

Ejercicios de la regla de L'Hôpital

1

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\operatorname{sen} x}$$

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\operatorname{sen} x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\operatorname{sen} x} = \lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{\cos x} = 2$$

2

$$\lim_{x \rightarrow \infty^+} \frac{x}{(\ln x)^3 + 2x}$$

$$\lim_{x \rightarrow \infty^+} \frac{x}{(\ln x)^3 + 2x} = \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow \infty^+} \frac{x}{(\ln x)^3 + 2x} = \lim_{x \rightarrow \infty^+} \frac{1}{3(\ln x)^2 \frac{1}{x} + 2}$$

3

$$\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\operatorname{sen} x} \right)$$

$$\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\operatorname{sen} x} \right) = \infty - \infty$$

$$\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\operatorname{sen} x} \right) = \lim_{x \rightarrow 0} \frac{\operatorname{sen} x - x}{x \operatorname{sen} x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{sen} x - x}{x \operatorname{sen} x} = \lim_{x \rightarrow 0} \frac{\cos x - 1}{\operatorname{sen} x + x \cos x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{\operatorname{sen} x + x \cos x} = \lim_{x \rightarrow 0} \frac{-\operatorname{sen} x}{\cos x + \cos x - x \operatorname{sen} x} = 0$$

4

$$\lim_{x \rightarrow \frac{\pi}{4}} (\operatorname{tg} x - 1) \sec 2x$$

$$\lim_{x \rightarrow \frac{\pi}{4}} (\operatorname{tg} x - 1) \sec 2x = 0 \cdot \infty$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{tg} x - 1}{\cos 2x} = \frac{0}{0}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{tg} x - 1}{\cos 2x} = \frac{1 + \operatorname{tg}^2 x}{-2 \operatorname{sen} 2x} = -1$$

5

$$\lim_{x \rightarrow 0} x^{\operatorname{tg} x}$$

$$A = x^{\operatorname{tg} x} \quad \ln A = \operatorname{tg} x \ln x \quad A = e^{\operatorname{tg} x \ln x}$$

$$\begin{aligned} \lim_{x \rightarrow 0} x^{\operatorname{tg} x} &= \lim_{x \rightarrow 0} e^{\operatorname{tg} x \ln x} = e^{\lim_{x \rightarrow 0} (\operatorname{tg} x \ln x)} = e^{\lim_{x \rightarrow 0} \frac{\ln x}{\operatorname{cotg} x}} = e^{\lim_{x \rightarrow 0} \left(-\frac{\frac{1}{x}}{\operatorname{cosec}^2 x} \right)} = \\ &= e^{\lim_{x \rightarrow 0} \frac{-\operatorname{sen}^2 x}{x}} = e^{\lim_{x \rightarrow 0} (-2 \operatorname{sen} x \cos x)} = e^0 = 1 \end{aligned}$$

6

$$\lim_{x \rightarrow 0} (\operatorname{cotg} x)^{\operatorname{sen} x}$$

$$A = (\operatorname{cotg} x)^{\operatorname{sen} x} \quad \ln A = \operatorname{sen} x \ln(\operatorname{cotg} x)$$

$$A = e^{\operatorname{sen} x \ln(\operatorname{cotg} x)}$$

$$\lim_{x \rightarrow 0} (\operatorname{cotg} x)^{\operatorname{sen} x} = \lim_{x \rightarrow 0} e^{\operatorname{sen} x \ln(\operatorname{cotg} x)} = e^{\lim_{x \rightarrow 0} \frac{\ln(\operatorname{cotg} x)}{\operatorname{cosec} x}} =$$

$$e^{\lim_{x \rightarrow 0} \frac{-\operatorname{cosec}^2 x}{\operatorname{cotg} x}} = e^{\lim_{x \rightarrow 0} \frac{-\operatorname{cosec}^2 x}{-\cos x}} = e^{\lim_{x \rightarrow 0} \frac{\operatorname{cosec}^2 x \operatorname{sen}^2 x}{\cos x \operatorname{cotg} x}} = e^{\lim_{x \rightarrow 0} \frac{1}{\cos x \frac{\cos x}{\operatorname{sen} x}}} = e^{\lim_{x \rightarrow 0} \frac{\operatorname{sen} x}{\cos^2 x}} = e^0 = 1$$

