1. Calcula las siguientes integrales por el método que resulte más conveniente:

- $\int \frac{2senx\cos x}{1+sen^2 x}dx$ (1 punto)

- $\int \frac{1+x}{1+\sqrt{x}}dx$ (1,25 puntos)

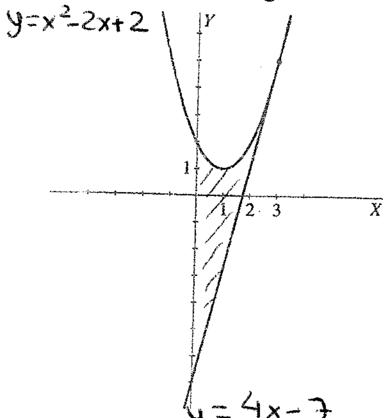
- $\int \ln(25+x^2)dx$ (1,25 puntos)

- $\int \frac{x^3+1}{x^2+4}dx$ (1,25 puntos)

- $\int \sqrt{16-x^2}dx$ (1,25 puntos)

2. Calcular el área de la región del plano encerrada por las gráficas de $f(x)=2x$ y $g(x)=6+3x-x^2$. (1,5 puntos)

3. Calcular el área de la región sombreada. (1 punto)



4. De la función $f\ (-1,\infty)\rightarrow \Re$ se sabe que $f'(x)=\frac{3}{(x+2)^2}$. (1,5 puntos)

   a) Determinar la expresión de $f(x)$ si $f(2)=0$.
   b) Halla la primitiva de $f'(x)$ cuya gráfica pasa por el punto $(0,1)$.

EXAMEN DE INTEGRALES - 2º BACHILLERATO - 11-I-18

• $\int \frac{2\operatorname{sen}x\cos x}{1+\operatorname{sen}^2 x}\,dx = \ln|1+\operatorname{sen}^2 x| + C$ $\left[\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|+C\right]$

• $\int \frac{1+x}{1+\sqrt{x}}\,dx = \int \frac{1+t^2}{1+\sqrt{t^2}}\cdot 2t\,dt = \int \frac{(1+t^2)\cdot 2t}{1+t}\,dt = \int \frac{2t^3+2t}{1+t}\,dt =$

cambio $\sqrt{x}=t$; $x=t^2$
$dx = 2t\,dt$

$= \int (2t^2-2t+4)\,dt - \int \frac{4}{t+1}\,dt = \frac{2t^3}{3} - \frac{2t^2}{2} + 4t - 4\ln|t+1| + C$

Deshacemos el cambio $\boxed{\sqrt{x}=t}$

$$\boxed{I = \frac{2\sqrt{x^3}}{3} - \frac{2(\sqrt{x})^2}{2} + 4\sqrt{x} - 4\ln|\sqrt{x}+1| + C = \frac{2\sqrt{x^3}}{3} - x + 4\sqrt{x} - 4\ln|\sqrt{x}+1| + C}$$

• $\int \ln(25+x^2)\,dx = x\ln(25+x^2) - \underbrace{\int \frac{x\cdot 2x}{25+x^2}\,dx}_{(*)}$

Por partes $u = \ln(25+x^2)$; $du = \frac{1}{25+x^2}\cdot 2x\,dx$

$dv = dx$; $v = x$

(*) $\int \frac{2x^2}{25+x^2}\,dx = \int \left(2 - \frac{50}{x^2+25}\right)dx =$

$= \int 2\,dx - 50\int \frac{dx}{x^2+25} = \int 2\,dx - 50\int \frac{\frac{1}{25}\,dx}{\left(\frac{x}{5}\right)^2+1} =$

$= \int 2\,dx - \frac{50\cdot 5}{25}\int \frac{\frac{1}{5}\,dx}{\left(\frac{x}{5}\right)^2+1} = 2x - 10\operatorname{arctg}\frac{x}{5} + C$

$$\boxed{\int \ln(25+x^2)\,dx = x\ln(25+x^2) - 2x + 10\operatorname{arctg}\frac{x}{5} + C}$$

$$\bullet \int \frac{x^3+1}{x^2+4}\,dx = \int \left(x + \left(\frac{-4x+1}{x^2+4} \right) \right) dx =$$

$$= \int x\,dx - \underbrace{\int \frac{4x-1}{x^2+4}\,dx}_{\circledast}$$

$$\circledast \int \frac{4x-1}{x^2+4}\,dx = 4 \int \frac{x-1/4}{x^2+4}\,dx =$$

$$= \frac{4}{2} \int \frac{2x-2/4}{x^2+4}\,dx = 2\left[\int \frac{2x}{x^2+4}\,dx - \frac{1}{2} \int \frac{dx}{x^2+4} \right]$$

$$= 2\left[\int \frac{2x}{x^2+4}\,dx - \frac{1}{2} \cdot \int \frac{\frac{dx}{4}}{\left(\frac{x}{2}\right)^2+1} \right] =$$

$$= 2\left[\int \frac{2x}{x^2+4}\,dx - \frac{1}{2} \cdot \frac{2}{4} \int \frac{\frac{dx}{2}}{\left(\frac{x}{2}\right)^2+1} \right] =$$

$$= 2 \ln |x^2+4| - \frac{2}{4} \operatorname{arctg} \frac{x}{2} + C.$$

$$\boxed{\int \frac{x^3+1}{x^2+4}\,dx = \frac{x^2}{2} - 2 \ln |x^2+4| + \frac{1}{2} \operatorname{arctg} \frac{x}{2} + C}$$

