| + C$$

e) $$\int \frac{3x-2}{(2-x)^2}\,dx = \int\left(\frac{A}{2-x}+\frac{B}{(2-x)^2}\right)dx$$

$$\frac{3x-2}{\cancel{(2-x)^2}} = \frac{A(2-x)+B}{\cancel{(2-x)^2}}$$

$$\left.\begin{array}{l}\text{Si } x=2 \to 4=B \\ \text{Si } x=1 \to 1=A+B\end{array}\right\} \to A=1-4=-3$$

$$I=\int\left[\frac{-3}{2-x}+\frac{4}{(2-x)^2}\right]dx = -3\,Ln|2-x| + 4\int(2-x)^{-2}\,dx$$

$$= -3\,Ln|2-x| + 4(-1)\int(-1)(2-x)^{-2}\,dx = -3\,Ln|2-x| - 4\,\frac{(2-x)^{-1}}{-1} + C =$$

$$= -3\,Ln|2-x| + \frac{4}{2-x} + C$$

$$f)\ \int \frac{-2x^2+1}{x^3+6x^2+12x+8}\,dx = \int \frac{-2x^2+1}{(x+2)^3}\,dx =$$

$$\begin{array}{r|rrrr} & 1 & 6 & 12 & 8 \\ -2 & & -2 & -8 & -8 \\ \hline & 1 & 4 & 4 & \lfloor 0 \\ -2 & & -2 & -4 & \\ \hline & 1 & 2 & \lfloor 0 & \\ -2 & & -2 & & \\ \hline & 1 & \lfloor 0 & & \end{array}$$

$$= \int \left[\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}\right] dx$$

$$-2x^2+1 = A(x+2)^2 + B(x+2) + C$$

$$\left.\begin{array}{l} \text{Si } x=-2 \rightarrow -7 = C \\ \text{Si } x=0 \rightarrow 1 = 4A+2B+C \\ \text{Si } x=-1 \rightarrow -1 = A+B+C \end{array}\right\} \begin{array}{l} \rightarrow \boxed{C=-7} \\ \left.\begin{array}{l} 4A+2B=8 \rightarrow 2A+B=4 \\ A+B=6 \end{array}\right\} \end{array}$$

$$\begin{array}{r} 2A+B=4 \\ A+B=6 \\ \hline \boxed{A=-2} \end{array} \rightarrow \boxed{B=8}$$

$$\Rightarrow I = \int \left[\frac{-2}{x+2} + \frac{8}{(x+2)^2} + \frac{-7}{(x+2)^3}\right] dx =$$

$$= -2\ln|x+2| + 8\int (x+2)^{-2}\,dx - 7\int (x+2)^{-3}\,dx =$$

$$= -2\ln|x+2| + 8\,\frac{(x+2)^{-1}}{-1} - 7\,\frac{(x+2)^{-2}}{-2} + C =$$

$$= -2\ln|x+2| - \frac{8}{x+2} + \frac{7}{2(x+2)^2} + C$$

g) $\displaystyle\int \frac{x-2}{x^4}\,dx = \int\left(\frac{x}{x^4} - \frac{2}{x^4}\right)dx = \int\left(\frac{1}{x^3} - \frac{2}{x^4}\right)dx$

No es racional

$$= \int\left(x^{-3} - 2x^{-4}\right)dx = \frac{x^{-2}}{-2} - 2\,\frac{x^{-3}}{-3} + C = \frac{-1}{2x^2} + \frac{2}{3x^3} + C$$

h) $\displaystyle\int \frac{4x^2-2x}{(x+2)(x-3)^2}\,dx = \int\left(\frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}\right)dx$

$$\frac{4x^2-2x}{\cancel{(x+2)(x-3)^2}} = \frac{A(x-3)^2 + B(x+2)(x-3) + C(x+2)}{\cancel{(x+2)(x-3)^2}}$$

Si $x=-2 \to 20 = 25A \to \boxed{A = \frac{20}{25} = \frac{4}{5}}$

Si $x=3 \to 30 = 5C \to \boxed{C=6}$

Si $x=0 \to 0 = 9A - 6B + 2C \to 0 = 9\cdot\frac{4}{5} - 6B + 2\cdot 6 \to \boxed{B = \frac{16}{5}}$

$$I = \int \frac{4/5}{x+2}\,dx + \int \frac{16/5}{x-3}\,dx + \int \frac{6}{(x-3)^2}\,dx =$$

$$= \frac{4}{5}\ln|x+2| + \frac{16}{5}\ln|x-3| + 6\int (x-3)^{-2}\,dx =$$

$$= \frac{4}{5}\ln|x+2| + \frac{16}{5}\ln|x-3| - \frac{6}{x-3} + C$$

i) $$\int \frac{-x^2+7x}{x^3-x^2-x+1}\,dx = \int \frac{-x^2+7x}{(x-1)^2(x+1)}\,dx =$$

$$\begin{array}{r|rrrr}
 & 1 & -1 & -1 & 1 \\
1 & & 1 & 0 & -1 \\
\hline
 & 1 & 0 & -1 & \llcorner 0 \\
1 & & 1 & 1 & \\
\hline
 & 1 & 1 & \llcorner 0 & \\
-1 & & -1 & & \\
\hline
 & 1 & 0 & &
\end{array}$$

$$= \int \left(\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \right) dx$$

$$\frac{-x^2+7x}{\cancel{(x-1)^2(x+1)}} = \frac{A(x-1)(x+1)+B(x+1)+C(x-1)^2}{\cancel{(x-1)^2(x+1)}}$$

Si $x=1 \rightarrow 6=2B \rightarrow \boxed{B=3}$

Si $x=-1 \rightarrow -8=4C \rightarrow \boxed{C=-2}$

Si $x=0 \rightarrow 0=-A+B+C \rightarrow A=B+C=1 \; \boxed{A=1}$

$$\Rightarrow I = \int \left(\frac{1}{x-1} + \frac{3}{(x-1)^2} + \frac{-2}{x+1} \right) dx =$$

$$= \ln|x-1| - \frac{3}{x-1} - 2\ln|x+1| + C$$