

SOLUCIONES

$$1a) \frac{2 \cdot \sqrt[3]{4} \cdot \sqrt[5]{8}}{\sqrt{2}\sqrt{2}} = \frac{2 \cdot \sqrt[3]{2^2} \cdot \sqrt[5]{2^3}}{(2 \cdot 2^{1/2})^{1/2}} = \frac{2 \cdot 2^{2/3} \cdot 2^{3/5}}{(2^{3/2})^{1/2}} = \frac{2^{34/15}}{2^{3/4}} = 2^{91/60} = \sqrt[60]{2^{91}}$$

$$1b) \frac{2}{\sqrt{2}} - \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{2\sqrt{2}}{\sqrt{2}\sqrt{2}} - \frac{(\sqrt{2}+1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{2\sqrt{2}}{2} - \frac{2+1+2\sqrt{2}}{2-1} = \sqrt{2} - 3 - 2\sqrt{2} = -3 - \sqrt{2}$$

$$1c) \sqrt[3]{54} - \sqrt[3]{2} - \sqrt[3]{16} + 2\sqrt[3]{150} = \sqrt[3]{3^3 \cdot 2} - \sqrt[3]{2} - \sqrt[3]{2^4} + 2\sqrt[3]{2 \cdot 3 \cdot 5^2} = 3\sqrt[3]{2} - \sqrt[3]{2} - 2\sqrt[3]{2} + 2\sqrt[3]{150} = 2\sqrt[3]{150}$$

$$2a) \log_3 \sqrt[3]{9} = x; 3^x = \sqrt[3]{9} = 3^{2/3} \text{ luego } \log_3 \sqrt[3]{9} = \frac{2}{3}$$

$$\log_5 125 = x; 5^x = 125 = 5^3 \text{ luego } \log_5 125 = 3$$

$$\log_{\sqrt{2}} 2 = x; \sqrt{2}^x = 2; 2^{x/2} = 2; \frac{x}{2} = 1; x = \log_{\sqrt{2}} 2 = 2$$

$$\text{El valor de la expresión } \log_3 \sqrt[3]{9} - \log_5 125 + \log_{\sqrt{2}} 2 \text{ es } \frac{2}{3} - 3 + 2 = -\frac{1}{3}$$

$$2b) \log x = 2\log 5 + 3\log 2 - 2 \rightarrow \log x = \log 5^2 + \log 2^3 - \log 100 \rightarrow$$

$$\log x = \log(25 \cdot 8) - \log 100 \rightarrow \log x = \log \frac{200}{100} \rightarrow x = 2$$

$$2c) \log \sqrt{\frac{10a^3}{b}} = \log \left(\frac{10a^3}{b} \right)^{1/2} = \frac{1}{2} \log \frac{10a^3}{b} = 0,5 \cdot (\log 10 + \log a^3 - \log b) =$$

$$0,5(\log 10 + 3\log a - \log b) = 0,5(1 + 1,2 - 1,2) = 0,5$$

$$3a) \log 8 = \log 2^3 = 3\log 2 = 0,9 \quad \log 0,8 = \log 8/10 = \log 8 - \log 10 = -0,1 \quad \log 0,2 = -0,7$$

$$3b) \text{ Tenemos que despejar } t \text{ en la expresión } 2 = 10 \cdot 0,8^t; \frac{2}{10} = 0,2 = 0,8^t$$

$$\text{Tomamos log: } \log 0,2 = \log 0,8^t = t \log 0,8 \rightarrow -0,7 = -0,1t \rightarrow t = 7 \text{ horas}$$

$$4) V_{10,2} = \frac{10!}{8!} = \frac{10 \cdot 9 \cdot 8!}{8!} = 10 \cdot 9 = 90; P_5 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120; C_{6,2} = \binom{6}{2} = \frac{6!}{2! \cdot 4!} = \frac{30}{2} = 15; VR_{2,3} = 8. \text{ Así pues, } \frac{V_{10,2}}{P_5} - \frac{C_{6,2}}{VR_{2,3}} = \frac{90}{120} - \frac{15}{8} = \frac{3}{4} - \frac{15}{8} = -\frac{9}{8}$$

$$5a) VR_{4,20} = 4^{20}$$

$$5b) C_{10,3} \cdot C_{8,3} = \binom{10}{3} \cdot \binom{8}{3} = \frac{10!}{7! \cdot 3!} \cdot \frac{8!}{5! \cdot 3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} \cdot \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = 120 \cdot 56 = 6720$$

5c) Cifras disponibles: 0,1,2,3,4,5,6,7,8,9 . Si han de estar entre 2000 y 3000, la primera cifra es un 2 fijo y si son múltiplos de 5, la última cifra ha de ser 0 o 5.

$$2 - - 0 \rightarrow V_{8,2} = \frac{8!}{6!} = 56 \quad 2 - - 5 \rightarrow V_{8,2} = \frac{8!}{6!} = 56$$

Hay pues 112 números

$$**5d)** PR_{A,R;C,N}^{3+3+1+1=8} = \frac{8!}{3!.3!.1!.1!} = 1120$$