

- ① Halla la tasa de variación media de $f(x) = \sqrt{x-1}$ en los intervalos $[1,2]$ y $[1,10]$

$$T.V.M. [a,b] = \frac{f(b) - f(a)}{b-a}$$

TASA DE VARIACIÓN
MEDIA EN $[a,b]$

$$T.V.M. [1,2] = \frac{f(2) - f(1)}{2-1} = \frac{\sqrt{1} - \sqrt{0}}{1} = 1$$

$$T.V.M. [1,10] = \frac{f(10) - f(1)}{10-1} = \frac{\sqrt{9} - \sqrt{0}}{9} = \frac{3}{9} = \frac{1}{3}$$

- ② Dadas las funciones $f(x) = \frac{1-x^2}{3}$ y $g(x) = \frac{2}{x+1}$ determina la T.V.M. de ambas en los intervalos $[2,3]$ y $[-3,-2]$

$$T.V.M_f [2,3] = \frac{f(3) - f(2)}{3-2} = \frac{\frac{1-3^2}{3} - \frac{1-2^2}{3}}{1} = -\frac{5}{3}$$

$$T.V.M_f [-3,-2] = \frac{f(-2) - f(-3)}{-2-(-3)} = \frac{\frac{1-(-2)^2}{3} - \frac{1-(-3)^2}{3}}{1} = \frac{5}{3}$$

$$T.V.M_g [2,3] = \frac{g(3) - g(2)}{3-2} = \frac{\frac{2}{3+1} - \frac{2}{2+1}}{1} = -\frac{1}{6}$$

$$T.V.M_g [-3,-2] = \frac{g(-2) - g(-3)}{-2-(-3)} = \frac{\frac{2}{-2+1} - \frac{2}{-3+1}}{1} = -1$$

- ③ Dada $f(x) = x^2 + 2x$, obtener $f'(2)$ utilizando la definición de derivada de una función en un punto.

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Derivada de una
función en el punto
 $x=a$

$$\begin{aligned}
 f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{(2+h)^2 + 2(2+h) - 8}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{4} + 4h + \cancel{h^2} + \cancel{4} + 2h - 8}{h} = \lim_{h \rightarrow 0} \frac{h^2 + 6h}{h} = \lim_{h \rightarrow 0} \frac{\cancel{h}(h+6)}{\cancel{h}} = \\
 &= \lim_{h \rightarrow 0} (h+6) = 6
 \end{aligned}$$

④ Utilizando la definición de derivada, calcula $f'(3)$ en las siguientes funciones:

a) $f(x) = \frac{3x-2}{7}$

b) $f(x) = x^2 - 4$

c) $f(x) = (x-5)^2$

d) $f(x) = \frac{2+x}{x}$

a) $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{\frac{3(3+h)-2}{7} - 1}{h} =$

$$= \lim_{h \rightarrow 0} \frac{\frac{7+3h}{7} - 1}{h} = \lim_{h \rightarrow 0} \frac{\cancel{7} + 3h - \cancel{7}}{7h} = \lim_{h \rightarrow 0} \frac{\cancel{3h}}{\cancel{7h}} = \frac{3}{7}$$

b) $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{(3+h)^2 - 4 - 5}{h} =$

$$= \lim_{h \rightarrow 0} \frac{\cancel{9} + 6h + \cancel{h^2} - \cancel{9}}{h} = \lim_{h \rightarrow 0} \frac{6h + h^2}{h} = \lim_{h \rightarrow 0} \frac{\cancel{h}(6+h)}{\cancel{h}} =$$

$$= \lim_{h \rightarrow 0} (6+h) = 6$$

$$\begin{aligned}
 c) f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{(3+h-5)^2 - 4}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{(h-2)^2 - 4}{h} = \lim_{h \rightarrow 0} \frac{h^2 - 4h + 4 - 4}{h} = \lim_{h \rightarrow 0} \frac{h^2 - 4h}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{h(h-4)}{h} = \lim_{h \rightarrow 0} (h-4) = -4
 \end{aligned}$$

$$\begin{aligned}
 d) f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{\frac{2+3+h}{3+h} - \frac{5}{3}}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{\frac{5+h}{3+h} - \frac{5}{3}}{h} = \lim_{h \rightarrow 0} \frac{\frac{15+3h-15-5h}{3(3+h)}}{h} = \lim_{h \rightarrow 0} \frac{-2h}{3h(3+h)} = \\
 &= \lim_{h \rightarrow 0} \frac{-2}{9+3h} = -\frac{2}{9}
 \end{aligned}$$

⑤ Utilizando la definición derivada, calcula las derivadas de las siguientes funciones:

$$a) f(x) = \frac{5x+1}{2} \quad b) f(x) = 3x^2-1 \quad c) f(x) = \frac{1}{x-2}$$

$$d) f(x) = \frac{x-1}{x} \quad e) f(x) = x^2+2x+3 \quad f) f(x) = \sqrt{x}$$

$$g) f(x) = \frac{3}{x} \quad h) f(x) = \sec(x) \quad i) f(x) = e^x$$

$$j) f(x) = \ln(x) \quad k) f(x) = \sqrt[3]{x} \quad l) f(x) = \lg(x)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