7

$$\lim_{x \rightarrow 2} \left(\frac{x}{2} \right)^{\frac{1}{x-2}}$$

$$\lim_{x \rightarrow 2} \left(\frac{x}{2}\right)^{\frac{1}{x-2}} = 1^\infty \quad A = \left(\frac{x}{2}\right)^{\frac{1}{x-2}}$$

$$\ln A = \frac{1}{x-2} \ln\left(\frac{x}{2}\right) \quad A = e^{\frac{1}{x-2} \ln\left(\frac{x}{2}\right)}$$

$$\lim_{x \rightarrow 2} \left(\frac{x}{2}\right)^{\frac{1}{x-2}} = e^{\lim_{x \rightarrow 2} \left[\frac{1}{x-2} \ln\left(\frac{x}{2}\right)\right]} = e^{\lim_{x \rightarrow 2} \left[\frac{\ln\left(\frac{x}{2}\right)}{x-2}\right]} =$$

$$e^{\lim_{x \rightarrow 2} \frac{\frac{1}{2}}{x-2}} = e^{\frac{1}{2}} = \sqrt{e}$$

8

$$\lim_{x \rightarrow 1} \frac{\ln x}{x-1}$$

$$\lim_{x \rightarrow 1} \frac{\ln x}{x-1} = \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{\ln x}{x-1} = \lim_{x \rightarrow 1} \frac{1}{x} = 1$$

9

$$\lim_{x \rightarrow 0} \frac{\text{sen } 3x}{x - \frac{3}{2} \text{sen } 2x}$$

$$\lim_{x \rightarrow 0} \frac{\text{sen } 3x}{x - \frac{3}{2} \text{sen } 2x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\text{sen } 3x}{x - \frac{3}{2} \text{sen } 2x} = \lim_{x \rightarrow 0} \frac{3 \cos 3x}{1 - 3 \cos 2x} = -\frac{3}{2}$$

10

$$\lim_{x \rightarrow 0} (\text{arc sen } x \cot g x)$$

$$\lim_{x \rightarrow 0} (\text{arc sen } x \cot g x) = 0 \cdot \infty$$

$$\lim_{x \rightarrow 0} (\text{arc sen } x \cot g x) = \lim_{x \rightarrow 0} \frac{\cos x \text{ arc sen } x}{\text{sen } x} =$$

$$= \lim_{x \rightarrow 0} \frac{-\operatorname{sen} x \operatorname{arc} \operatorname{sen} x + \frac{\cos x}{\sqrt{1-x^2}}}{\cos x} = 1$$

11

$$\lim_{x \rightarrow 0} \left[\frac{1}{2} \frac{\operatorname{sen} x}{\operatorname{tg} x} (1 + \operatorname{tg} 2x)^{\frac{4}{x}} \right]$$

$$\lim_{x \rightarrow 0} \left[\frac{1}{2} \frac{\operatorname{sen} x}{\frac{\operatorname{sen} x}{\cos x}} (1 + \operatorname{tg} 2x)^{\frac{4}{x}} \right] = \frac{1}{2} \lim_{x \rightarrow 0} (\cos x) \lim_{x \rightarrow 0} (1 + \operatorname{tg} 2x)^{\frac{4}{x}} =$$

$$= \frac{1}{2} \lim_{x \rightarrow 0} (1 + \operatorname{tg} 2x)^{\frac{4}{x}} = 1^\infty$$

$$A = (1 + \operatorname{tg} 2x)^{\frac{4}{x}} \quad \ln A = \frac{4}{x} \ln(1 + \operatorname{tg} 2x) \quad A = e^{\frac{4}{x} \ln(1 + \operatorname{tg} 2x)}$$

$$\frac{1}{2} \lim_{x \rightarrow 0} (1 + \operatorname{tg} 2x)^{\frac{4}{x}} = \frac{1}{2} e^{\lim_{x \rightarrow 0} \frac{4 \ln(1 + \operatorname{tg} 2x)}{x}} = \frac{1}{2} e^{\lim_{x \rightarrow 0} \frac{4 \cdot 2(1 + \operatorname{tg}^2 2x)}{\ln(1 + \operatorname{tg} 2x)}} = \frac{1}{2} e^8$$

12

$$\lim_{x \rightarrow 0} \frac{\ln(1+x) - \operatorname{sen} x}{x \operatorname{sen} x}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+x) - \operatorname{sen} x}{x \operatorname{sen} x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+x) - \operatorname{sen} x}{x \operatorname{sen} x} = \lim_{x \rightarrow 0} \frac{\frac{1}{1+x} - \cos x}{\operatorname{sen} x + x \cos x} =$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{1}{(1+x)^2} + \operatorname{sen} x}{\cos x + \cos x - x \operatorname{sen} x} = -\frac{1}{2}$$

