4 Sean $A=\begin{pmatrix}3&0\\5&-1\end{pmatrix}$ y $B=\begin{pmatrix}0&6\\1&-3\end{pmatrix}$.

Encuentra X que cumpla: $3\cdot X-2\cdot A=5\cdot B$. $\longrightarrow 3X=5B+2A \rightarrow X=\frac{1}{3}(5B+2A)$

$5B=\begin{pmatrix}0&30\\5&-15\end{pmatrix}$ $\quad 2A=\begin{pmatrix}6&0\\10&-2\end{pmatrix}$ $\quad 5B-2A=\begin{pmatrix}6&30\\15&-17\end{pmatrix}$

$X=\frac{1}{3}\begin{pmatrix}6&30\\15&-17\end{pmatrix}=\begin{pmatrix}2&10\\5&-17/3\end{pmatrix}$

5 Encuentra dos matrices, A y B, de dimensión 2×2 que cumplan:

$2A+B=\begin{pmatrix}1&4\\2&0\end{pmatrix}$ $\qquad A-B=\begin{pmatrix}-1&2\\1&0\end{pmatrix}$

$C=\begin{pmatrix}1&4\\2&0\end{pmatrix}$ $\qquad D=\begin{pmatrix}-1&2\\1&0\end{pmatrix}$

$\left.\begin{array}{l}2A+\cancel{B}=C\\ A-\cancel{B}=D\end{array}\right\}$

$3A=C+D$

$A=\frac{1}{3}(C+D)$ $\qquad C+D=\begin{pmatrix}0&6\\3&0\end{pmatrix}$

$A=\frac{1}{3}\begin{pmatrix}0&6\\3&0\end{pmatrix}=\boxed{\begin{pmatrix}0&2\\1&0\end{pmatrix}=A}$

$A-B=D \Rightarrow A-D=B$

$B=\begin{pmatrix}0&2\\1&0\end{pmatrix}-\begin{pmatrix}-1&2\\1&0\end{pmatrix}=\boxed{\begin{pmatrix}1&0\\0&0\end{pmatrix}=B}$

1 Calcula el rango de las siguientes matrices:

$A=\begin{pmatrix}1&4&-1\\-1&3&2\\2&2&0\end{pmatrix}$ $\qquad C=\begin{pmatrix}1&-2&0&-3\\-1&3&1&4\\2&1&5&-1\end{pmatrix}$

Triangulamos por Gauss

$-2\cdot 1F+3F$
$1F+2F$

$A=\begin{pmatrix}1&4&-1\\-1&3&2\\2&2&0\end{pmatrix}=\begin{pmatrix}1&4&-1\\0&7&1\\0&-5&2\end{pmatrix}=$

$5\cdot 2F+7\cdot 3F$

$5-14=-9$

$\begin{pmatrix}1&4&-1\\0&7&1\\0&0&-9\end{pmatrix}$ $\qquad \text{Ran}(A)=3$

$1F+2F \qquad -2\cdot 1F+3F \qquad -5\cdot 2F+3F$

$C=\begin{pmatrix}1&-2&0&-3\\-1&3&1&4\\2&1&5&-1\end{pmatrix}=\begin{pmatrix}1&-2&0&-3\\0&1&1&1\\0&5&5&5\end{pmatrix}=\begin{pmatrix}1&-2&0&-3\\0&1&1&1\\0&0&0&0\end{pmatrix}$ $\qquad \text{Ran}(C)=2$

1 Expresa en forma matricial y resuelve los siguientes sistemas de ecuaciones:

b) $\begin{cases} 2x - y = 7 \\ x - 2y = 11 \end{cases}$

$$\underbrace{\begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}}_{A} \cdot \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 7 \\ 11 \end{pmatrix}}_{B}$$

$$AX = B$$

$$\underbrace{A^{-1} \cdot A}_{I} \cdot X = A^{-1} B$$

$$X = A^{-1} \cdot B$$

$$A^{-1} = \frac{[adj(A)]^T}{|A|}$$

$$Adj(A) = \begin{pmatrix} -2 & -1 \\ +1 & 2 \end{pmatrix}$$

$$[Adj(A)]^T = \begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix}$$

$$|A| = -4 + 1 = -3$$

$$A^{-1} = \begin{pmatrix} 2/3 & -1/3 \\ 1/3 & -2/3 \end{pmatrix}$$

$$X = A^{-1} \cdot B = \begin{pmatrix} 2/3 & -1/3 \\ 1/3 & -2/3 \end{pmatrix} \begin{pmatrix} 7 \\ 11 \end{pmatrix} = \begin{pmatrix} \frac{14}{3} - \frac{11}{3} \\ \frac{7}{3} - \frac{22}{3} \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$$

