1.- a) **(1,5 puntos)** Resolver por el Método de Gauss el sistema: $\begin{cases} 2x - y + 3z = 2 \\ 5x - y + z = 6 \\ x + y + 2z = 2 \end{cases}$

b) **(1 punto)** Calcular el término 19 del desarrollo por Newton de $(a^3\,b - a^2)^{20}$

2.- **(2,5 puntos)** Resolver las ecuaciones:

a) $\log x^2 - \log\dfrac{10x+11}{10} = 1$

b) $2 \cdot \binom{2x}{x} = 7 \cdot \binom{2x-2}{x-1}$

3.- **(2,5 puntos)**

a) Calcular sec 280° si sabemos que cotg 10° = h

b) Demostrar la identidad trigonométrica: $\dfrac{\cos^2 x - \cos 2x}{1 - \cos x} = 1 + \cos x$

4.- **(2,5 puntos)**

a) Si β es un ángulo del segundo cuadrante y cos β = -3/4, calcular las razones trigonométricas (solo sen y cos) de β/2 y 2β

b) Resolver la ecuación: $2 \cdot sen x + \sqrt{3} \cdot tg\ x = 0$

① a) $\begin{cases} 2x - y + 3z = 2 \\ 5x - y + z = 6 \\ x + y + 2z = 2 \end{cases} \xrightarrow{E_3 \to E_1} \begin{cases} x + y + 2z = 2 \\ 2x - y + 3z = 2 \\ 5x - y + z = 6 \end{cases} \xrightarrow[E_3 - 5E_1]{E_2 - 2E_1} \begin{cases} x + y + 2z = 2 \\ / \; -3y - z = -2 \\ / \; -6y - 9z = -4 \end{cases}$

$\xrightarrow{E_3 - 2E_2} \begin{cases} x + y + 2z = 2 \\ / \; -3y - z = -2 \\ / \; / \; -7z = 0 \end{cases}$

$\longrightarrow x = 2 - \frac{2}{3} - 2 \cdot 0 = \frac{4}{3}$

$\longrightarrow -3y - 0 = -2 \Rightarrow 3y = 2 \Rightarrow$

$\Rightarrow y = \frac{2}{3}$

$-7z = 0 \Rightarrow z = 0$

$$\boxed{S: \left(\frac{4}{3}, \frac{2}{3}, 0\right)}$$

b) $T_K = (-1)^{K-1} \binom{n}{K-1} a^{n-(K-1)} \cdot b^{K-1}$ con $(a^3 b - a^2)^{20}$ ¿T_{19}?

$T_{19} = (-1)^{19-1} \binom{20}{19-1} (a^3 b)^{20-(19-1)} \cdot (a^2)^{19-1} =$

$= (-1)^{18} \binom{20}{18} (a^3 b)^2 \cdot (a^2)^{18} = 1 \cdot 190 \cdot a^6 b^2 \cdot a^{36} = \boxed{190\, a^{42} b^2}$

② a) $\log x^2 - \log \frac{10x + 11}{10} = 1 \Rightarrow \log \frac{x^2}{\frac{10x + 11}{10}} = \log 10 \Rightarrow$

$\Rightarrow \frac{10x^2}{10x + 11} = 10 \Rightarrow 10x^2 = 100x + 110 \Rightarrow 10x^2 - 100x - 110 = 0$

$\Rightarrow$ Simplificando: $x^2 - 10x - 11 = 0 \begin{cases} \boxed{x_1 = 11} \\ \boxed{x_2 = -1} \end{cases}$

Comprobación:

$x_1 = 11 \Rightarrow \log 121 - \log \frac{121}{10} = \log 121 - (\log 121 - \log 10) = \cancel{\log 121} - \cancel{\log 121} +$

$+ \log 10 = \underline{\underline{1}}$

$x_1 = -1 \Rightarrow \log 1 - \log \frac{-10 + 11}{10} = 0 - \log \frac{1}{10} = 0 - (-1) = \underline{\underline{1}}$

② b) $2\binom{2x}{x} = 7\binom{2x-2}{x-1}$ $\quad x \neq 0,1$ pues no tendría sentido

$$\frac{2\cdot(2x)!}{x!\,(2x-x)!} = \frac{7\,(2x-2)!}{(x-1)!\,(2x-2-x+1)!} \longrightarrow \frac{2\cdot(2x)!}{x!\cdot x!} = \frac{7\,(2x-2)!}{(x-1)!\cdot(x-1)!} \longrightarrow$$

$$\rightarrow 2\cdot(2x)!\cdot(x-1)!\cdot(x-1)! = 7\,(2x-2)!\cdot x!\cdot x! \longrightarrow$$

$$\rightarrow 2\cdot 2x\cdot(2x-1)\cdot\cancel{(2x-2)!}\cdot\cancel{(x-1)!}\cdot\cancel{(x-1)!} = 7\cdot\cancel{(2x-2)!}\, x\cdot\cancel{(x-1)!}\cdot x\cdot\cancel{(x-1)!} \rightarrow$$

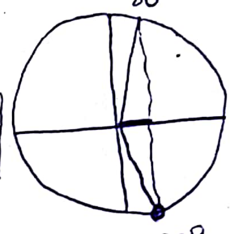
$$\rightarrow 4x\cdot(2x-1) = 7x^2 \Rightarrow 8x^2-4x = 7x^2 \Rightarrow 8x^2-7x^2-4x = 0$$

$$\Rightarrow x^2-4x=0 \Rightarrow x(x-4)=0 \Rightarrow \begin{cases} \boxed{\cancel{x=0}} \text{ NO VALE !! (por enunciado)} \\ x-4=0 \Rightarrow \boxed{x=4} \end{cases}$$

③ a) ¿ $\sec 280°$? siendo $\operatorname{cotg} 10° = h$

$$\sec 280° = \frac{1}{\cos 280°} = \frac{1}{\cos 80°} \underset{\text{(complementarios)}}{=} \frac{1}{\operatorname{sen} 10°} \overset{(*)}{=} \boxed{+\sqrt{1+h^2}}$$

$10 \in I$



(*) $1+\operatorname{cotg}^2 10 = \operatorname{cosec}^2 10 \Rightarrow 1+h^2 = \dfrac{1}{\operatorname{sen}^2 10°}$

b) $\dfrac{\cos^2 x - \cos 2x}{1-\cos x} = 1+\cos x$

$$\frac{\cos^2 x - \cos 2x}{1-\cos x} = \frac{\cos^2 x - (\cos^2 x - \operatorname{sen}^2 x)}{1-\cos x} = \frac{\cancel{\cos^2 x} - \cancel{\cos^2 x} + \operatorname{sen}^2 x}{1-\cos x} =$$

$$= \frac{1-\cos^2 x}{1-\cos x} = \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{(1-\cos x)}} = \underline{\underline{1+\cos x}}$$

④ a) $\beta \in II \quad \cos\beta = -\frac{3}{4}$

$$\operatorname{sen}\frac{\beta}{2} = \pm\sqrt{\frac{1-\cos\beta}{2}} \Rightarrow \beta\in II \Rightarrow \frac{\beta}{2}\in I \Rightarrow \operatorname{sen}\frac{\beta}{2} = +\sqrt{\frac{1-\left(-\frac{3}{4}\right)}{2}} = \sqrt{\frac{1+\frac{3}{4}}{2}} =$$

$$= \sqrt{\frac{7/4}{2}} = \sqrt{\frac{7}{8}} = \frac{1}{2}\sqrt{\frac{7}{2}} = \frac{1}{2}\,\frac{\sqrt{7}}{\sqrt{2}} = \frac{1}{2}\,\frac{\sqrt{7}\cdot\sqrt{2}}{\underbrace{\sqrt{2}\cdot\sqrt{2}}_{2}} = \boxed{\frac{\sqrt{14}}{4}}$$

$$\cos\frac{\beta}{2} = \pm\sqrt{\frac{1+\cos\beta}{2}} \Rightarrow \cos\frac{\beta}{2} = +\sqrt{\frac{1+\left(-\frac{3}{4}\right)}{2}} = \sqrt{\frac{\frac{1}{4}}{2}} = \sqrt{\frac{1}{8}} = \frac{1}{2}\cdot\sqrt{\frac{1}{2}} =$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}}{4}}$$

$$\operatorname{sen} 2\beta = 2 \operatorname{sen}\beta \cos\beta = 2\cdot\frac{\sqrt{7}}{4}\cdot\left(-\frac{3}{4}\right) = -\frac{6\sqrt{7}}{16} = \boxed{-\frac{3\sqrt{7}}{8}}$$

Como $\cos\beta = -\frac{3}{4} \Rightarrow \operatorname{sen}^2\beta + \cos^2\beta = 1 \Rightarrow \operatorname{sen}^2\beta = 1 - \cos^2\beta = 1 - \left(-\frac{3}{4}\right)^2 = 1 - \frac{9}{16} =$

$$= \frac{7}{16} \Rightarrow \operatorname{sen}\beta = \pm\sqrt{\frac{7}{16}} = +\frac{\sqrt{7}}{4}$$

$\beta \in \mathrm{II}$

$$\cos 2\beta = \cos^2\beta - \operatorname{sen}^2\beta = \left(-\frac{3}{4}\right)^2 - \left(\frac{\sqrt{7}}{4}\right)^2 =$$

$$= \frac{9}{16} - \frac{7}{16} = \frac{2}{16} = \boxed{\frac{1}{8}}$$

4 b) $2\operatorname{sen}x + \sqrt{3}\operatorname{tg}x = 0 \Rightarrow 2\operatorname{sen}x + \sqrt{3}\frac{\operatorname{sen}x}{\cos x} = 0 \Rightarrow$

$$\Rightarrow \frac{2\operatorname{sen}x\cos x + \sqrt{3}\operatorname{sen}x}{\cos x} = 0 \Rightarrow \operatorname{sen}x(2\cos x + \sqrt{3}) = 0 \Rightarrow$$

$$\Rightarrow \begin{cases} \operatorname{sen}x = 0 \Rightarrow x = \operatorname{arc\,sen} 0 \Rightarrow x = \begin{cases} 0^\circ + 2\pi K \\ 180^\circ + 2\pi K \end{cases} \rightarrow \boxed{0^\circ + \pi K} \quad K \in \mathbb{Z} \\ 2\cos x = -\sqrt{3} \Rightarrow \cos x = \frac{-\sqrt{3}}{2} \Rightarrow x = \operatorname{arc\,cos}\frac{-\sqrt{3}}{2} = \boxed{\begin{cases} x = 150^\circ + 2\pi K \\ x = 210^\circ + 2\pi K \end{cases} \quad K \in \mathbb{Z}} \end{cases}$$