

1.- a) (1 pto) Demuestra la propiedad de los logaritmos que dice que:

$$\log_a \left(\frac{M}{N} \right) = \log_a M - \log_a N$$

b) (1 pto) Calcula $\sqrt{2 \cdot \sqrt{2 \cdot \sqrt{\log_2 \left(\frac{0,16}{10^{-2}} \right)}}}$

2.- a) (2 ptos) Desarrollar por el Binomio de Newton: $\left(3x^4 y^2 - \frac{1}{6} y^3 x^5 \right)^5 =$

b) (1 pto) Racionalizar: $\frac{4}{\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}}$

3.- a) (1 pto) Desarrolla el valor de: $2 \cdot |3 - x| + 5x - 4 + 3 \cdot |x + 1| =$

b) (1 pto) Resuelve la ecuación: $\frac{(x-1)^2 - (x-2)^2}{1-x^2} = \frac{x-1}{x+1} - \frac{x+1}{x-1}$

4.- a) (2 ptos) Resolver por el método de Gauss y clasifica:
$$\begin{cases} 2x - y - z = 0 \\ x - 2y + 7z = 3 \\ x - y + 2z = 1 \end{cases}$$

b) (1 pto) Calcular: $\{ |2x + 3| \leq 5 \} \cap E\left(\frac{1}{2}, 3\right)$

① a) $\log_a \left(\frac{M}{N} \right) = \log_a M - \log_a N$ (Ver apuntes de teoría)

b) $\sqrt{2 \sqrt{2 \cdot \sqrt{\log_2 \left(\frac{0,16}{10^{-2}} \right)}}} \stackrel{(*)}{=} \sqrt{2 \cdot \sqrt{2 \cdot \sqrt{4}}} = \sqrt{2 \cdot \sqrt{2 \cdot 2}} = \sqrt{2 \cdot \sqrt{4}} = \sqrt{2 \cdot 2} = \sqrt{4} = \boxed{2}$

(*) $\frac{0,16}{10^{-2}} = 0,16 \cdot 10^2 = 16 \cdot \frac{10^{-2} \cdot 10^2}{10^0 = 1} = 16 = 2^4 \Rightarrow \log_2 2^4 = 4$

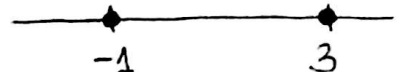
② a) $(3x^4y^2 - \frac{1}{6}y^3x^5)^5 = \binom{5}{0}(3x^4y^2)^5 - \binom{5}{1}(3x^4y^2)^4(\frac{1}{6}y^3x^5) +$
 $+ \binom{5}{2}(3x^4y^2)^3 \cdot (\frac{1}{6}y^3x^5)^2 - \binom{5}{3}(3x^4y^2)^2(\frac{1}{6}y^3x^5)^3 + \binom{5}{4}(3x^4y^2)(\frac{1}{6}y^3x^5)^4 - \binom{5}{5}(\frac{1}{6}y^3x^5)^5$
 $= 1 \cdot 243 \cdot x^{20}y^{10} - 5 \cdot 81 \cdot x^{16}y^8 \cdot \frac{1}{6}y^3x^5 + 10 \cdot 27 \cdot x^{12}y^6 \cdot \frac{1}{36}y^6x^{10} - 10 \cdot 9 \cdot x^8y^4 \cdot \frac{1}{216}y^9x^{15} +$
 $+ 5 \cdot 3 \cdot x^4y^2 \cdot \frac{1}{1296}y^{12}x^{20} - 1 \cdot \frac{1}{7776}y^{15}x^{25} =$
 $= \boxed{243x^{20}y^{10} - \frac{135}{2}x^{21}y^{11} + \frac{15}{2}x^{22}y^{12} - \frac{5}{12}x^{23}y^{13} + \frac{5}{432}x^{24}y^{14} - \frac{1}{7776}x^{25}y^{15}}$

b) $\frac{4}{\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}} = \frac{4 \cdot (\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})}{(\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}})(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})} = \frac{4 \cdot (\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})}{(\sqrt{4+2\sqrt{3}})^2 - (\sqrt{4-2\sqrt{3}})^2}$
 $= \frac{4 \cdot (\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})}{4+2\sqrt{3} - (4-2\sqrt{3})} = \frac{4 \cdot (\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})}{4\sqrt{3}} = \frac{\sqrt{3} \cdot (\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})}{\sqrt{3} \cdot \sqrt{3}} =$
 $= \boxed{\frac{\sqrt{12+6\sqrt{3}} + \sqrt{12-6\sqrt{3}}}{3}}$

③ a) $2 \cdot |3-x| + 5x - 4 + 3 \cdot |x+1| = *$

$|3-x| = \begin{cases} 3-x & \text{si } 3-x \geq 0 \Rightarrow 3 \geq x \Rightarrow x \leq 3 \\ x-3 & \text{si } 3-x < 0 \Rightarrow 3 < x \Rightarrow x > 3 \end{cases}$

