

① Opera y simplifica:

$$a) \frac{3}{3x^2+6x} + \frac{1}{x} - \frac{2-x}{6x+12} = \left\{ \begin{array}{l} 3x^2+6x = 3x(x+2) \\ x = x \\ 6x+12 = 6(x+2) \end{array} \right\} \text{mcm} = 6x(x+2)$$

$$= \frac{3 \cdot 2}{6x(x+2)} + \frac{1 \cdot 6(x+2)}{6x(x+2)} - \frac{(2-x) \cdot x}{6x(x+2)} = \frac{6+6x+12-2x+x^2}{6x(x+2)} =$$

$$= \frac{x^2+4x+18}{6x(x+2)} = \frac{x^2+4x+18}{6x(x+2)}$$

$$x^2+4x+18=0$$

$$x = \frac{-4 \pm \sqrt{16-72}}{2} = \nexists x \in \mathbb{R}$$

$$b) \frac{x-1}{x^2+2x+1} : \frac{x^2-3x+2}{x+1} = \left\{ \begin{array}{l} x^2-3x+2=0 \begin{cases} x=2 \\ x=1 \end{cases} (x-2)(x-1) \\ x^2+2x+1=0 \rightarrow x=-1 \text{ (Doble)} \Rightarrow (x+1)^2 \end{array} \right.$$

$$= \frac{(x-1) \cdot (x+1)}{(x^2+2x+1) \cdot (x^2-3x+2)} = \frac{\cancel{(x-1)} \cdot \cancel{(x+1)}}{(x+1)^2 (x-2) \cancel{(x-1)}} = \frac{1}{(x+1)(x-2)}$$

$$c) \frac{2-x}{x^2-3x} - \frac{1}{4x-12} + \frac{5}{6x-18} = \left\{ \begin{array}{l} x^2-3x = x(x-3) \\ 4x-12 = 4(x-3) \\ 6x-18 = 6(x-3) \end{array} \right\} \text{mcm} = 24x(x-3)$$

$$= \frac{(2-x) \cdot 24}{24x(x-3)} - \frac{1 \cdot 6x(x-3)}{24x(x-3)} + \frac{5 \cdot 4x(x-3)}{24x(x-3)} = * (\text{signe...})$$

$$* = \frac{48 - 24x - 6x^2 + 18x + 20x^2 - 60x}{24x(x-3)} = \frac{14x^2 - 66x + 48}{24x(x-3)}$$

$$14x^2 - 66x + 48 = 0 \rightarrow \frac{66 \pm \sqrt{4356 - 2688}}{28} = \frac{66 \pm \sqrt{1668}}{28} =$$

$$= \frac{66 \pm 2\sqrt{417}}{28} = \frac{33 \pm \sqrt{417}}{14}$$

y como no tiene factores iguales a los del denominador, se deja así.

$$d) \frac{4-x^2}{x+2} + \frac{9-x^2}{x+3} = \frac{(2+x)(2-x)}{(x+2)} + \frac{(3+x)(3-x)}{(x+3)} = 2-x+3-x = 5-2x$$

② Resuelve las siguientes ecuaciones:

$$a) \frac{x+1}{x-1} - 1 = \frac{1}{x} \Rightarrow \frac{(x+1) \cdot x}{x(x-1)} - \frac{1 \cdot x(x-1)}{x(x-1)} = \frac{(x-1)}{x(x-1)} \Rightarrow$$

$$\Rightarrow \cancel{x^2} + x - \cancel{x^2} + x = x - 1 \Rightarrow 2x = x - 1 \Rightarrow x = -1$$

$$b) \frac{2x^4+4}{x^4} = \frac{x^2-3}{x^2} + 2 \Rightarrow \frac{2x^4+4}{\cancel{x^4}} = \frac{(x^2-3) \cdot x^2}{\cancel{x^4}} + \frac{2x^4}{\cancel{x^4}} \Rightarrow$$

$$\Rightarrow \cancel{2x^4} + 4 = x^4 - 3x^2 + \cancel{2x^4} \Rightarrow x^4 - 3x^2 - 4 = 0$$

$$\left. \begin{array}{l} x^2 = t \\ x^4 = t^2 \end{array} \right\} \rightarrow t^2 - 3t - 4 = 0 \rightarrow \begin{array}{l} t = 4 \rightarrow \begin{array}{l} x = -2 \\ x = +2 \end{array} \\ t = -1 \rightarrow \nexists x \in \mathbb{R} \end{array}$$

$$c) 1 + 2\sqrt{x+1} = 3\sqrt{4x-3} - 4$$

$$(2\sqrt{x+1})^2 = (3\sqrt{4x-3} - 5)^2 \Rightarrow 4(x+1) = 9(4x-3) - 30\sqrt{4x-3} + 25$$

$$\Rightarrow 4x+4 = 36x-27-30\sqrt{4x-3} + 25 \Rightarrow$$

$$\Rightarrow 30\sqrt{4x-3} = 36x-2-4x-4 \Rightarrow (30\sqrt{4x-3})^2 = (32x-6)^2 \Rightarrow$$

$$\Rightarrow 900(4x-3) = 1024x^2 - 384x + 36 \Rightarrow 3600x - 2700 = 1024x^2 - 384x + 36$$

$$\Rightarrow 1024x^2 - 3984x + 2736 = 0 \begin{cases} x=3 \checkmark \rightarrow \text{válida} \\ x=\frac{57}{64} \times \rightarrow \text{No válida.} \end{cases}$$

$$d) (x^2+x-2) \cdot (x^2-9) = 0$$

$$\begin{cases} x^2+x-2=0 \begin{cases} x=1 \\ x=-2 \end{cases} \\ x^2-9=0 \begin{cases} x=-3 \\ x=+3 \end{cases} \end{cases}$$

$$e) \log(x-2) + \log(2x-1) = \log(3x-4)$$

$$\cancel{\log}[(x-2) \cdot (2x-1)] = \cancel{\log}(3x-4) \Rightarrow 2x^2 - x - 4x + 2 = 3x - 4$$

$$\Rightarrow 2x^2 - 8x + 6 = 0 \begin{cases} x=3 \checkmark \rightarrow \text{válida} \\ x=1 \times \rightarrow \text{No válida} \end{cases}$$

$$f) \log_2 (3x-2) = 2$$

$$3x-2 = 2^2 \Rightarrow 3x = 6 \Rightarrow x = 2$$

$$g) \log_x 12 + \log_x 18 = 3$$

$$\log_x (12 \cdot 18) = 3 \rightarrow 216 = x^3 \Rightarrow x = \sqrt[3]{216} = 6$$

$$h) 2^{\frac{3x+1}{4}} = \frac{1}{32} \rightarrow \cancel{2^{\frac{3x+1}{4}}} = \cancel{2}^{-5} \Rightarrow \frac{3x+1}{4} = -5 \Rightarrow$$

$$\Rightarrow 3x+1 = -20 \Rightarrow 3x = -21 \Rightarrow x = -7$$

$$i) 4^{x-6} = 32 \rightarrow (2^2)^{x-6} = 2^5 \Rightarrow \cancel{2^{2x-12}} = \cancel{2^5} \Rightarrow$$

$$\Rightarrow 2x-12 = 5 \Rightarrow 2x = 17 \Rightarrow x = \frac{17}{2}$$

$$j) 5^{x^2+4} = 25^{\frac{5x^2}{2}} \rightarrow 5^{x^2+4} = (5^2)^{\frac{5x^2}{2}} \Rightarrow \cancel{5^{x^2+4}} = \cancel{5^{5x^2}} \Rightarrow$$

$$\Rightarrow x^2+4 = 5x^2 \Rightarrow 4x^2 = 4 \Rightarrow x^2 = 1 \begin{cases} \rightarrow x = -1 \\ \rightarrow x = +1 \end{cases}$$

$$k) \sqrt[3]{3^{x^2}} = 9^{x-1} \Rightarrow 3^{\frac{x^2}{3}} = (3^2)^{x-1} \Rightarrow \cancel{3^{\frac{x^2}{3}}} = \cancel{3^{2x-2}} \Rightarrow$$

$$\Rightarrow \frac{x^2}{3} = 2x-2 \Rightarrow x^2 = 6x-6 \Rightarrow$$

$$\Rightarrow x^2 - 6x + 6 = 0, x = \frac{6 \pm \sqrt{36-24}}{2} = \frac{6 \pm 2\sqrt{3}}{2}$$

$$\begin{cases} \rightarrow 3+\sqrt{3} \\ \rightarrow 3-\sqrt{3} \end{cases}$$

$$l) \frac{x^3+x^2+7x+2}{x^2+2x+4} = x-2 \Rightarrow x^3+x^2+7x+2 = (x-2)(x^2+2x+4) \Rightarrow$$

$$\Rightarrow \cancel{x^3} + x^2 + 7x + 2 = \cancel{x^3} + \cancel{2x^2} + \cancel{4x} - \cancel{2x^2} - \cancel{4x} - 8 \Rightarrow$$

$$\Rightarrow x^2 + 7x + 2 = -8 \Rightarrow x^2 + 7x + 10 = 0 \begin{cases} x = -2 \\ x = -5 \end{cases}$$

$$m) (\sqrt{x^2-3})^2 = (1)^2 \Rightarrow x^2-3 = 1 \Rightarrow x^2 = 4 \begin{cases} x = -2 \checkmark \rightarrow \text{Válida} \\ x = +2 \checkmark \rightarrow \text{Válida} \end{cases}$$

$$n) (x)^2 = (\sqrt{x+6})^2 \Rightarrow x^2 = x+6 \Rightarrow x^2-x-6 = 0 \begin{cases} x = 3 \checkmark \rightarrow \text{Válida} \\ x = -2 \times \rightarrow \text{No válida} \end{cases}$$

$$o) \sqrt{x^2-5} + \sqrt{x-2} = 3 \Rightarrow (\sqrt{x^2-5})^2 = (3-\sqrt{x-2})^2 \Rightarrow$$

$$\Rightarrow x^2-5 = 9 - 6\sqrt{x-2} + x-2 \Rightarrow x^2-5 = 7+x-6\sqrt{x-2} \Rightarrow$$

$$\Rightarrow (6\sqrt{x-2})^2 = (x-x^2+12)^2 \Rightarrow 36(x-2) = (x-x^2)^2 + 24(x-x^2) + 144 \Rightarrow$$

$$\Rightarrow 36x-72 = x^2-2x^3+x^4+24x-24x^2+144 \Rightarrow$$

$$\Rightarrow x^4-2x^3-23x^2-12x+216 = 0 \Rightarrow \text{Ruffoni} \Rightarrow$$

1	-2	-23	-12	216
3	3	3	-60	-216
1	1	-20	-72	0

$$x^3+x^2-20x-72=0$$

$$x = 5.314$$

$$x = \text{Complejas}$$

$$x = 3 \checkmark \rightarrow \text{Válida}$$

$$x = 5.314 \times \rightarrow \text{No válida}$$

$$p) (\sqrt{3x+1} + \sqrt{x^2+3x+9})^2 = (2x+1)^2 \Rightarrow$$

$$\Rightarrow 3x+1 + 2\sqrt{3x+1} \cdot \sqrt{x^2+3x+9} + x^2+3x+9 = 4x^2+4x+1 \Rightarrow$$

$$\Rightarrow 2\sqrt{(3x+1)(x^2+3x+9)} = 4x^2+4x-3x-x^2-3x-9 \Rightarrow$$

$$\Rightarrow (2\sqrt{(3x+1)(x^2+3x+9)})^2 = (3x^2-2x-9)^2$$

$$\Rightarrow 4(3x+1)(x^2+3x+9) = (3x^2-2x)^2 - 18(3x^2-2x) + 81 \Rightarrow$$

$$\Rightarrow 4(3x^3+9x^2+27x+x^2+3x+9) = 9x^4-12x^3+4x^2-54x^2+36x+81$$

$$\Rightarrow 12x^3+36x^2+108x+4x^2+12x+36 = 9x^4-12x^3-50x^2+36x+81 \Rightarrow$$

$$\Rightarrow 12x^3+40x^2+120x+36 = 9x^4-12x^3-50x^2+36x+81$$

$$\Rightarrow 9x^4-24x^3-90x^2-84x+45=0 \Rightarrow \text{Ruffini:}$$

5	9	-24	-90	-84	45
		45	105	75	-45
	9	21	15	-9	0

$9x^3+21x^2+15x-9=0$
 $x=0'373$
 $x=\text{Complexos}$

$\swarrow$ $x=5 \checkmark \rightarrow \text{Válida}$
 $\searrow$ $x=0'373 \times \rightarrow \text{No válida}$

$$q) 25x^4 - 101x^2 + 4 = 0$$

$$x^2 = t \rightarrow 25t^2 - 101t + 4 = 0$$

$t = 4$
 $t = \frac{1}{25}$

$\swarrow$
 $\searrow$

$x = -2$
 $x = +2$
 $x = -\frac{1}{5}$
 $x = +\frac{1}{5}$

$$r) x^4 - 25x^2 + 144 = 0$$

$$x^2 = t \rightarrow t^2 - 25t + 144 = 0$$

$t = 16$
 $t = 9$

$\swarrow$
 $\searrow$

$x = -4$
 $x = +4$
 $x = -3$
 $x = +3$

$$s) x^8 + 3x^4 - 4 = 0$$

$$x^4 = t \rightarrow t^2 + 3t - 4 = 0$$

$t = 1$
 $t = -4$

$\swarrow$
 $\searrow$

$x = -1$
 $x = 1$
 $\nexists x$

$$t) x^2 - 3x - 4 + \frac{12}{x} = 0 \Rightarrow \frac{x^3 - 3x^2 - 4x + 12}{x} = 0 \Rightarrow$$

$$\Rightarrow x^3 - 3x^2 - 4x + 12 = 0 \Rightarrow$$

1	-3	-4	12	}	$x = -2$ $x = 3$ $x = 2$
-2	-2	+10	-12		
1	-5	6	0		
3	3	-6	0		
1	-2	0	0		
2	2	0	0		
1	0	0	0		

$$u) \frac{2x}{3x-4} - \frac{x}{x-1} = 0 \Rightarrow \frac{2x(x-1) - x(3x-4)}{(3x-4)(x-1)} = 0 \Rightarrow$$

$$\Rightarrow 2x^2 - 2x - 3x^2 + 4x = 0 \Rightarrow -x^2 + 2x = 0 \Rightarrow x(-x + 2) = 0$$

$\swarrow$
 $\searrow$

$x = 0$
 $-x + 2 = 0 \Rightarrow x = 2$

$$v) \frac{1}{x-1} + 1 = \frac{x^2}{x^2+3x-4} \Rightarrow \left\{ \begin{array}{l} x^2+3x-4=0 \begin{cases} x=1 \\ x=-4 \end{cases} \\ \quad \quad \quad \hookrightarrow (x-1)(x+4) \\ (x-1) = (x-1) \end{array} \right\} \text{mcm} = (x-1)(x+4)$$

$$\Rightarrow \frac{x+4}{\cancel{(x-1)(x+4)}} + \frac{\cancel{(x-1)}(x+4)}{\cancel{(x-1)}(x+4)} = \frac{x^2}{\cancel{(x-1)}(x+4)} \Rightarrow$$

$$\Rightarrow x+4 + x^2+3x-4 = x^2 \Rightarrow 4x=0 \Rightarrow x=0$$

$$w) \sqrt{x^2+9} - 1 = x \Rightarrow$$

$$\Rightarrow (\sqrt{x^2+9})^2 = (x+1)^2 \Rightarrow x^2+9 = x^2+2x+1 \Rightarrow 2x=8 \Rightarrow x=4$$

$$\Rightarrow x=4 \checkmark \rightarrow \text{Válida}$$

$$x) \sqrt{x^2+4x+4} + \sqrt{x+3} = 5 \Rightarrow (\sqrt{x^2+4x+4})^2 = (5-\sqrt{x+3})^2 \Rightarrow$$

$$\Rightarrow x^2+4x+4 = 25-10\sqrt{x+3}+x+3 \Rightarrow$$

$$\Rightarrow 10\sqrt{x+3} = 28+x-x^2-4x-4 \Rightarrow (10\sqrt{x+3})^2 = (24-3x-x^2)^2 \Rightarrow$$

$$\Rightarrow 100(x+3) = (24-3x)^2 - 2x^2(24-3x) + x^4 \Rightarrow$$

$$\Rightarrow 100x+300 = 576-144x+9x^2-48x^2+6x^3+x^4 \Rightarrow$$

$$\Rightarrow x^4+6x^3-39x^2-244x+276=0 \Rightarrow \text{Ruffini}$$

	1	6	-39	-244	276
1		1	7	-32	-276
	1	7	-32	-276	0
6		6	78	276	
	1	13	46		0

$$x^2 + 13x + 46 = 0$$

$x = \text{Complexas}$

$x = 1 \checkmark \rightarrow \text{Válida}$
 $x = 6 \times \rightarrow \text{No válida}$

$$y) \sqrt{x - \sqrt{1-x}} + \sqrt{x} = 1 \Rightarrow (\sqrt{x - \sqrt{1-x}})^2 = (1 - \sqrt{x})^2$$

$$\cancel{x} - \sqrt{1-x} = 1 - 2\sqrt{x} + \cancel{x} \Rightarrow (2\sqrt{x})^2 = (1 + \sqrt{1-x})^2 \Rightarrow$$

$$\Rightarrow 4x = 1 + 2\sqrt{1-x} + 1 - x \Rightarrow 4x = 2 - x + 2\sqrt{1-x} \Rightarrow$$

$$\Rightarrow (5x - 2)^2 = (2\sqrt{1-x})^2 \Rightarrow 25x^2 - 20x + 4 = 4(1-x) \Rightarrow$$

$$\Rightarrow 25x^2 - 20x + \cancel{4} = \cancel{4} - 4x \Rightarrow 25x^2 - 16x = 0$$

$$\Rightarrow x(25x - 16) = 0 \begin{cases} x = 0 \times \rightarrow \text{No válida} \\ 25x - 16 = 0 \Rightarrow x = \frac{16}{25} \checkmark \rightarrow \text{Válida} \end{cases}$$

$$z) x^2 \cdot (x^2 + 1) + 2x^3 + 36 = 12x(x+1)$$

$$x^4 + x^2 + 2x^3 + 36 = 12x^2 + 12x$$

$$x^4 + 2x^3 - 11x^2 - 12x + 36 = 0 \Rightarrow \text{Ruffini}$$

1	2	-11	-12	36
2	2	8	-6	-36
1	4	-3	-18	0
-3	-3	-3	+18	0
1	1	-6	0	0

$$x^2 + x - 6 = 0 \begin{cases} x=2 \\ x=-3 \end{cases}$$

$$\Rightarrow x=2 \text{ (Doble)} \text{ y } x=-3 \text{ (Doble)}$$

③ Resuelve las siguientes ecuaciones y sistemas:

$$a) x^3 - 6x^2 + 11x - 6 = 0$$

1	-6	11	-6
1	1	-5	6
1	-5	6	0

$$x^2 - 5x + 6 = 0 \begin{cases} x=2 \\ x=3 \end{cases}$$

$$\Rightarrow x=1 \vee x=2 \vee x=3$$

$$b) \log_2 4^{x+4} = -2$$

$$\log_2 (2^2)^{x+4} = -2 \rightarrow \log_2 2^{2x+8} = -2 \Rightarrow 2x+8 = -2$$

$$\begin{aligned} 2x &= -10 \\ x &= -5 \end{aligned}$$

$$c) \log_3 27^{3x+4} = -2$$

$$\log_3 (3^3)^{3x+4} = -2 \Rightarrow \log_3 3^{9x+12} = -2 \Rightarrow 9x+12 = -2 \Rightarrow$$

$$\Rightarrow 9x = -14 \Rightarrow x = -\frac{14}{9}$$

$$d) \log_2 \left(\frac{3x-1}{4} \right) = 2 + \log_2 \frac{1}{16}$$

$$\log_2 \left(\frac{3x-1}{4} \right) = \log_2 2^2 + \log_2 \frac{1}{16} \Rightarrow \cancel{\log_2} \left(\frac{3x-1}{4} \right) = \cancel{\log_2} \frac{2^2}{16}$$

$$\Rightarrow \frac{3x-1}{\cancel{4}} = \frac{1}{\cancel{4}} \Rightarrow 3x-1=1 \Rightarrow 3x=2 \Rightarrow x = \frac{2}{3}$$

$$e) \log_3 \sqrt{x-5} + \log_3 \sqrt{2x-3} = 1$$

$$\cancel{\log_3} (\sqrt{x-5} \cdot \sqrt{2x-3}) = \cancel{\log_3} 3^1 \Rightarrow (\sqrt{(x-5) \cdot (2x-3)})^2 = (3)^2 \Rightarrow$$

$$\Rightarrow 2x^2 - 3x - 10x + 15 = 9 \Rightarrow 2x^2 - 13x + 6 = 0 \begin{cases} x=6 \checkmark \rightarrow \text{válida} \\ x=1/2 \times \rightarrow \text{No válida} \end{cases}$$

$$f) \log_{1/2} (x^2 - 3x + 3) = 0$$

$$\cancel{\log_{1/2}} (x^2 - 3x + 3) = \cancel{\log_{1/2}} 1 \Rightarrow x^2 - 3x + 3 = 1 \Rightarrow$$

$$\Rightarrow x^2 - 3x + 2 = 0 \begin{cases} x=1 \checkmark \rightarrow \text{Válida} \\ x=2 \checkmark \rightarrow \text{Válida} \end{cases}$$

$$g) \log_2 \sqrt{x} - \log_2 \sqrt[3]{x} = \frac{2}{3}$$

$$\cancel{\log_2} \frac{\sqrt{x}}{\sqrt[3]{x}} = \cancel{\log_2} 2^{\frac{2}{3}} \Rightarrow \frac{\sqrt{x}}{\sqrt[3]{x}} = \sqrt[3]{4} \Rightarrow \frac{\sqrt[6]{x^3}}{\sqrt[6]{x^2}} = \sqrt[6]{4^2} \Rightarrow$$

$$\Rightarrow \sqrt[6]{\frac{x^3}{x^2}} = \sqrt[6]{16} \Rightarrow \sqrt[6]{x} = \sqrt[6]{16} \Rightarrow x = 16$$

$$h) 3 \cdot 27^{x-2} = 9^x$$

$$3 \cdot (3^3)^{x-2} = (3^2)^x \rightarrow 3 \cdot 3^{3x-6} = 3^{2x} \Rightarrow \cancel{3}^{3x-5} = \cancel{3}^{2x} \Rightarrow$$

$$\Rightarrow 3x-5 = 2x \Rightarrow x = 5$$

$$i) 256^x = 4 \cdot 4^{2x-3}$$

$$(2^8)^x = 2^2 \cdot (2^2)^{2x-3} \Rightarrow 2^{8x} = 2^2 \cdot 2^{4x-6} \Rightarrow \cancel{2}^{8x} = \cancel{2}^{4x-4}$$

$$\Rightarrow 8x = 4x - 4 \Rightarrow 4x = -4 \Rightarrow x = -1$$

$$j) \frac{2^{x^3-x^2-5x}}{8} = 1$$

$$\frac{2^{x^3-x^2-5x}}{2^3} = 2^0 \Rightarrow \cancel{2}^{x^3-x^2-5x-3} = \cancel{2}^0 \Rightarrow x^3-x^2-5x-3 = 0$$

$$\begin{array}{r|rrrr} & 1 & -1 & -5 & -3 \\ 3 & & 3 & 6 & 3 \\ \hline & 1 & 2 & 1 & 0 \\ -1 & & -1 & -1 & \\ \hline & 1 & 1 & 0 & \\ -1 & & -1 & & \\ \hline & 1 & 0 & & \end{array}$$

$$\Rightarrow x = 3 \vee x = -1 \text{ (Doble)}$$

$$k) \left(\frac{1}{7}\right)^{5-x^3} \cdot \sqrt[7]{7^{13x^2+13x}} = 1$$

$$(7^{-1})^{5-x^3} \cdot 7^{\frac{13x^2+13x}{2}} = 7^0 \Rightarrow 7^{x^3-5} \cdot 7^{\frac{13x^2+13x}{2}} = 7^0 \Rightarrow$$

$$\Rightarrow \cancel{7}^{x^3-5+\frac{13x^2+13x}{2}} = \cancel{7}^0 \Rightarrow x^3-5+\frac{13x^2+13x}{2} = 0 \Rightarrow$$

$$\Rightarrow \frac{2x^3-10+13x^2+13x}{2} = 0 \Rightarrow 2x^3+13x^2+13x-10=0$$

$$\begin{array}{r|rrrr} & 2 & 13 & 13 & -10 \\ -5 & & -10 & -15 & +10 \\ \hline & 2 & 3 & -2 & 0 \\ -2 & & -4 & +2 & \\ \hline & 2 & -1 & 0 & \end{array}$$

$$2x-1=0 \Rightarrow x=1/2$$

$$\text{Sol: } x=-5 \vee x=-2 \vee x=1/2$$

$$e) 3^{3x} + 5 \cdot 3^{2x-1} - 11 \cdot 3^{x-1} + 1 = 0$$

$$3^{3x} + \frac{5 \cdot 3^{2x}}{3} - \frac{11 \cdot 3^x}{3} + 1 = 0 \Rightarrow 3^x = t \Rightarrow t^3 + \frac{5}{3}t^2 - \frac{11}{3}t + 1 = 0$$

$$\Rightarrow \frac{3t^3+5t^2-11t+3}{3} = 0 \Rightarrow 3t^3+5t^2-11t+3=0$$

$$\begin{array}{r|rrrr} & 3 & 5 & -11 & 3 \\ -3 & & -9 & +12 & -3 \\ \hline & 3 & -4 & 1 & 0 \\ 1 & & 3 & -1 & \\ \hline & 3 & -1 & 0 & \end{array}$$

$$3t-1=0 \rightarrow t=1/3$$

$$3^x = t \begin{cases} 3^x = -3 \rightarrow \cancel{x} \\ 3^x = 1 \rightarrow x=0 \\ 3^x = \frac{1}{3} \rightarrow x=-1 \end{cases}$$

$$m) 2 \cdot 3^x - 3^{2x} + 3 = 0$$

$$3^x = t \rightarrow 2t - t^2 + 3 = 0 \Rightarrow -t^2 + 2t + 3 = 0 \begin{cases} t=3 \\ t=-1 \end{cases}$$

$$3^x = t \begin{cases} 3^x = 3 \Rightarrow x=1 \\ 3^x = -1 \Rightarrow \nexists x \end{cases}$$

$$n) 2^{x+2} + 2^{x+3} + 2^{x+4} + 2^{x+5} + 2^{x+6} = 31$$

$$2^x \cdot 2^2 + 2^x \cdot 2^3 + 2^x \cdot 2^4 + 2^x \cdot 2^5 + 2^x \cdot 2^6 = 31$$

$$4 \cdot 2^x + 8 \cdot 2^x + 16 \cdot 2^x + 32 \cdot 2^x + 64 \cdot 2^x = 31$$

$$124 \cdot 2^x = 31 \Rightarrow 2^x = \frac{31}{124} \Rightarrow 2^x = \frac{1}{4} \Rightarrow 2^x = 2^{-2} \Rightarrow x = -2$$

$$o) \left. \begin{array}{l} 6x + 6y = 5 \\ x^2 - y^2 = \frac{5}{36} \end{array} \right\} \begin{array}{l} 6(x+y) = 5 \rightarrow x+y = \frac{5}{6} \\ (x+y)(x-y) = \frac{5}{36} \rightarrow \frac{5}{6}(x-y) = \frac{5}{36} \end{array} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} x+y = \frac{5}{6} \\ x-y = \frac{1}{6} \end{array} \right\} \begin{array}{l} E_1 + E_2 \rightarrow 2x = 1 \Rightarrow x = \frac{1}{2} \\ y = \frac{5}{6} - x = \frac{5}{6} - \frac{1}{2} = \frac{1}{3} \end{array}$$

$$\text{Sol: } (x, y) = \left(\frac{1}{2}, \frac{1}{3} \right)$$

$$p) \left\{ \begin{array}{l} \sqrt{x+y+4} + x = y+2 \\ y^2 - x^2 = 2(x+y) \end{array} \right\} \left\{ \begin{array}{l} \sqrt{(y+x)+4} - 2 = (y-x) \\ (y+x)(y-x) = 2(y+x) \end{array} \right\}$$

Cambio variable

$$\left. \begin{array}{l} y+x = z \\ y-x = t \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \sqrt{z+4} - 2 = t \\ z \cdot t = 2z \end{array} \right\} \begin{array}{l} zt - 2z = 0 \\ z(t-2) = 0 \end{array} \begin{array}{l} z=0 \\ t=2 \end{array}$$

Si $z=0$

$$\sqrt{0+4} - 2 = t \Rightarrow t=0 \Rightarrow \left\{ \begin{array}{l} y+x=0 \\ y-x=0 \end{array} \right\} \begin{array}{l} E_1 + E_2 \rightarrow 2y=0 \\ y=0 \end{array}$$

$\rightarrow x=y=0$

Sol: $(x,y) = (0,0)$

Si $t=2$

$$\sqrt{z+4} - 2 = 2 \Rightarrow \sqrt{z+4} = 4 \Rightarrow z+4 = 16 \Rightarrow z=12$$

$$\Rightarrow \left\{ \begin{array}{l} y+x = 12 \\ y-x = 2 \end{array} \right\} \begin{array}{l} E_1 + E_2 \rightarrow 2y = 14 \Rightarrow y=7 \\ 7-x=2 \rightarrow x=5 \end{array}$$

Sol: $(x,y) = (5,7)$

$$q) \left\{ \begin{array}{l} x+y = 23 \\ x^2+y^2 = 289 \end{array} \right\} \begin{array}{l} y = 23-x \\ \xrightarrow{\quad} x^2 + (23-x)^2 = 289 \Rightarrow \end{array}$$

$$\Rightarrow x^2 + 529 - 46x + x^2 = 289 \Rightarrow 2x^2 - 46x + 240 = 0$$

$$2x^2 - 46x + 240 = 0 \begin{cases} x = 15 \rightarrow y = 23 - 15 = 8 \rightarrow (x, y) = (15, 8) \\ x = 8 \rightarrow y = 23 - 8 = 15 \rightarrow (x, y) = (8, 15) \end{cases}$$

$$r) \begin{cases} x + \frac{5}{y} = 2 \\ 4x + y = 9 \end{cases} \begin{cases} xy + 5 = 2y \\ 4x + y = 9 \end{cases} \xrightarrow{y = 9 - 4x} x(9 - 4x) + 5 = 2(9 - 4x) \Rightarrow$$

$$\Rightarrow 9x - 4x^2 + 5 = 18 - 8x \Rightarrow$$

$$\Rightarrow 4x^2 - 17x + 13 = 0 \begin{cases} x = \frac{13}{4} \rightarrow y = 9 - 4 \cdot \frac{13}{4} = -4 \\ x = 1 \rightarrow y = 9 - 4 \cdot 1 = 5 \end{cases}$$

$$\text{Sol. } \begin{cases} (x, y) = \left(\frac{13}{4}, -4\right) \\ (x, y) = (1, 5) \end{cases}$$

$$s) \begin{cases} 3x - y = 2 \\ \frac{2}{x-2} - \frac{1}{y} = -3 \end{cases} \begin{cases} 3x - y = 2 \\ 2y - (x-2) = -3y(x-2) \end{cases} \xrightarrow{y = 3x - 2}$$

$$2 \cdot (3x - 2) - x + 2 = -3(3x - 2)(x - 2)$$

$$6x - 4 - x + 2 = -3(3x^2 - 6x - 2x + 4) \Rightarrow 5x - 2 = -3(3x^2 - 8x + 4) \Rightarrow$$

$$\Rightarrow 5x - 2 = -9x^2 + 24x - 12 \Rightarrow 9x^2 - 19x + 10 = 0$$

$$\begin{cases} x = \frac{10}{9} \rightarrow y = 3 \cdot \frac{10}{9} - 2 = \frac{4}{3} \rightarrow (x, y) = \left(\frac{10}{9}, \frac{4}{3}\right) \\ x = 1 \rightarrow y = 3 \cdot 1 - 2 = 1 \rightarrow (x, y) = (1, 1) \end{cases}$$

$$t) \left. \begin{aligned} 2^x \cdot 2^y &= 16 \\ 9^x &= \frac{3^y}{3} \end{aligned} \right\} \left. \begin{aligned} 2^{x+y} &= 2^4 \\ 3^{2x} &= 3^{y-1} \end{aligned} \right\} \left. \begin{aligned} x+y &= 4 \\ 2x &= y-1 \end{aligned} \right\} \left. \begin{aligned} x+y &= 4 \\ 2x-y &= -1 \end{aligned} \right\} E_1 + E_2 \rightarrow$$

$$3x = 3 \Rightarrow x = 1$$

$$y = 4 - x = 4 - 1 = 3$$

$$\Rightarrow (x, y) = (1, 3)$$

$$u) \left. \begin{aligned} x \cdot y &= 2 \\ \frac{125^x}{625} &= 25^y \end{aligned} \right\} \left. \begin{aligned} x \cdot y &= 2 \\ \frac{5^{3x}}{5^4} &= 5^{2y} \end{aligned} \right\} \left. \begin{aligned} x \cdot y &= 2 \\ 5^{3x-4} &= 5^{2y} \end{aligned} \right\} \left. \begin{aligned} x \cdot y &= 2 \\ 3x-4 &= 2y \end{aligned} \right\}$$

$$\Rightarrow y = \frac{3x-4}{2}$$

$$x \cdot \left(\frac{3x-4}{2} \right) = 2 \Rightarrow 3x^2 - 4x = 4 \Rightarrow$$

$$\Rightarrow 3x^2 - 4x - 4 = 0 \begin{cases} x=2 \rightarrow y = \frac{3 \cdot 2 - 4}{2} = 1 \rightarrow (x, y) = (2, 1) \\ x = -\frac{2}{3} \rightarrow y = \frac{3 \cdot (-\frac{2}{3}) - 4}{2} = -3 \rightarrow (x, y) = (-\frac{2}{3}, -3) \end{cases}$$

$$v) \left. \begin{aligned} \log(2x) &= 1 + \log(y) \\ \sqrt{x-1} &= y+1 \end{aligned} \right\} \left. \begin{aligned} \log(2x) &= \log 10 + \log(y) \\ x-1 &= (y+1)^2 \end{aligned} \right\} \left. \begin{aligned} 2x &= 10y \\ x-1 &= y^2 + 2y + 1 \end{aligned} \right\}$$

$$\Rightarrow x = 5y$$

$$5y - 1 = y^2 + 2y + 1 \Rightarrow y^2 - 3y + 2 = 0$$

$$\begin{cases} y=1 \rightarrow x = 5 \cdot 1 = 5 \rightarrow (x, y) = (5, 1) \checkmark \rightarrow \text{Válida} \\ y=2 \rightarrow x = 5 \cdot 2 = 10 \rightarrow (x, y) = (10, 2) \checkmark \rightarrow \text{Válida} \end{cases}$$

$$w) \left. \begin{aligned} 25^x \cdot 5^y &= \frac{1}{5} \\ \sqrt{x+11} &= y \end{aligned} \right\} \left. \begin{aligned} 5^{2x} \cdot 5^y &= 5^{-1} \\ x+11 &= y^2 \end{aligned} \right\} \left. \begin{aligned} 5^{2x+y} &= 5^{-1} \\ x+11 &= y^2 \end{aligned} \right\} \left. \begin{aligned} 2x+y &= -1 \\ x+11 &= y^2 \end{aligned} \right\}$$

$$\Rightarrow x = y^2 - 11$$

$$\Rightarrow 2(y^2 - 11) + y = -1 \Rightarrow 2y^2 - 22 + y = -1 \Rightarrow$$

$$\Rightarrow 2y^2 + y - 21 = 0 \begin{cases} y = 3 \rightarrow x = 3^2 - 11 = -2 \rightarrow (x, y) = (-2, 3) \checkmark \rightarrow \text{válida} \\ y = -\frac{7}{2} \rightarrow x = \left(-\frac{7}{2}\right)^2 - 11 = \frac{5}{4} \rightarrow (x, y) = \left(\frac{5}{4}, -\frac{7}{2}\right) \times \rightarrow \text{No válida} \end{cases}$$

$$x) \left. \begin{aligned} xy - 12 &= 4y \\ 3y - x &= 1 \end{aligned} \right\} x = 3y - 1$$

$$\Rightarrow (3y - 1) \cdot y - 12 = 4y \Rightarrow 3y^2 - y - 12 = 4y \Rightarrow$$

$$\Rightarrow 3y^2 - 5y - 12 = 0 \begin{cases} y = 3 \rightarrow x = 3 \cdot 3 - 1 = 8 \rightarrow (x, y) = (8, 3) \\ y = -\frac{4}{3} \rightarrow x = 3 \cdot \left(-\frac{4}{3}\right) - 1 = -5 \rightarrow (x, y) = (-5, -\frac{4}{3}) \end{cases}$$

$$y) \left. \begin{aligned} \sqrt{x} + \sqrt{y} &= 6 \\ \frac{x}{y} &= y \end{aligned} \right\} x = y^2$$

$$\Rightarrow (\sqrt{y^2} + \sqrt{y})^2 = (6)^2 \rightarrow y^2 + 2\sqrt{y^2} \cdot \sqrt{y} + y = 36 \rightarrow y^2 + 2\sqrt{y^3} + y = 36$$

$$\Rightarrow 2\sqrt{y^3} = 36 - y^2 - y \Rightarrow (2\sqrt{y^3})^2 = (36 - (y^2 + y))^2 \Rightarrow$$

$$\Rightarrow 4y^3 = 1296 - 72(y^2 + y) + (y^2 + y)^2 \Rightarrow$$

$$\Rightarrow 4y^3 = 1296 - 72y^2 - 72y + y^4 + 2y^3 + y^2 \Rightarrow$$

$$\Rightarrow y^4 - 2y^3 - 71y^2 - 72y + 1296 = 0$$

	1	-2	-71	-72	1296
4		4	8	-252	-1296
	1	2	-63	-324	0
9		9	99	324	
	1	11	36	0	

$$y^2 + 11y + 36 = 0$$

$y = \text{Complejas}$

$$y = 4 \rightarrow x = 4^2 = 16 \rightarrow (x, y) = (16, 4) \checkmark \rightarrow \text{Válida}$$

$$y = 9 \rightarrow x = 9^2 = 81 \rightarrow (x, y) = (81, 9) \times \rightarrow \text{No válida}$$

$$z) \left. \begin{array}{l} (x-3)(y+1) = 50 \\ \frac{x+2}{y-4} = 2 \end{array} \right\} \left. \begin{array}{l} xy + x - 3y - 3 = 50 \\ x + 2 = 2y - 8 \end{array} \right\} \left. \begin{array}{l} xy + x - 3y = 53 \\ x = 2y - 10 \end{array} \right\}$$

$$\Rightarrow (2y - 10) \cdot y + 2y - 10 - 3y = 53$$

$$\Rightarrow 2y^2 - 10y - 10 - y = 53$$

$$\Rightarrow 2y^2 - 11y - 63 = 0$$

$$y = 9 \rightarrow x = 2 \cdot 9 - 10 = 8 \rightarrow (x, y) = (8, 9)$$

$$y = -\frac{7}{2} \rightarrow x = 2 \cdot \left(-\frac{7}{2}\right) - 10 = -17 \rightarrow (x, y) = \left(-17, -\frac{7}{2}\right)$$

④ Resuelve los sistemas utilizando Gauss:

$$a) \begin{cases} x+y-z=1 \\ x-2y+z=11 \\ 2x+y-2z=4 \end{cases} \xrightarrow{\begin{matrix} -E_1+E_2 \rightarrow \\ -2E_1+E_2 \rightarrow \end{matrix}} \begin{cases} x+y-z=1 \\ -3y+2z=10 \\ -y+2z=2 \end{cases} \xrightarrow{\begin{matrix} -3E_3+E_2 \end{matrix}} \begin{cases} x+y-z=1 \\ -3y+2z=10 \\ -4z=4 \end{cases}$$

$$-4z=4 \Rightarrow z=-1$$

$$-3y+2(-1)=10 \Rightarrow -3y=12 \Rightarrow y=-4$$

$$x+(-4)-(-1)=1 \Rightarrow x=4$$

$(x, y, z) = (4, -4, -1)$ Sistema Compatible Determinado.

$$b) \begin{cases} x+y+z=2 \\ x+2y=5 \\ 2x-y-2z=2 \end{cases} \xrightarrow{\begin{matrix} -E_1+E_2 \rightarrow \\ -2E_1+E_3 \rightarrow \end{matrix}} \begin{cases} x+y+z=2 \\ y-z=3 \\ -3y-4z=-2 \end{cases} \xrightarrow{3E_2+E_3} \begin{cases} x+y+z=2 \\ y-z=3 \\ -7z=7 \end{cases}$$

$$-7z=7 \Rightarrow z=-1$$

$$y-(-1)=3 \Rightarrow y=2$$

$$x+2+(-1)=2 \Rightarrow x=1$$

$(x, y, z) = (1, 2, -1)$ Sistema Compatible Determinado.

$$c) \begin{cases} 3x+2y+2z=1 \\ 2x+y+z=0 \\ 3y+2z=-1 \end{cases} \xrightarrow{\begin{matrix} -2E_1+3E_2 \rightarrow \end{matrix}} \begin{cases} 3x+2y+2z=1 \\ -y-z=-2 \\ 3y+2z=-1 \end{cases} \xrightarrow{3E_2+E_3} \begin{cases} 3x+2y+2z=1 \\ -y-z=-2 \\ -z=-7 \end{cases}$$

$$z = 7$$

$$-y - 7 = -2 \Rightarrow y = -5$$

$$3x + 2(-5) + 2 \cdot 7 = 1 \Rightarrow 3x = -3 \Rightarrow x = -1$$

$(x, y, z) = (-1, -5, 7)$ Sistema Compatible Determinado

$$d) \begin{cases} x + y - z = 3 \\ 2x - y - 5z = 0 \\ x + 6y + 4z = 13 \end{cases} \xrightarrow{\begin{matrix} -2E_1 + E_2 \\ -E_1 + E_3 \end{matrix}} \begin{cases} x + y - z = 3 \\ -3y - 3z = -6 \\ 5y + 5z = 10 \end{cases} \xrightarrow{\begin{matrix} 5E_2 + 3E_3 \end{matrix}} \begin{cases} x + y - z = 3 \\ -3y - 3z = -6 \\ 0 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x + y - z = 3 \\ -3y - 3z = -6 \end{cases} \xrightarrow{-\frac{1}{3}E_2} \begin{cases} x + y - z = 3 \\ y + z = 2 \end{cases} \quad z = \lambda \in \mathbb{R}$$

$$y = 2 - \lambda$$

$$x + 2 - \lambda - \lambda = 3 \Rightarrow x = 1 + 2\lambda$$

$(x, y, z) = (1 + 2\lambda, 2 - \lambda, \lambda) \quad \forall \lambda \in \mathbb{R}$ Sistema Compatible Indeterminado

$$e) \begin{cases} -x + y + z = 5 \\ 2x - y - 4z = -5 \\ x + y - 5z = 5 \end{cases} \xrightarrow{\begin{matrix} 2E_1 + E_2 \\ E_1 + E_3 \end{matrix}} \begin{cases} -x + y + z = 5 \\ y - 2z = 5 \\ 2y - 4z = 10 \end{cases} \xrightarrow{-2E_2 + E_3} \begin{cases} -x + y + z = 5 \\ y - 2z = 5 \\ 0 = 0 \end{cases}$$

$$\begin{cases} -x + y + z = 5 \\ y - 2z = 5 \end{cases} \quad z = \lambda \in \mathbb{R} \Rightarrow y = 5 + 2\lambda$$

$$\xrightarrow{\text{green arrow}} x = y + z - 5 = 5 + 2\lambda + \lambda - 5 = 3\lambda$$

$(x, y, z) = (3\lambda, 5 + 2\lambda, \lambda) \quad \forall \lambda \in \mathbb{R}$ Sistema Compatible Indeterminado

$$f) \begin{cases} x - y + z = 3 \\ 2x + 4z = 2 \\ 2x - y + 3z = 1 \end{cases} \xrightarrow{\substack{-2E_1 + E_2 \\ -2E_1 + E_3}} \begin{cases} x - y + z = 3 \\ 2y + 2z = -4 \\ y + z = -5 \end{cases} \xrightarrow{\substack{-2E_3 + E_2}} \begin{cases} x - y + z = 3 \\ 2y + 2z = -4 \\ 0 = 6 \end{cases}$$

$0 \neq 6 \Rightarrow \text{Sistema Incompatible}$