• $\int \sqrt{16-x^2}\, dx = \int \sqrt{16-(4\,\text{sen}t)^2} \cdot 4\cos t\, dt =$

Cambio $x = 4\,\text{sen}t \quad dx = 4\cos t\, dt$

$= \int \sqrt{16-16\,\text{sen}^2 t} \cdot 4\cos t\, dt = \int 4 \cdot \sqrt{1-\text{sen}^2 t} \cdot 4 \cdot \cos t \cdot dt =$

$= \int 16 \cdot \sqrt{\cos^2 t} \cdot \cos t \cdot dt = 16 \cdot \int \cos^2 t\, dt =$

$= 16 \int \frac{1+\cos 2t}{2}\, dt = \frac{16}{2} \cdot \int (1+\cos 2t)\, dt =$

$= 8 \cdot \left[t + \frac{\text{sen}\, 2t}{2} + C \right] = 8t + 4\,\text{sen}\, 2t + C.$

Deshacemos el cambio: $\left\{ \begin{array}{l} x = 4\,\text{sen}t \\ \frac{x}{4} = \text{sen}t \ ; \ t = \text{arcsen}\frac{x}{4} \end{array} \right.$

$I = 8\,\text{arcsen}\frac{x}{4} + 4\,\text{sen}\,2\left(\text{arcsen}\frac{x}{4}\right) + C$

2° $f(x) = 2x \ ; \ g(x) = 6+3x-x^2$

1 Corte $f(x)$ y $g(x)$ $\rightarrow$ $2x = 6+3x-x^2$; $x^2+2x-3x-6=0$

$x^2-x-6=0$; $x = \frac{1\pm\sqrt{1-4(-6)}}{2} = \frac{1\pm\sqrt{25}}{2} \left\langle \begin{array}{l} \frac{1+5}{2} = \frac{6}{2} = 3 \\ \frac{1-5}{2} = \frac{-4}{2} = -2 \end{array} \right.$

2 En $(-2,3)$ vemos qué función queda por encima:

$\left. \begin{array}{l} f(0)=0 \\ g(0)=6 \end{array} \right\} g(x) > f(x)$ en $(-2,3)$

3 $A = \int_{-2}^{3} |g(x) - f(x)|\, dx = \int_{-2}^{3} |6+3x-x^2-2x|\, dx$

$= \left| \int_{-2}^{3} -x^2+x+6\, dx \right| = \left| \left[\frac{-x^3}{3} + \frac{x^2}{2} + 6x \right]_{-2}^{3} \right| =$

$$\left| \frac{-3^3}{3} + \frac{3^2}{2} + 6\cdot 3 - \left[-\frac{(-2)^3}{3} + \frac{(-2)^2}{2} + 6\cdot(-2)\right]\right| =$$

$$\left| -9 + \frac{9}{2} + 18 - \left[\frac{8}{3} + 2 - 12\right]\right| = \left|9 + \frac{9}{2} - \left[\frac{8}{3} - 10\right]\right|$$

$$= \left|\frac{27}{2} - \left[\frac{-22}{3}\right]\right| = \left|\frac{27}{2}+\frac{22}{3}\right| = \left|\frac{81+44}{6}\right| = \boxed{\frac{125}{6}u^2}$$

3)

$$A = \int_0^3 \left|x^2-2x+2-(4x-7)\right| dx =$$

$$= \int_0^3 \left|x^2-2x+2-4x+7\right| dx = \left|\int_0^3 x^2-6x+9\,dx\right| =$$

$$= \left|\left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_0^3\right| = \left|\frac{3^3}{3} - \frac{6\cdot 3^2}{2} + 9\cdot 3\right| = \left|9-27+27\right| =$$

$$= \boxed{9u^2}$$

4)

$$f'(x) = \frac{3}{(x+2)^2}$$

a)

$$F(x) = \int f'(x)\,dx = \int 3(x+2)^{-2}\,dx = 3\int (x+2)^{-2}dx$$

$$= 3\,\frac{(x+2)^{-2+1}}{-1} + C = \frac{3(x+2)^{-1}}{-1} + C = \boxed{\frac{-3}{(x+2)^1} + C = F(x)}$$

F(x) son todas las primitivas de f'(x)

Como $f(2)=0 \Rightarrow 0 = \frac{-3}{(2+2)^1} + C$; $0 = \frac{-3}{4} + C$;

$$C = \frac{3}{4}$$

Así $\boxed{f(x) = \frac{-3}{(x+2} + \frac{3}{4}}$ Solución del apartado a

b) Haciendo ahora $F(0)=1$ obtenemos la función primitiva de $f'(x)$ que verifica la 2ª condición.

$$\frac{-3}{(0+2)^4} + C = 1 \; ; \; -\frac{3}{2} + C = 1 \; ; \; C = 1 + \frac{3}{2}$$

$$C = \frac{5}{2}$$

$\boxed{f(x) = \frac{-3}{(x+2} + \frac{5}{2}}$ Solución del apartado b