FUNCIÓN DERIVADA

$$a) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{5(x+h)+1}{2} - \frac{5x+1}{2}}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\frac{5x+5h+1-5x-1}{2}}{h} = \lim_{h \rightarrow 0} \frac{5h}{2h} = \frac{5}{2}$$

$$b) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 1 - (3x^2 - 1)}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 1 - 3x^2 + 1}{h} = \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h} = 6x$$

$$c) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\frac{x-2 - (x+h-2)}{(x+h-2)(x-2)}}{h} = \lim_{h \rightarrow 0} \frac{-h}{h(x+h-2)(x-2)} =$$

$$= \lim_{h \rightarrow 0} \frac{-1}{(x+h-2)(x-2)} = \frac{-1}{(x-2)^2}$$

$$d) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{x+h-1}{x+h} - \frac{x-1}{x}}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\cancel{x^2} + \cancel{xh} - \cancel{x} - \cancel{x^2} - \cancel{xh} + x + h}{x(x+h)}}{h} = \lim_{h \rightarrow 0} \frac{\cancel{h}}{x \cancel{h}(x+h)} =$$

$$= \lim_{h \rightarrow 0} \frac{1}{x(x+h)} = \frac{1}{x^2}$$

$$e) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) + 3 - (x^2 + 2x + 3)}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2\cancel{xh} + \cancel{h^2} + 2\cancel{x} + 2\cancel{h} + 3 - \cancel{x^2} - 2\cancel{x} - 3}{h} = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 2h}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(2x + 2 + h)}{\cancel{h}} = \lim_{h \rightarrow 0} (2x + 2 + h) = 2x + 2$$

$$f) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} =$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h} - \sqrt{x}) \cdot (\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} =$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$\begin{aligned}
 g) f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{\frac{\cancel{3x} - \cancel{3x} - 3h}{x(x+h)}}{h} = \lim_{h \rightarrow 0} \frac{-3\cancel{h}}{x\cancel{h}(x+h)} = \lim_{h \rightarrow 0} \frac{-3}{x(x+h)} = \\
 &= -\frac{3}{x^2}
 \end{aligned}$$

$$\text{sen}(\alpha + \beta) = \text{sen} \alpha \cos \beta + \cos \alpha \text{sen} \beta$$

$$\begin{aligned}
 h) f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\text{sen}(x+h) - \text{sen} x}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{\text{sen} x \cdot \cancel{\cos h} + \cos x \text{sen} h - \text{sen} x}{h} = \lim_{h \rightarrow 0} \frac{\cos x \text{sen} h}{h} = \\
 &= \lim_{h \rightarrow 0} \cos x \cdot \lim_{h \rightarrow 0} \frac{\text{sen} h}{h} = \cos x
 \end{aligned}$$

$$\begin{aligned}
 i) f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x \cdot e^h - e^x}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{e^x (e^h - 1)}{h} = \lim_{h \rightarrow 0} e^x \cdot \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = \\
 &= \lim_{h \rightarrow 0} e^x \cdot \lim_{h \rightarrow 0} \frac{(1+h)^{1/h} - 1}{h} = \lim_{h \rightarrow 0} e^x \cdot \lim_{h \rightarrow 0} \frac{1+h-1}{h} = e^x
 \end{aligned}$$

$e^h = \lim_{h \rightarrow 0} \left[\frac{(1+h)^{1/h}}{e} \right]^h$

$$\begin{aligned}
 j) f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\ln(x+h) - \ln(x)}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \cdot \ln\left(\frac{x+h}{x}\right) = \lim_{h \rightarrow 0} \ln\left(\frac{x+h}{x}\right)^{1/h} \\
 &= \ln \left(\lim_{h \rightarrow 0} \left(\frac{x+h}{x}\right)^{1/h} \right) = \ln e^{\lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{x+h}{x} - 1\right)} = \\
 &= \ln e^{\lim_{h \rightarrow 0} \frac{1}{h} \left(\frac{x+h-x}{x}\right)} = \ln e^{\lim_{h \rightarrow 0} \frac{1}{x}} = \ln e^{1/x} = \frac{1}{x}
 \end{aligned}$$

$$\begin{aligned}
 k) f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} = \\
 &= \lim_{h \rightarrow 0} \frac{(\sqrt[3]{x+h} - \sqrt[3]{x}) \cdot (\sqrt[3]{(x+h)^2} + \sqrt[3]{(x+h)x} + \sqrt[3]{x^2})}{h (\sqrt[3]{(x+h)^2} + \sqrt[3]{(x+h)x} + \sqrt[3]{x^2})} = \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{\sqrt[3]{(x+h)^3}} + \cancel{\sqrt[3]{(x+h)^2} \cdot x} + \cancel{\sqrt[3]{(x+h)x^2}} - \cancel{\sqrt[3]{(x+h)^2} x} - \cancel{\sqrt[3]{(x+h)x^2}} - \sqrt[3]{x^3}}{h (\sqrt[3]{(x+h)^2} + \sqrt[3]{(x+h)x} + \sqrt[3]{x^2})} = \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{x+h-x}}{h (\sqrt[3]{(x+h)^2} + \sqrt[3]{(x+h)x} + \sqrt[3]{x^2})} = \lim_{h \rightarrow 0} \frac{\cancel{h}}{\cancel{h} (\sqrt[3]{(x+h)^2} + \sqrt[3]{(x+h)x} + \sqrt[3]{x^2})} = \\
 &= \lim_{h \rightarrow 0} \frac{1}{\sqrt[3]{(x+h)^2} + \sqrt[3]{(x+h)x} + \sqrt[3]{x^2}} = \frac{1}{3\sqrt[3]{x^2}}
 \end{aligned}$$