$|x+1| = \begin{cases} x+1 & \text{si } x+1 \geq 0 \Rightarrow x \geq -1 \\ -x-1 & \text{si } x+1 < 0 \Rightarrow x < -1 \end{cases}$



$$= (*) = \begin{cases} 2 \cdot (3-x) + 5x - 4 + 3(-x-1) & \text{si } x < -1 \\ 2 \cdot (3-x) + 5x - 4 + 3(x+1) & \text{si } -1 \leq x \leq 3 \\ 2 \cdot (x-3) + 5x - 4 + 3(x+1) & \text{si } x > 3 \end{cases} \Rightarrow$$

$$= \begin{cases} -1 & \text{si } x < -1 \\ 6x+5 & \text{si } -1 \leq x \leq 3 \\ 10x-7 & \text{si } x > 3 \end{cases}$$

$$b) \frac{(x-1)^2 - (x-2)^2}{(1-x^2)} = \frac{x-1}{x+1} - \frac{x+1}{x-1} \Rightarrow \frac{x^2-2x+1 - x^2+4x-4}{(1-x^2)} = \frac{(x-1)^2 - (x+1)^2}{x^2-1} \Rightarrow$$

$$\Rightarrow \frac{2x-3}{1-x^2} = \frac{x^2-2x+1 - x^2-2x-4}{x^2-1} \Rightarrow \frac{2x-3}{1-x^2} = \frac{-4x}{x^2-1} \Rightarrow \frac{2x-3}{1-x^2} = \frac{4x}{1-x^2}$$

$$\Rightarrow (2x-3)(1-x^2) = 4x(1-x^2) \Rightarrow (2x-3)(1-x^2) - 4x(1-x^2) = 0$$

$$\Rightarrow (1-x^2)(2x-3-4x) = 0 \Rightarrow (1-x^2) \cdot (-2x-3) = 0 \Rightarrow$$

$$\Rightarrow \begin{cases} 1-x^2=0 \Rightarrow 1=x^2 \Rightarrow \boxed{x = \pm\sqrt{1} = \pm 1} \text{ NO VALE} \\ -2x-3=0 \Rightarrow -3=2x \Rightarrow \boxed{x = -\frac{3}{2}} \end{cases} \text{ (habían 0 alguno de los 3 denominadores de la ec. de partida)}$$

Comprobación:

$$\text{Si } x = -\frac{3}{2} \Rightarrow \frac{\left(-\frac{5}{2}\right)^2 - \left(-\frac{7}{2}\right)^2}{-\frac{5}{4}} = \frac{-\frac{5}{2}}{-\frac{1}{2}} - \frac{-\frac{1}{2}}{-\frac{5}{2}} \Rightarrow \frac{-\frac{24}{4}}{-\frac{5}{4}} = 5 - \frac{1}{5} \Rightarrow$$

$$\frac{24}{5} = \frac{24}{5} \Rightarrow \underline{\underline{\text{SÍ}} \text{ es solución}}$$

$$\textcircled{4} a) \begin{cases} 2x-y-z=0 \\ x-2y+7z=3 \\ x-y+2z=1 \end{cases} \xrightarrow{E_1 \leftrightarrow E_3} \begin{cases} x-y+2z=1 \\ x-2y+7z=3 \\ 2x-y-z=0 \end{cases} \xrightarrow{\begin{matrix} E_2-E_1 \\ E_3-2E_1 \end{matrix}} \begin{cases} x-y+2z=1 \\ -y+5z=2 \\ y-5z=-2 \end{cases} \xrightarrow{E_3+E_2} \begin{cases} x-y+2z=1 \\ -y+5z=2 \\ 0=0 \end{cases}$$

Sistema Compatible Indeterminado: $z=t$

$$\begin{cases} x-y+2t=1 \\ -y+5t=2 \end{cases} \xrightarrow{\text{sumando}} x-(5t-2)+2t=1 \Rightarrow x-3t+2=1 \Rightarrow \underline{\underline{x=3t-1}}$$

$$\underline{\underline{S = \{(3t-1, 5t-2, t) \mid \forall t \in \mathbb{R}\}}}$$

$$\textcircled{4} \text{ b) } \{ |2x+3| \leq 5 \} \cap E\left(\frac{1}{2}, 3\right) = \boxed{\left(-\frac{5}{2}, 1\right]}$$

$$|2x+3| \leq 5 \Rightarrow \left|x + \frac{3}{2}\right| \leq \frac{5}{2} \Rightarrow E\left[-\frac{3}{2}, \frac{5}{2}\right] = \left[-\frac{3}{2} - \frac{5}{2}, -\frac{3}{2} + \frac{5}{2}\right] = \underline{\underline{[-4, 1]}}$$

$$E\left(\frac{1}{2}, 3\right) = \left(\frac{1}{2} - 3, \frac{1}{2} + 3\right) = \underline{\underline{\left(-\frac{5}{2}, \frac{7}{2}\right)}}$$

Graficamente:

