1.- a) **(1,5 puntos)** Resolver por el Método de Gauss: $\begin{cases} 3x + 4y - 6z = -25 \\ x + 2y - 3z = -11 \\ 5x + 8y - 12z = -47 \end{cases}$

b) **(1 punto)** Desarrollar por el binomio de Newton: $\left(\sqrt{3} - 3x^3\right)^5 =$

2.- **(2,5 puntos)**

a) Desarrollar la expresión: $|x - 3| - |2x - 8| =$

b) Si se sabe que $log_2 A = \frac{1}{2}$ y $log_2 B = -3$ calcular: $log_2 \sqrt{\frac{8}{A^2 \cdot B^2}}$

3.- **(2,5 puntos)**

a) Calcular cos 220° si se sabe que tg 40° = h

b) Resolver la ecuación: sen 3x – sen x = cos 2x

4.- **(2,5 puntos)**

a) Demostrar la identidad trigonométrica: $\frac{\cos 2a + \cos a}{sen\ 2a + sen\ a} = \sqrt{\frac{1 + \cos 3a}{1 - \cos 3a}}$

b) Sabiendo que sen a = m calcular cos 4a en función de m.

① a) $\begin{cases} 3x+4y-6z=-25 \\ x+2y-3z=-11 \\ 5x+8y-12z=-47 \end{cases} \xrightarrow{E_2 \to E_1} \begin{cases} x+2y-3z=-11 \\ 3x+4y-6z=-25 \\ 5x+8y-12z=-47 \end{cases} \xrightarrow[E_3-5E_1]{E_2-3E_1} \begin{cases} x+2y-3z=-11 \\ / \ -2y+3z=8 \\ / \ -2y+3z=8 \end{cases}$

$\xrightarrow{E_2=E_3} \begin{cases} x+2y-3z=-11 & (1) \\ -2y+3z=8 & (2) \end{cases} \longrightarrow \underline{z=t}$

$(2)\ -2y+3t=8 \Rightarrow 3t-8=2y$

$\Rightarrow \underline{y=\dfrac{3t-8}{2}}$

$(1)\ x+\cancel{2}\cdot\dfrac{\cancel{3t}-8}{\cancel{2}}\cancel{-3t}=-11 \Rightarrow$

$\Rightarrow x-8=-11 \Rightarrow \underline{x=-11+8=-3}$

$\boxed{S:\left(-3, \dfrac{3t-8}{2}, t\right)\ \forall t\in\mathbb{R}}$

b) $(\sqrt{3}-3x^3)^5 = \binom{5}{0}(\sqrt{3})^5 - \binom{5}{1}(\sqrt{3})^4\cdot(3x^3) + \binom{5}{2}(\sqrt{3})^3\cdot(3x^3)^2 \ \binom{5}{3}(\sqrt{3})^2\cdot(3x^3)^3 + \binom{5}{4}\sqrt{3}(3x^3)^4$

$\binom{5}{5}(3x^3)^5 = 1\cdot\sqrt{3^5} - 5\cdot\sqrt{3^4}\cdot 3x^3 + 10\cdot\sqrt{3^3}\cdot 3^2\cdot x^6 - 10\sqrt{3^2}\cdot 3^3x^9 + 5\cdot\sqrt{3}\cdot 3^4x^{12} - 1\cdot 3^5\cdot x^{15} =$

$= 3^2\cdot\sqrt{3} - 5\cdot 3^2\cdot 3x^3 + 10\cdot 3\cdot\sqrt{3}\cdot 9x^6 - 10\cdot 3\cdot 27x^9 + 405\sqrt{3}\,x^{12} - 243x^{15} =$

$= \boxed{9\sqrt{3} - 135x^3 + 270\sqrt{3}\,x^6 - 810x^9 + 405\sqrt{3}\,x^{12} - 243x^{15}}$

② a) $|x-3|-|2x-8|=$

$|x-3| = \begin{cases} x-3 & \text{si } x-3\geq 0 \Rightarrow x\geq 3 \\ -x+3 & \text{si } x-3<0 \Rightarrow x<3 \end{cases}$

$|2x-8| = \begin{cases} 2x-8 & \text{si } 2x-8\geq 0 \Rightarrow 2x\geq 8 \Rightarrow x\geq 4 \\ -2x+8 & \text{si } 2x-8<0 \Rightarrow x<4 \end{cases}$

3 4

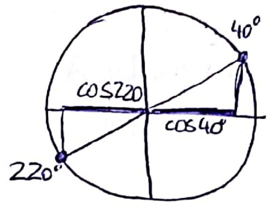
$|x-3|-|2x-8| = \begin{cases} -x+3-(-2x+8) & \text{si } x<3 \\ x-3-(-2x+8) & \text{si } 3\leq x<4 \\ x-3-(2x-8) & \text{si } x\geq 4 \end{cases} = \boxed{\begin{cases} x-5 & \text{si } x<3 \\ 3x-11 & \text{si } 3\leq x<4 \\ -x+5 & \text{si } x\geq 4 \end{cases}}$

b) $\log_2\sqrt{\dfrac{8}{A^2\cdot B^2}} = \log_2\left(\dfrac{8}{A^2\cdot B^2}\right)^{1/2} = \dfrac{1}{2}\log_2\dfrac{8}{A^2\cdot B^2} = \dfrac{1}{2}\left(\log_2 8 - \log_2(A^2\cdot B^2)\right) =$

$= \dfrac{1}{2}\left(3-(\log_2 A^2+\log_2 B^2)\right) = \dfrac{1}{2}\left(3-2\log_2 A - 2\log_2 B\right) \xrightarrow[\log_2 B=-3]{\log_2 A=\frac{1}{2}}$

$= \dfrac{1}{2}\left(3-2\cdot\dfrac{1}{2}-2\cdot(-3)\right) = \dfrac{1}{2}(3-1+6) = \dfrac{1}{2}\cdot 8 = \boxed{4}$

3) a) ¿$\cos 220°$? si se conoce $tg\,40° = h$



$$\cos 220° = -\cos 40° \underset{(*)}{=} \boxed{-\frac{1}{\sqrt{1+h^2}}}$$

$(*)$ $1 + tg^2 40° = \sec^2 40 = \dfrac{1}{\cos^2 40°} \Rightarrow \cos^2 40° = \dfrac{1}{1+tg^2 40} = \dfrac{1}{1+h^2} \Rightarrow$

$$\Rightarrow \underset{40° \in I}{\cos 40°} = +\sqrt{\frac{1}{1+h^2}} = \frac{1}{\sqrt{1+h^2}}$$

b) $\operatorname{sen} 3x - \operatorname{sen} x = \cos 2x$

$$2 \operatorname{sen} \frac{3x-x}{2} \cos \frac{3x+x}{2} = \cos 2x$$

$$2 \operatorname{sen} x \cos 2x = \cos 2x \Rightarrow 2 \operatorname{sen} x \cos 2x - \cos 2x = 0$$

$\cos 2x (2\operatorname{sen} x - 1) = 0$

- $\cos 2x = 0 \Rightarrow 2x = \arccos 0 = \begin{cases} 90° + 2\pi K \\ 270° + 2\pi K \end{cases}$

  $2x = 90° + \pi K \Rightarrow \boxed{x = 45° + \frac{\pi}{2} K \quad K \in \mathbb{Z}}$

- $2\operatorname{sen} x - 1 = 0 \Rightarrow 2\operatorname{sen} x = 1 \Rightarrow \operatorname{sen} x = \frac{1}{2}$

  $\Rightarrow x = \operatorname{arc\,sen} \frac{1}{2} = \boxed{\begin{cases} 30° + 2\pi K \\ 150° + 2\pi K \end{cases}}$

4) a) $\dfrac{\cos 2a + \cos a}{\operatorname{sen} 2a + \operatorname{sen} a} = \sqrt{\dfrac{1+\cos 3a}{1-\cos 3a}}$

$$\frac{\cos 2a + \cos a}{\operatorname{sen} 2a + \operatorname{sen} a} = \frac{\cancel{2} \cos \frac{2a+a}{2} \cancel{\cos \frac{2a-a}{2}}}{\cancel{2} \operatorname{sen} \frac{2a+a}{2} \cdot \cancel{\cos \frac{2a-a}{2}}} = \frac{\cos \frac{3a}{2}}{\operatorname{sen} \frac{3a}{2}} = \operatorname{cotg} \frac{3a}{2} =$$

$$= \sqrt{\frac{1+\cos 3a}{1-\cos 3a}} \qquad \left(\text{pues } \operatorname{cotg} \frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}\right)$$

b) $\operatorname{sen} a = m \Rightarrow \cos^2 a = 1 - \operatorname{sen}^2 a = 1 - m^2$

$\cos 4a = \cos(2a+2a) = \cos 2a \cos 2a - \operatorname{sen} 2a \operatorname{sen} 2a = \cos^2 2a - \operatorname{sen}^2 2a$

$= (\cos^2 a - \operatorname{sen}^2 a)^2 - (2 \operatorname{sen} a \cos a)^2 = \cos^4 a + \operatorname{sen}^4 a - 2\cos^2 a \operatorname{sen}^2 a -$

$- 4 \operatorname{sen}^2 a \cos^2 a = \cos^4 a + \operatorname{sen}^4 a - 6 \operatorname{sen}^2 a \cos^2 a = (1-m^2)^2 + m^4 - 6m^2(1-m^2)$

$= 1 + m^4 - 2m^2 + m^4 - 6m^2 + 6m^4 = \boxed{8m^4 - 8m^2 + 1}$