$$e) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\text{tg}(x+h) - \text{tg}(x)}{h} =$$

$$\text{tg}(\alpha + \beta) = \frac{\text{tg} \alpha + \text{tg} \beta}{1 - \text{tg} \alpha \cdot \text{tg} \beta}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\text{tg}(x) + \text{tg}(h)}{1 - \text{tg}(x) \cdot \text{tg}(h)} - \text{tg}(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{\text{tg}(x) + h}{1 - h \cdot \text{tg}(x)} - \text{tg}(x)}{h} =$$

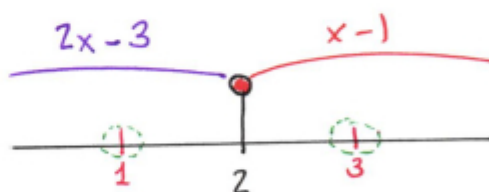
$$\text{Si } h \rightarrow 0 \Rightarrow \text{tg}(h) \approx h$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{\text{tg}(x)} + h - \cancel{\text{tg}(x)} + h \text{tg}^2 x}{h(1 - h \cdot \text{tg}(x))} = \lim_{h \rightarrow 0} \frac{\cancel{h}(1 + \text{tg}^2(x))}{\cancel{h}(1 - h \cdot \text{tg}(x))} = 1 + \text{tg}^2(x)$$

⑥ Dada $f(x) = \begin{cases} 2x-3 & \text{si } x < 2 \\ x-1 & \text{si } x \geq 2 \end{cases}$ halla $f'(1)$ y $f'(3)$ utilizando

la definición de derivada

$$f(x) = \begin{cases} 2x-3 & \text{si } x < 2 \\ x-1 & \text{si } x \geq 2 \end{cases}$$

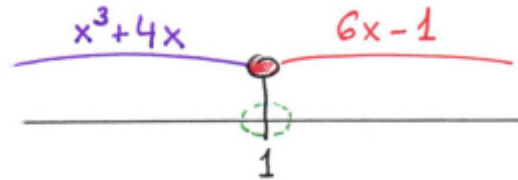


$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{2(1+h) - 3 + 1}{h} = \lim_{h \rightarrow 0} \frac{2h}{h} = 2$$

$$f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{3+h - 1 - 2}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = 1$$

⑦ Dada la función $f(x) = \begin{cases} x^3 + 4x & \text{si } x < 1 \\ 6x - 1 & \text{si } x \geq 1 \end{cases}$, ¿existe $f'(1)$?

$$f(x) = \begin{cases} x^3 + 4x & \text{si } x < 1 \\ 6x - 1 & \text{si } x \geq 1 \end{cases}$$



$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} \Rightarrow$$

$$\Rightarrow \begin{cases} \lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0^+} \frac{6(1+h) - 1 - 5}{h} = \lim_{h \rightarrow 0^+} \frac{6h}{h} = 6 \\ \lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0^-} \frac{(1+h)^3 + 4(1+h) - 5}{h} = \\ = \lim_{h \rightarrow 0^-} \frac{1 + 3h + 3h^2 + h^3 + 4 + 4h - 5}{h} = \lim_{h \rightarrow 0^-} \frac{h(7 + 3h + h^2)}{h} = 7 \end{cases}$$

Como los laterales son distintos $\Rightarrow \nexists \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} \Rightarrow \nexists f'(1)$

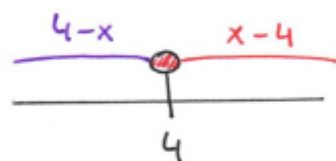
Nota Esos límites laterales que hemos evaluado se llaman **DERIVADAS LATERALES**

$$\left. \begin{aligned} f'(a^-) &= \lim_{h \rightarrow 0^-} \frac{f(a+h) - f(a)}{h} \\ f'(a^+) &= \lim_{h \rightarrow 0^+} \frac{f(a+h) - f(a)}{h} \end{aligned} \right\} \exists f'(a) \iff f'(a^-) = f'(a^+)$$

⑧ Expresa la función $f(x) = |x-4|$ y luego calcula $f'(3)$, $f'(5)$ y $f'(4)$ utilizando la definición de derivada

$$f(x) = |x-4|$$

$$x-4=0 \Rightarrow x=4$$



$$\Rightarrow f(x) = \begin{cases} 4-x & \text{si } x < 4 \\ x-4 & \text{si } x > 4 \end{cases}$$

$$f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{4 - (3+h) - 1}{h} = \lim_{h \rightarrow 0} -\frac{h}{h} = -1$$

$$f'(5) = \lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h} = \lim_{h \rightarrow 0} \frac{(5+h) - 4 - 1}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = +1$$

$$f'(4) = \lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h} \Rightarrow$$

$$\Rightarrow \begin{cases} \lim_{h \rightarrow 0^-} \frac{f(4+h) - f(4)}{h} = \lim_{h \rightarrow 0^-} \frac{4 - (4+h) - 0}{h} = \lim_{h \rightarrow 0^-} -\frac{h}{h} = -1 \\ \lim_{h \rightarrow 0^+} \frac{f(4+h) - f(4)}{h} = \lim_{h \rightarrow 0^+} \frac{(4+h) - 4 - 0}{h} = \lim_{h \rightarrow 0^+} \frac{h}{h} = +1 \end{cases}$$

Como los laterales son distintos $\Rightarrow \nexists \lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h} \Rightarrow \nexists f'(4)$

⑨ Calcular la derivada de las siguientes funciones:

$$a) f(x) = 6 \quad b) f(x) = x^4 \quad c) f(x) = \sqrt[5]{x}$$

$$d) f(x) = \sqrt[4]{x^3} \quad e) f(x) = \frac{1}{x^5} \quad f) f(x) = \sqrt[7]{x^4}$$

$$g) f(x) = 6x^3 \quad h) f(x) = \sqrt[3]{3x^4} \quad i) f(x) = \frac{3x}{x^2 \cdot \sqrt[3]{x}}$$

$$j) f(x) = 5x^2 - 4x^3 \quad k) f(x) = x^2 + \sqrt{x} \quad l) f(x) = (x^3 + x^2) \cdot (1 - x^3)$$

$$m) f(x) = \frac{5x}{x^2 - 1} \quad n) f(x) = \frac{x^2 + 2}{x + 1} \quad o) f(x) = \frac{x^3 - 5x^2}{4}$$

$$p) f(x) = \frac{x}{e} \quad q) f(x) = \frac{2}{x} \quad r) f(x) = \frac{x^4 - 5x^3 + x^2}{2}$$

$$s) f(x) = (x^2 + 5)^3 \quad t) f(x) = \frac{(x+8)^3}{(x-2)^2} \quad u) f(x) = \frac{x^2 + 3}{(1-x)^2}$$

$$v) f(x) = \frac{x+2}{(x-3)^2} \quad w) f(x) = \sqrt{\frac{x+2}{x-2}} \quad x) f(x) = \frac{6}{(x^2-3)^2}$$

$$y) f(x) = \left(1 + \frac{3}{x}\right)^3 \quad z) f(x) = \sqrt{\frac{24 - \sqrt{3}}{\pi^2 + 5}}$$

$$a) f(x) = 6 \rightarrow f'(x) = 0$$

$$b) f(x) = x^4 \rightarrow f'(x) = 4x^3$$

$$c) f(x) = \sqrt[5]{x} \rightarrow f'(x) = \frac{1}{5\sqrt[5]{x^4}}$$

$$d) f(x) = \sqrt[4]{x^3} = x^{3/4} \rightarrow f'(x) = \frac{3}{4} x^{-1/4} = \frac{3}{4 x^{1/4}} = \frac{3}{4 \sqrt[4]{x}}$$

$$e) f(x) = \frac{1}{x^5} = x^{-5} \rightarrow f'(x) = -5 x^{-6} = -\frac{5}{x^6}$$

$$f) f(x) = \sqrt[7]{x^4} = x^{4/7} \rightarrow f'(x) = \frac{4}{7} x^{-3/7} = \frac{4}{7 x^{3/7}} = \frac{4}{7 \sqrt[7]{x^3}}$$

$$g) f(x) = 6x^3 \rightarrow f'(x) = 18x^2$$

$$h) f(x) = \sqrt[3]{3x^4} = \sqrt[3]{3} \cdot \sqrt[3]{x^4} = \sqrt[3]{3} \cdot x^{4/3}$$

$$f'(x) = \sqrt[3]{3} \cdot \frac{4}{3} x^{1/3} = \frac{4 \sqrt[3]{3} \cdot \sqrt[3]{x}}{3} = \frac{4 \sqrt[3]{3x}}{3}$$

$$i) f(x) = \frac{3x}{x^2 \cdot \sqrt[3]{x}} = \frac{3}{x \sqrt[3]{x}} = \frac{3}{\sqrt[3]{x^4}} = 3 \cdot x^{-4/3}$$

$$f'(x) = 3 \cdot \left(-\frac{4}{3}\right) \cdot x^{-7/3} = -\frac{4}{x^{7/3}} = -\frac{4}{\sqrt[3]{x^7}} = -\frac{4}{x^2 \sqrt[3]{x}}$$

$$j) f(x) = 5x^2 - 4x^3 \rightarrow f'(x) = 10x - 12x^2$$

$$k) f(x) = x^2 + \sqrt{x} \rightarrow f'(x) = 2x + \frac{1}{2\sqrt{x}}$$

$$l) f(x) = (x^3 + x^2) \cdot (1 - x^3) = x^3 - x^6 + x^2 - x^5$$

$$f'(x) = 3x^2 - 6x^5 + 2x - 5x^4$$

$$m) f(x) = \frac{5x}{x^2 - 1}$$

$$f'(x) = \frac{5(x^2 - 1) - 5x \cdot 2x}{(x^2 - 1)^2} = \frac{5x^2 - 5 - 10x^2}{(x^2 - 1)^2} = \frac{-5x^2 - 5}{(x^2 - 1)^2}$$

$$n) f(x) = \frac{x^2 + 2}{x + 1}$$

$$f'(x) = \frac{2x(x+1) - (x^2 + 2) \cdot 1}{(x+1)^2} = \frac{2x^2 + 2x - x^2 - 2}{(x+1)^2} = \frac{x^2 + 2x - 2}{(x+1)^2}$$

$$o) f(x) = \frac{x^3 - 5x^2}{4} \longrightarrow f'(x) = \frac{3x^2 - 10x}{4}$$

$$p) f(x) = \frac{x}{2} \longrightarrow f'(x) = \frac{1}{2}$$

$$q) f(x) = \frac{2}{x} \longrightarrow f'(x) = \frac{0 \cdot x - 2 \cdot 1}{x^2} = -\frac{2}{x^2}$$

$$r) f(x) = \frac{x^4 - 5x^3 + x^2}{2} \longrightarrow f'(x) = \frac{4x^3 - 15x^2 + 2x}{2}$$

$$s) f(x) = (x^2 + 5)^3 \longrightarrow f'(x) = 3(x^2 + 5)^2 \cdot 2x = 6x \cdot (x^2 + 5)^2$$

$$t) f(x) = \frac{(x+8)^3}{(x-2)^2}$$

$$f'(x) = \frac{3(x+8)^2 \cdot (x-2) - (x+8)^3 \cdot 2(x-2)}{(x-2)^4} = \frac{3(x+8)^2(x-2) - 2(x+8)^3}{(x-2)^3} =$$

$$= \frac{(x+8)^2 \cdot [3(x-2) - 2(x+8)]}{(x-2)^3} = \frac{(x+8)^2 \cdot (x-22)}{(x-2)^3} = \frac{x^3 - 6x^2 - 288x - 1408}{(x-2)^3}$$

$$u) f(x) = \frac{x^2 + 3}{(1-x)^2}$$

$$f'(x) = \frac{2x(1-x)^2 - (x^2+3) \cdot 2(1-x) \cdot (-1)}{(1-x)^4} = \frac{2x(1-x) + 2(x^2+3)}{(1-x)^3} =$$

$$= \frac{2x - 2x^2 + 2x^2 + 6}{(1-x)^3} = \frac{2x+6}{(1-x)^3}$$

$$v) f(x) = \frac{x+2}{(x-3)^2}$$

$$f'(x) = \frac{1 \cdot (x-3)^2 - (x+2) \cdot 2(x-3) \cdot 1}{(x-3)^4} = \frac{x-3-2(x+2)}{(x-3)^3} = \frac{-x-7}{(x-3)^3}$$

$$w) f(x) = \sqrt{\frac{x+2}{x-2}}$$

$$f'(x) = \frac{1}{2 \cdot \sqrt{\frac{x+2}{x-2}}} \cdot \frac{1 \cdot (x-2) - (x+2) \cdot 1}{(x-2)^2} = \frac{1}{2 \sqrt{\frac{x+2}{x-2}}} \cdot \frac{-4}{(x-2)^2} =$$

$$= \frac{-2}{(x-2)^2 \cdot \sqrt{\frac{x+2}{x-2}}} = \frac{-2}{(x-2) \cdot \sqrt{\frac{(x-2)^2(x+2)}{(x-2)}}} = \frac{-2}{(x-2) \cdot \sqrt{x^2-4}}$$

$$x) f(x) = \frac{6}{(x^2-3)^2} \rightarrow f'(x) = \frac{0 \cdot (x^2-3)^2 - 6 \cdot 2(x^2-3) \cdot 2x}{(x^2-3)^4} = \frac{-24x}{(x^2-3)^3}$$

$$y) f(x) = \left(1 + \frac{3}{x}\right)^3$$

$$f'(x) = 3 \cdot \left(1 + \frac{3}{x}\right)^2 \cdot \left(-\frac{3}{x^2}\right) = -\frac{9}{x^2} \left(1 + \frac{3}{x}\right)^2 = \frac{-9x^2 - 54x - 81}{x^4}$$

$$z) f(x) = \sqrt{\frac{24 - \sqrt{3}}{\pi^2 + 5}} \rightarrow f'(x) = 0 \quad (f(x) \text{ es constante!!})$$

