

6. a) Siendo $z = 1 - 3i$ y $w = \sqrt[3]{8}_{45^\circ}$, calcular: $z + w, z \cdot w, 2z, \frac{z^3}{w}$
- b) Calcula: $\frac{i^{35} - i^5}{2i^{10}}$
7. a) Opera: $\frac{(1 - 2i)(-2 + i)}{3i(1 - i)}$
- b) Resuelve la siguiente ecuación y expresa el resultado en forma binómica:

$$\frac{3 - zi + 2i}{2} = z + i$$
8. Siendo $z = 4\sqrt{3} - 4i$, se pide:
- a) z^5
- b) Las raíces cúbicas de z
9. Halla el valor de x para que el cociente $(1 + 3xi) : (3 - 4i)$
- a) Sea un número imaginario puro.
- b) Sea un número real.
- c) Tenga módulo 1
10. Encontrar a y b para que $\frac{a - 6i}{3 + bi} = \sqrt{2}_{315}$

1. Calcular: $\sqrt{-2 + 2\sqrt{3}i} =$

2. Calcular: $a) \left(1 - \frac{\sqrt{3} - i}{2}\right)^{12}$ $b) \left(\frac{1 + \sqrt{3}i}{1 - i}\right)^{\frac{1}{4}}$

3. Expresar en forma binómica el resultado de: $a) \left(\frac{3\sqrt{3}}{2} + \frac{3}{2}i\right)^5$ $b) (-1 + i)^{10}$

4. Calcular: $\sqrt[5]{\frac{-8 - 8\sqrt{3}i}{(-2\sqrt{3})^2}} + 2i$

5. Resolver:

$$\begin{cases} z^4 + 27z = 0 \\ z^5 + 125z^2 = 0 \end{cases}$$

6. Resolver, expresando el resultado de forma binómica y trigonométrica: $\sqrt[3]{\frac{5+i}{2+3i}}$

7. Resolver: $a) \sqrt[3]{\frac{1-i}{1+i}}$ $b) \sqrt[3]{\frac{-1+i}{1+\sqrt{3}i}}$ $c) \sqrt[4]{81(\cos 120^\circ + i \sin 120^\circ)}$

8. Siendo $z = 1 - 3i$ y $w = \sqrt{8}_{45^\circ}$, calcular: $z + w$, $z \cdot w$, $2z$, $\frac{z^3}{w}$

9. Calcular: $\frac{i^{35} - i^5}{2 \cdot i^{10}}$

10. Opera: $\frac{(1-2i)(-2+i)}{3i(1-i)}$

11. Resuelve la siguiente ecuación y expresa el resultado en forma binómica: $\frac{3 - zi + 2i}{2} = z + i$

12. Siendo $z = 4\sqrt{3} - 4i$, se pide:

a) z^5

b) Las raíces cúbicas de z .

13. Hallar el valor de x para que el cociente $\frac{1+3xi}{3-4i}$:

- a) Sea un número imaginario puro.
- b) Sea un número real.
- c) Tenga módulo 1.

14. Encontrar a y b para que $\frac{a-6i}{3+bi} = \sqrt{2}_{315^\circ}$

NÚMEROS COMPLEJOS

⑥ a) $z = 1 - 3i$, $w = \sqrt{8} \angle 45^\circ = 2 + 2i$

↓

$$z = \sqrt{10} \angle 281.4^\circ$$

$$z + w = (1 - 3i) + (2 + 2i) = 3 - i$$

$$z \cdot w = (1 - 3i) \cdot (2 + 2i) = 8 - 4i$$

$$2z = 2 - 6i$$

$$\frac{z^3}{w} = \frac{(1 - 3i)^3}{2 + 2i} = \frac{-26 + 18i}{2 + 2i} = -2 + 11i$$

b) $\frac{i^{35} - i^5}{2 \cdot i^{10}} = \frac{(-i) - i}{2 \cdot (-1)} = \frac{-2i}{-2} = i$

$$\begin{array}{r} 35 \cancel{14} \\ 3 \quad 8 \end{array} \quad \begin{array}{r} 5 \cancel{14} \\ 1 \quad 1 \end{array} \quad \begin{array}{r} 10 \cancel{14} \\ 2 \quad 2 \end{array}$$

$i^1 = i$	R
$i^2 = -1$	1
$i^3 = -i$	2
$i^4 = 1$	3
	0

⑦ a) $\frac{(1 - 2i)(1 - 2i)}{3i(1 - i)} = \frac{5i}{3 + 3i} = \frac{5}{6} + \frac{5}{6}i$

b) $\frac{3 - 2i + 2i}{2} = z + i \rightarrow 3 - 2i + 2i = 2z + 2i \rightarrow 3 = 2z + 2i$

$$3 = (2 + i)z \rightarrow z = \frac{3}{2 + i} \cdot \frac{2 - i}{2 - i} = \frac{6 - 3i}{5} = \frac{6}{5} - \frac{3}{5}i$$

⑧ $z = 4\sqrt{3} - 4i = 8 \angle 330^\circ$

a) $z^5 = (8 \angle 330^\circ)^5 = 8^5 \angle (330 \cdot 5) = 32768 \angle 1650^\circ = 32768 \angle 210^\circ$

b) $\sqrt[3]{z} = \sqrt[3]{8 \angle 330^\circ} = \sqrt[3]{8} \angle \frac{330 + k360}{3} = 2 \begin{cases} 110^\circ \\ 230^\circ \\ 350^\circ \end{cases}$

$$(9) \quad \frac{1+3xi}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{(3-12x) + (9x+4)i}{25}$$

$$a) \quad \frac{3-12x}{25} = 0 \quad x = \frac{1}{4}$$

$$b) \quad \frac{9x+4}{25} = 0 \quad x = -\frac{4}{9}$$

$$c) \quad \sqrt{\left(\frac{3-12x}{25}\right)^2 + \left(\frac{9x+4}{25}\right)^2} = 1 \rightarrow x = \pm \sqrt{\frac{8}{3}}$$

$$(10) \quad \frac{a-6i}{3+bi} = \sqrt{2} \angle 315^\circ$$

$$\frac{a-6i}{3+bi} \cdot \frac{3-bi}{3-bi} = \frac{3a-18i-abi+6bi^2}{9+b^2} = \frac{(3a-6b)}{9+b^2} + \frac{(-18-ab)i}{9+b^2}$$

$$\sqrt{2} \angle 315^\circ = \sqrt{2} \cos 315^\circ + i \sqrt{2} \sin 315^\circ = \sqrt{2} \cdot \frac{\sqrt{2}}{2} - \sqrt{2} \cdot \frac{\sqrt{2}}{2} i = 1 - i$$

$$\Rightarrow \begin{cases} \frac{3a-6b}{9+b^2} = 1 \\ \frac{-18-ab}{9+b^2} = -1 \end{cases} \Rightarrow \begin{cases} 3a-6b = 9+b^2 \\ -18-ab = -9-b^2 \end{cases} \Rightarrow \begin{cases} a = \frac{9+b^2+6b}{3} \\ -18 - \left(\frac{9+b^2+6b}{3}\right)b = -9-b^2 \end{cases}$$

$$54 + 9b + b^3 + 6b^2 = -27 - 3b^2 \rightarrow b^3 + 9b^2 + 9b + 81 = 0$$

$$\begin{array}{r|rrrr} 1 & 9 & 9 & 81 \\ -9 & & & \\ \hline & 1 & 0 & 9 & 0 \end{array}$$

$$\begin{array}{l} b = -9 \rightarrow a = 12 \\ b = 3i \rightarrow a = 6i \\ b = -3i \rightarrow a = -6i \end{array}$$

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = \pm \sqrt{-9} = \pm 3i$$

(2)

$$\textcircled{1} \sqrt{-2+2\sqrt{3}i} = \sqrt{4} \sqrt{120^\circ} = \sqrt{4} \left\{ \begin{array}{l} \frac{120+12360}{2} \\ k=0,1 \end{array} \right. =$$

$$= 260^\circ, 2240^\circ$$

$$\textcircled{2} a) \left(1 - \frac{\sqrt{3}-i}{2}\right)^{12} = \left(\frac{2-\sqrt{3}+i}{2}\right)^{12} = \left(\frac{2+\sqrt{3}}{2} + \frac{1}{2}i\right)^{12} =$$

$$= \left(\sqrt{2+\sqrt{3}} \sqrt{75^\circ}\right)^{12} = \left(\sqrt{2+\sqrt{3}}\right)^{12} \sqrt{75^\circ \cdot 12} = (2+\sqrt{3})^6 900^\circ =$$

$$= (2+\sqrt{3})^6 180^\circ$$

$$b) \left(\frac{1+\sqrt{3}i}{1-i}\right)^{16} = \sqrt[4]{\frac{260^\circ}{\sqrt{2} 315^\circ}} = \sqrt[4]{\frac{2}{\sqrt{2}} - 255^\circ} = \sqrt[4]{\sqrt{2} 105^\circ} =$$

$$= \sqrt[8]{2} \left\{ \begin{array}{l} \frac{105+12360}{4} \\ k=0,1,2,3 \end{array} \right. \quad \sqrt[8]{2} 26'25^\circ, \sqrt[8]{2} 116'25^\circ$$

$$\sqrt[8]{2} 206'25^\circ, \sqrt[8]{2} 296'25^\circ$$

$$\textcircled{3} \text{ a) } \left(\frac{3\sqrt{3}}{2} + \frac{3}{2}i \right)^5 = \left(3_{30^\circ} \right)^5 = 3^5_{150^\circ} =$$

$$243_{150^\circ} = 243 (\cos 150^\circ + i \sin 150^\circ) =$$

$$= 243 \left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = -\frac{243\sqrt{3}}{2} + \frac{243}{2}i$$

$$\text{b) } (-1+i)^{10} = (\sqrt{2}_{135^\circ})^{10} = (\sqrt{2})^{10}_{1350^\circ} =$$

$$= 2^5_{270^\circ} = 32_{270^\circ} = 32 (\cos 270^\circ + i \sin 270^\circ) = 0 - 32i$$

$$= -32i$$

$$\textcircled{4} \quad x = \sqrt[5]{\frac{-8-8\sqrt{3}i}{(-2\sqrt{3})^2}} + 2i$$

Calculamos a parte da raíz:

$$y = \sqrt[5]{\frac{16_{240^\circ}}{12}} = \sqrt[5]{\frac{4}{3}_{240^\circ}} = \sqrt[5]{\frac{4}{3}} \left\{ \begin{array}{l} \frac{240 + k360}{5} \\ k=0,1,2,3,4 \end{array} \right.$$

$$x_1 = y_1 + 2i = \sqrt[5]{\frac{4}{3}}_{48^\circ} + 2i = \sqrt[5]{\frac{4}{3}} (\cos 48^\circ + i \sin 48^\circ) + 2i$$

$$x_2 = \sqrt[5]{\frac{4}{3}}_{120^\circ} + 2i = \dots$$

$$x_3 = \sqrt[5]{\frac{4}{3}}_{192^\circ} + 2i = \dots$$

$$x_4 = \sqrt[5]{\frac{4}{3}}_{264^\circ} + 2i = \dots$$

$$x_5 = \sqrt[5]{\frac{4}{3}}_{336^\circ} + 2i = \dots$$

Calculamos y decimales

i.e. cómo se encuentra estos

problemas para un examen !!! $\textcircled{4}$

$$\textcircled{5} \begin{cases} z^4 + 27z = 0 \\ z^5 + 125z^2 = 0 \end{cases}$$

$$\bullet z^4 + 27z = 0 \quad z(z^3 + 27) = 0 \quad \begin{matrix} z = 0 \\ z^3 + 27 = 0 \end{matrix}$$

$$z = \sqrt[3]{-27} = \sqrt[3]{27 \cdot 180^\circ} = \sqrt[3]{27} \left\{ \frac{180 + k360}{3} \right. =$$

$$= \begin{cases} 360^\circ \\ 3180^\circ \\ 3360^\circ \end{cases}$$

$$\bullet z^5 + 125z^2 = 0 \quad z^2 = 0$$

$$z^2(z^3 + 125) = 0 \quad z^3 = -125 \quad z = \sqrt[3]{-125} = \sqrt[3]{125 \cdot 180^\circ}$$

$$= 5 \left\{ \frac{180 + k360}{3} \right. = \begin{cases} 560^\circ \\ 5180^\circ \\ 5360^\circ \end{cases}$$

Solución del sistema: $z = 0$

$$\textcircled{6} \cdot \sqrt[3]{\frac{5+i}{2+3i}} = \sqrt[3]{\sqrt{26}} = \sqrt[3]{\sqrt{2} \cdot 13} = \sqrt[6]{2} \left\{ \frac{315^\circ + k360^\circ}{3} \right. =$$

$$= \sqrt[6]{2} 105^\circ, \sqrt[6]{2} 225^\circ, \sqrt[6]{2} 345^\circ$$

— 0 —

⑤

$$\textcircled{7} a) \sqrt[3]{\frac{1-i}{1+i}} = \sqrt[3]{-i} = \sqrt[3]{1_{270}} = \sqrt[3]{1} \left\{ \frac{270+k360}{3} \right. =$$

$$= 1_{90}, 1_{210}, 1_{330}$$

$$b) \sqrt[3]{\frac{-1+i}{1+\sqrt{3}i}} = \sqrt[3]{\frac{\sqrt{2}_{135}}{2_{60}}} = \sqrt[3]{\frac{\sqrt{2}}{2}_{75}} = \sqrt[3]{\frac{\sqrt{2}}{2}} \left\{ \frac{75+k360}{3} =$$

$$= \sqrt[3]{\frac{\sqrt{2}}{2}} \left\{ \begin{matrix} 25^\circ \\ 145^\circ \\ 265^\circ \end{matrix} \right.$$

$$c) \sqrt[4]{81(\cos 120^\circ + i \sin 120^\circ)} = \sqrt[4]{81}_{120} = \sqrt[4]{81} \left\{ \frac{120+k360}{4} =$$

$$= 3_{30}, 3_{110}, 3_{210}, 3_{300}$$

$$\textcircled{8} z = 1-3i, w = \sqrt{8}_{45} = \sqrt{8}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2+2i$$

$$z+w = (1-3i) + (2+2i) = 3-i$$

$$z \cdot w = 8-4i$$

$$2z = 2-6i$$

$$\frac{z^3}{w} = \frac{(1-3i)^3}{2+2i} = \frac{-26+18i}{2+2i} = -2+11i$$

$$\textcircled{9} \frac{i^{35} - i^5}{2 \cdot i^{10}} = \frac{(-i) - i}{2(-1)} = \frac{-2i}{-2} = i$$

$$\textcircled{10} \frac{(1-2i)(-2+i)}{3i(1-i)} = \frac{5i}{3+3i} = \frac{5}{6} + \frac{5}{6}i$$

⑥

$$(11) \quad \frac{3-2i+2i}{2} = z+i \rightarrow 3-2i+2i = 2z+2i \rightarrow$$

$$2z+2i = 3+2i-2i \rightarrow (2+i)z = 3 \rightarrow z = \frac{3}{2+i} \rightarrow$$

$$z = \frac{3(2-i)}{(2+i)(2-i)} = \frac{6-3i}{5} = \frac{6}{5} - \frac{3}{5}i$$

$$(12) \quad z = 4\sqrt{3} - 4i = 8_{330^\circ}$$

$$a) \quad z^5 = (8_{330^\circ})^5 = 8^5_{330^\circ \cdot 5} = 32768_{1650^\circ} = 32768_{210^\circ}$$

$$b) \quad \sqrt[3]{4\sqrt{3}-4i} = \sqrt[3]{8_{330^\circ}} = \sqrt[3]{8} \left\{ \frac{330^\circ + k \cdot 360^\circ}{3} \right. \\ \left. k=0,1,2 \right.$$

$$= 2_{165^\circ}, 2_{285^\circ}, 2_{405^\circ}$$

$$(13) \quad \frac{1+3xi}{3-4i} \cdot \frac{(3+4i)}{(3+4i)} = \frac{3+9xi+4i+12xi^2}{9+16} = \frac{(3-12x)}{25} + \frac{(9x+4)i}{25}$$

$$a) \quad \frac{3-12x}{25} = 0 \rightarrow x = \frac{1}{4}; \quad b) \quad \frac{9x+4}{25} = 0 \quad x = -\frac{4}{9}$$

$$c) \quad \sqrt{\left(\frac{3-12x}{25}\right)^2 + \left(\frac{9x+4}{25}\right)^2} = 1 \rightarrow x = \pm \sqrt{\frac{8}{3}}$$

(14) Igual que el 10 de la otra hoja.