$$\boxed{\begin{matrix} x = 1 \\ y = -5 \end{matrix}}$$

1 Escribe las matrices traspuestas de:

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 5 \\ 7 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 5 & 7 \\ 4 & 1 & 0 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & 2 & 4 & 1 \\ 6 & 1 & 0 & 3 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 3 & 2 & 7 \\ 1 & 5 & 6 \end{pmatrix} \qquad B^T = \begin{pmatrix} 2 & 4 \\ 5 & 1 \\ 7 & 0 \end{pmatrix} \qquad C^T = \begin{pmatrix} 1 & 0 & 6 \\ 3 & 2 & 1 \\ 5 & 4 & 0 \\ -1 & 1 & 3 \end{pmatrix}$$

1 Dadas las siguientes matrices:

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 4 & 1 & -3 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 0 & 1 \\ -4 & 1 & 3 \end{pmatrix} \qquad C = \begin{pmatrix} 7 & 1 & -1 \\ 8 & -10 & 0 \end{pmatrix} \qquad D = \begin{pmatrix} -3 & 1 & 5 \\ 6 & 2 & 4 \end{pmatrix}$$

calcula $E = 2A - 3B + C - 2D$.

$$2A = \begin{pmatrix} 2 & 0 & -4 \\ 8 & 2 & -6 \end{pmatrix} \qquad 3B = \begin{pmatrix} -3 & 0 & 3 \\ -12 & 3 & 9 \end{pmatrix} \qquad 2D = \begin{pmatrix} -6 & 2 & 10 \\ 12 & 4 & 8 \end{pmatrix}$$

$$E = \begin{pmatrix} 2 & 0 & -4 \\ 8 & 2 & -6 \end{pmatrix} + \begin{pmatrix} 3 & 0 & -3 \\ 12 & -3 & -9 \end{pmatrix} + \begin{pmatrix} 7 & 1 & -1 \\ 8 & -10 & 0 \end{pmatrix} + \begin{pmatrix} 6 & -2 & -10 \\ -12 & -4 & -8 \end{pmatrix} = \begin{pmatrix} 18 & -1 & -18 \\ 16 & -15 & -23 \end{pmatrix}$$

1 Calcula, utilizando el método de Gauss, la inversa de cada una de las siguientes matrices en el supuesto de que la tengan:

a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\quad A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\qquad$ b) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $\quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

a)

$$A = \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right) \xrightarrow{-1 \cdot 2F + 1F} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{array}\right) \Rightarrow A^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

b)

$$B = \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right) \xRightarrow{-3 \cdot 1F + 2F} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right) \xRightarrow{-1/2 \cdot 2F} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right) \xrightarrow{-2 \cdot 2F + 1F} \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right)$$

$$B^{-1} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$$

2 Calcula la inversa de cada una de las siguientes matrices o averigua que no la tiene:

a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $\qquad$ b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$ $\qquad$ c) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 1 \\ 2 & 0 & 0 \end{pmatrix}$

a) $|A| = 1.5.9 + 2.6.7 + 3.4.8 - 7.5.3 - 8.6.1 - 9.4.2 = 0$

$|A| = 0$ No tiene inversa

b) $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix} \Rightarrow |A| = 1$ $\qquad$ $Adj(A) = \begin{pmatrix} 0 & 2 & -1 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$

$$[Adj(A)]^T = \begin{pmatrix} 0 & -2 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{pmatrix} \qquad A^{-1} = \frac{[adj(A)]^T}{|A|} = \boxed{\begin{pmatrix} 0 & -2 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{pmatrix}}$$

c) $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 1 \\ 2 & 0 & 0 \end{pmatrix}$ $\qquad$ $Adj(A) = \begin{pmatrix} 0 & 2 & -4 \\ 0 & -6 & 2 \\ -5 & 2 & 1 \end{pmatrix}$ $\qquad$ $\{Adj(A)]^T = \begin{pmatrix} 0 & 0 & -5 \\ 2 & -6 & 2 \\ -4 & 2 & 1 \end{pmatrix}$

$|A| = -10$ $\qquad$ $A^{-1} = \dfrac{[adj(A)]^T}{|A|} = \boxed{\begin{pmatrix} 0 & 0 & 1/2 \\ -1/5 & 3/5 & -1/5 \\ 2/5 & -1/5 & -1/10 \end{pmatrix}}$