⑩ Calcular la derivada de las siguientes funciones:

$$a) f(x) = 2^x \quad b) f(x) = e^x \quad c) f(x) = \log_3(x)$$

$$d) f(x) = \ln(x) \quad e) f(x) = \sqrt[3]{x} + \sqrt[4]{x} \quad f) f(x) = -\frac{1}{x} + x$$

$$g) f(x) = 3^x - \arctg(x) \quad h) f(x) = 2\operatorname{sen}(x) - 3\cos(x)$$

$$i) f(x) = \operatorname{sen} x \cdot \cos x \quad j) f(x) = x \cdot e^x \quad k) f(x) = x^3 \cdot \ln x$$

$$l) f(x) = (x^2 + 2x) \cdot \operatorname{sen} x \quad m) f(x) = (e^x - x) \cdot \log_3(x) \quad n) f(x) = x \cdot \operatorname{sen}(x)$$

$$o) f(x) = 4x \cdot \operatorname{sen} x + x^3 \cos x \quad p) f(x) = e^x \cdot \operatorname{tg} x \quad q) f(x) = \frac{\operatorname{sen}(x)}{x}$$

$$r) f(x) = \frac{\sqrt{x}}{\cos x} \quad s) f(x) = \frac{\operatorname{sen} x + 2x}{x - 3} \quad t) f(x) = \sqrt{x^2 + x}$$

$$u) f(x) = (\operatorname{arcsen}(x))^3 \quad v) f(x) = \ln(x^2 + 3) \quad w) f(x) = e^{x^2 + 7x - 4}$$

$$x) f(x) = \operatorname{tg}\left(\frac{x}{x-5}\right) \quad y) f(x) = \operatorname{arctg}\left(\frac{x}{x+1}\right) \quad z) f(x) = \ln\left(\frac{1+x}{1-x}\right)$$

$$a) f(x) = 2^x \longrightarrow f'(x) = 2^x \cdot \ln(2)$$

$$b) f(x) = e^x \longrightarrow f'(x) = e^x$$

$$c) f(x) = \log_3(x) \longrightarrow f'(x) = \frac{1}{x \cdot \ln(3)}$$

$$d) f(x) = \ln(x) \longrightarrow f'(x) = \frac{1}{x}$$

$$e) f(x) = \sqrt[3]{x} + \sqrt[4]{x} \longrightarrow f'(x) = \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{4\sqrt[4]{x^3}}$$

$$f) f(x) = -\frac{1}{x} + x \longrightarrow f'(x) = \frac{1}{x^2} + 1 = \frac{1+x^2}{x^2}$$

$$g) f(x) = 3^x - \arctg(x) \longrightarrow f'(x) = 3^x \ln 3 - \frac{1}{1+x^2}$$

$$h) f(x) = 2\operatorname{sen}(x) - 3\cos x \longrightarrow f'(x) = 2\cos(x) + 3\operatorname{sen}(x)$$

$$i) f(x) = \operatorname{sen} x \cdot \cos x \longrightarrow f'(x) = \cos^2 x - \operatorname{sen}^2 x = \cos(2x)$$

$$j) f(x) = x \cdot e^x \longrightarrow f'(x) = 1 \cdot e^x + x \cdot e^x = e^x(1+x)$$

$$k) f(x) = x^3 \cdot \ln x \longrightarrow f'(x) = 3x^2 \cdot \ln x + x^{\overset{2}{3}} \cdot \frac{1}{\cancel{x}} = x^2(3\ln x + 1)$$

$$l) f(x) = (x^2 + 2x) \operatorname{sen} x \longrightarrow f'(x) = (2x + 2) \operatorname{sen} x + (x^2 + 2x) \cos x$$

$$m) f(x) = (e^x - x) \cdot \log_3(x) \longrightarrow f'(x) = (e^x - 1) \log_3(x) + (e^x - x) \cdot \frac{1}{x \ln(3)}$$

$$n) f(x) = x \cdot \operatorname{sen}(x) \rightarrow f'(x) = \operatorname{sen} x + x \cdot \cos x$$

$$o) f(x) = 4x \operatorname{sen} x + x^3 \cos x$$

$$\begin{aligned} f'(x) &= 4 \operatorname{sen} x + 4x \cos x + 3x^2 \cos x - x^3 \operatorname{sen} x = \\ &= \operatorname{sen} x \cdot (4 - x^3) + \cos x \cdot (4x + 3x^2) \end{aligned}$$

$$p) f(x) = e^x \cdot \operatorname{tg} x \rightarrow f'(x) = e^x \cdot \operatorname{tg} x + e^x (1 + \operatorname{tg}^2 x) = e^x (1 + \operatorname{tg} x + \operatorname{tg}^2 x)$$

$$q) f(x) = \frac{\operatorname{sen}(x)}{x} \rightarrow f'(x) = \frac{x \cos x - \operatorname{sen} x}{x^2}$$

$$r) f(x) = \frac{\sqrt{x}}{\cos x} \rightarrow f'(x) = \frac{\frac{1}{2\sqrt{x}} \cdot \cos x + x \cdot \operatorname{sen}(x)}{\cos^2 x}$$

$$s) f(x) = \frac{\operatorname{sen} x + 2x}{x-3}$$

$$\begin{aligned} f'(x) &= \frac{(\cos x + 2)(x-3) - (\operatorname{sen} x + 2x) \cdot 1}{(x-3)^2} = \frac{(x-3)\cos x + 2x - 6 - \operatorname{sen} x - 2x}{(x-3)^2} = \\ &= \frac{(x-3)\cos x - \operatorname{sen} x - 6}{(x-3)^2} \end{aligned}$$

$$t) f(x) = \sqrt{x^2 + x} \rightarrow f'(x) = \frac{1}{2\sqrt{x^2 + x}} \cdot (2x + 1) = \frac{2x + 1}{2\sqrt{x^2 + x}}$$

$$u) f(x) = (\operatorname{arcsen}(x))^3 \rightarrow f'(x) = 3(\operatorname{arcsen}(x))^2 \cdot \frac{1}{\sqrt{1-x^2}}$$

$$v) f(x) = \ln(x^2 + 3) \rightarrow f'(x) = \frac{1}{x^2 + 3} \cdot 2x = \frac{2x}{x^2 + 3}$$

$$w) f(x) = e^{x^2+7x-4} \rightarrow f'(x) = e^{x^2+7x-4} \cdot (2x+7)$$

$$x) f(x) = \operatorname{tg}\left(\frac{x}{x-5}\right)$$

$$f'(x) = \frac{1}{\cos^2\left(\frac{x}{x-5}\right)} \cdot \frac{1(x-5) - x}{(x-5)^2} = \frac{-5}{(x-5)^2 \cdot \cos^2\left(\frac{x}{x-5}\right)}$$

$$y) f(x) = \operatorname{arctg}\left(\frac{x}{x+1}\right)$$

$$f'(x) = \frac{1}{1 + \left(\frac{x}{x+1}\right)^2} \cdot \frac{1 \cdot (x+1) - x \cdot 1}{(x+1)^2} = \frac{1}{\frac{(x+1)^2 + x^2}{(x+1)^2}} \cdot \frac{1}{\cancel{(x+1)^2}} =$$

$$= \frac{1}{2x^2 + 2x + 1}$$

$$z) f(x) = \ln\left(\frac{1+x}{1-x}\right)$$

$$f'(x) = \frac{1}{\frac{1+x}{\cancel{1-x}}} \cdot \frac{1 \cdot (1-x) + (1+x)}{(1-x)^2} = \frac{2}{(1+x)(1-x)} = \frac{2}{1-x^2}$$

⑪ Calcular las derivadas de las siguientes funciones:

$$a) f(x) = \frac{x^2-3}{x^2+3}$$

$$b) f(x) = \frac{x+1}{(2-x)^2}$$

$$c) f(x) = \frac{3x^2}{x+\sqrt{x}}$$

$$d) f(x) = \left(0.5 - \frac{x}{10}\right)^4$$

$$e) f(x) = \frac{x^3}{(x+1)^2}$$

$$f) f(x) = \left(\frac{x^2+1}{x}\right)^3$$

$$g) f(x) = \frac{x}{(2x+1)^3}$$

$$h) f(x) = e^{4x} \cdot (x-1)$$

$$i) f(x) = \frac{(1-x)^2}{e^x}$$

$$j) f(x) = \sqrt{2^x}$$

$$k) f(x) = \ln(2x-1)$$

$$l) f(x) = \ln(x^2-1)$$

$$m) f(x) = \ln(\sqrt{1-x})$$

$$n) f(x) = \frac{\ln x}{e^x}$$

$$o) f(x) = e^{x^2+1}$$

$$p) f(x) = \sec x \cdot \cos^2 x$$

$$q) f(x) = \frac{\sec^2 x}{1+\cos^2 x}$$

$$r) f(x) = \sec^2(x^2)$$

$$s) f(x) = \cos^3(2x+1)$$

$$t) f(x) = \log_2 \left| \frac{1}{x} \right|$$

$$u) f(x) = \sqrt[3]{\sec(x^2)}$$

$$v) f(x) = \sqrt{\frac{1+2x}{1-2x}}$$

$$w) f(x) = \sqrt{x+\sqrt{x}}$$

$$x) f(x) = \ln \sqrt{\frac{x}{x+1}}$$

$$y) f(x) = \ln(\sec(\sqrt{e^x}))$$

$$z) f(x) = \ln \sqrt{\frac{x-1}{x+1}}$$

$$a) f(x) = \frac{x^2-3}{x^2+3}$$

$$f'(x) = \frac{2x(x^2+3) - (x^2-3) \cdot 2x}{(x^2+3)^2} = \frac{2x(x^2+3 - x^2+3)}{(x^2+3)^2} = \frac{12x}{(x^2+3)^2}$$

$$b) f(x) = \frac{x+1}{(2-x)^2}$$

$$f'(x) = \frac{1 \cdot (2-x)^2 - (x+1) \cdot 2(2-x) \cdot (-1)}{(2-x)^4} = \frac{2-x+2x+2}{(2-x)^3} = \frac{x+4}{(2-x)^3}$$