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$$\lim_{x \rightarrow 0} \frac{1 + \operatorname{sen} x - e^x}{(\operatorname{arc} \operatorname{tg} x)^2}$$

$$\lim_{x \rightarrow 0} \frac{1 + \operatorname{sen} x - e^x}{(\operatorname{arc} \operatorname{tg} x)^2} = \frac{0}{0}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 + \operatorname{sen} x - e^x}{(\operatorname{arc} \operatorname{tg} x)^2} &= \lim_{x \rightarrow 0} \frac{\cos x - e^x}{\frac{2 \operatorname{arc} \operatorname{tg} x}{1+x^2}} = \\ &= \lim_{x \rightarrow 0} \frac{\operatorname{sen} x - e^x}{\frac{2 - 4x \operatorname{arc} \operatorname{tg} x}{(1+x^2)^2}} = -\frac{1}{2} \end{aligned}$$

14

$$\lim_{x \rightarrow 0} \left[\frac{1}{\ln(1+x)} - \frac{1}{x} \right]$$

$$\lim_{x \rightarrow 0} \left[\frac{1}{\ln(1+x)} - \frac{1}{x} \right] = \infty - \infty$$

$$\lim_{x \rightarrow 0} \left[\frac{1}{\ln(1+x)} - \frac{1}{x} \right] = \lim_{x \rightarrow 0} \frac{x - \ln(1+x)}{x \ln(1+x)} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{x - \ln(1+x)}{x \ln(1+x)} = \lim_{x \rightarrow 0} \frac{1 - \frac{1}{1+x}}{\ln(1+x) + \frac{x}{1+x}} =$$

$$\lim_{x \rightarrow 0} \frac{\frac{1+x-1}{1+x}}{\frac{(1+x) \ln(1+x) + x}{1+x}} = \lim_{x \rightarrow 0} \frac{x}{\ln(1+x) + x \ln(1+x) + x} =$$

$$= \lim_{x \rightarrow 0} \frac{1}{\frac{1}{1+x} + \ln(1+x) + \frac{x}{1+x} + 1} = \frac{1}{2}$$

15

$$\lim_{x \rightarrow 0} \left(\frac{1 + \operatorname{tg} x}{1 + \operatorname{sen} x} \right)^{\frac{1}{\operatorname{sen} x}}$$

$$\lim_{x \rightarrow 0} \left(\frac{1 + \operatorname{tg} x}{1 + \operatorname{sen} x} \right)^{\frac{1}{\operatorname{sen} x}} = 1^\infty$$

$$A = \left(\frac{1 + \operatorname{tg} x}{1 + \operatorname{sen} x} \right)^{\frac{1}{\operatorname{sen} x}} \quad \ln A = \frac{1}{\operatorname{sen} x} \ln \left(\frac{1 + \operatorname{tg} x}{1 + \operatorname{sen} x} \right)$$

Aplicando las **propiedades de los logaritmos** en el segundo miembro tenemos:

$$\ln A = \frac{1}{\operatorname{sen} x} [\ln(1 + \operatorname{tg} x) - \ln(1 + \operatorname{sen} x)]$$

$$\ln A = \frac{\ln(1 + \operatorname{tg} x) - \ln(1 + \operatorname{sen} x)}{\operatorname{sen} x}$$

$$A = e^{\frac{\ln(1 + \operatorname{tg} x) - \ln(1 + \operatorname{sen} x)}{\operatorname{sen} x}}$$

$$\lim_{x \rightarrow 0} \left(\frac{1 + \operatorname{tg} x}{1 + \operatorname{sen} x} \right)^{\frac{1}{\operatorname{sen} x}} = e^{\lim_{x \rightarrow 0} \frac{\ln(1 + \operatorname{tg} x) - \ln(1 + \operatorname{sen} x)}{\operatorname{sen} x}} =$$

$$= e^{\lim_{x \rightarrow 0} \frac{1 + \operatorname{tg}^2 x}{1 + \operatorname{tg} x} \cdot \frac{\cos x}{1 + \operatorname{sen} x}} = e^0 = \mathbf{1}$$

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$$\lim_{x \rightarrow 0} x^{\operatorname{sen} x}$$

$$\lim_{x \rightarrow 0} x^{\operatorname{sen} x} = 0^0$$

$$A = x^{\operatorname{sen} x} \quad \ln A = \operatorname{sen} x \ln x \quad A = e^{\operatorname{sen} x \ln x}$$

$$\lim_{x \rightarrow 0} x^{\operatorname{sen} x} = e^{\lim_{x \rightarrow 0} (\operatorname{sen} x \ln x)} = e^{\lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{\operatorname{sen} x}}} = e^{\lim_{x \rightarrow 0} \frac{\frac{1}{x}}{-\cos x}} =$$

$$e^{\lim_{x \rightarrow 0} \frac{-\operatorname{sen}^2 x}{x \cos x}} = e^{\lim_{x \rightarrow 0} \frac{-2 \operatorname{sen} x \cos x}{\cos x - x \operatorname{sen} x}} = e^0 = \mathbf{1}$$