$$c) f(x) = \frac{3x^2}{x + \sqrt{x}}$$

$$\begin{aligned} f'(x) &= \frac{6x(x + \sqrt{x}) - 3x^2(1 + \frac{1}{2}\sqrt{x})}{(x + \sqrt{x})^2} = \frac{6x^2 + 6x\sqrt{x} - 3x^2 - \frac{3x^2}{2\sqrt{x}}}{(x + \sqrt{x})^2} = \\ &= \frac{3x^2 + 6x\sqrt{x} - \frac{3x^2}{2\sqrt{x}}}{(x + \sqrt{x})^2} = \frac{6x^2\sqrt{x} + 12x^2 - 3x^2}{2\sqrt{x}(x + \sqrt{x})^2} = \frac{6x^2\sqrt{x} + 9x^2}{2\sqrt{x}(x + \sqrt{x})^2} = \\ &= \frac{6x^2\sqrt{x} + 9x^2}{2\sqrt{x}(x^2 + 2x\sqrt{x} + x)} = \frac{\cancel{x}(6x\sqrt{x} + 9x)}{\cancel{x} \cdot 2\sqrt{x} \cdot (x + 2\sqrt{x} + 1)} = \frac{\cancel{\sqrt{x}}(6x + 9\sqrt{x})}{2\cancel{\sqrt{x}}(\sqrt{x} + 1)^2} = \\ &= \frac{6x + 9\sqrt{x}}{2(\sqrt{x} + 1)^2} \end{aligned}$$

$$d) f(x) = \left(0.5 - \frac{x}{10}\right)^4$$

$$\begin{aligned} f'(x) &= 4 \left(0.5 - \frac{x}{10}\right)^3 \cdot \left(-\frac{1}{10}\right) = -\frac{2}{5} \cdot \left(0.5 - \frac{x}{10}\right)^3 = -\frac{2(5-x)^3}{5000} = \\ &= -\frac{(5-x)^3}{2500} \end{aligned}$$

$$e) f(x) = \frac{x^3}{(x+1)^2}$$

$$f'(x) = \frac{3x^2 \cdot (x+1)^2 - x^3 \cdot 2(x+1)}{(x+1)^4} = \frac{3x^3 + 3x^2 - 2x^3}{(x+1)^3} = \frac{x^3 + 3x^2}{(x+1)^3}$$

$$f) f(x) = \left(\frac{x^2+1}{x}\right)^3$$

$$f'(x) = 3 \left(\frac{x^2+1}{x}\right)^2 \cdot \frac{2x \cdot x - (x^2+1)}{x^2} = \frac{3(x^2+1)^2}{x^2} \cdot \frac{x^2-1}{x^2} = \frac{3(x^2+1)(x^2-1)}{x^4}$$

$$g) f(x) = \frac{x}{(2x+1)^3}$$

$$f'(x) = \frac{1 \cdot (2x+1)^3 - x \cdot 3(2x+1)^2 \cdot 2}{(2x+1)^6} = \frac{2x+1-6x}{(2x+1)^4} = \frac{-4x+1}{(2x+1)^4}$$

$$h) f(x) = e^{4x} \cdot (x-1)$$

$$f'(x) = e^{4x} \cdot 4 \cdot (x-1) + e^{4x} = e^{4x} (4x-4+1) = e^{4x} (4x-3)$$

$$i) f(x) = \frac{(1-x)^2}{e^x}$$

$$f'(x) = \frac{2(1-x) \cdot (-1) \cdot e^{-x} - (1-x)^2 \cdot e^{-x}}{(e^x)^2} = \frac{-2+2x - (1-2x+x^2)}{e^x} = \frac{-x^2+4x-3}{e^x}$$

$$j) f(x) = \sqrt{2^x} = 2^{x/2}$$

$$f'(x) = 2^{x/2} \cdot \frac{1}{2} \cdot \ln(2) = \frac{1}{2} \cdot \ln 2 \cdot \sqrt{2^x}$$

$$k) f(x) = \ln(2x-1) \longrightarrow f'(x) = \frac{1}{2x-1} \cdot 2 = \frac{2}{2x-1}$$

$$e) f(x) = \ln(x^2-1) \rightarrow f'(x) = \frac{1}{x^2-1} \cdot 2x = \frac{2x}{x^2-1}$$

$$m) f(x) = \ln(\sqrt{1-x})$$

$$f'(x) = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1-x}} \cdot (-1) = \frac{-1}{2(1-x)} = \frac{1}{2(x-1)}$$

$$n) f(x) = \frac{\ln x}{e^x}$$

$$f'(x) = \frac{1/x \cdot \cancel{e^x} - \ln x \cdot \cancel{e^x}}{(e^x)^2} = \frac{\frac{1}{x} - \ln x}{e^x} = \frac{1 - x \cdot \ln x}{x \cdot e^x}$$

$$o) f(x) = e^{x^2+1} \rightarrow f'(x) = e^{x^2+1} \cdot 2x$$

$$p) f(x) = \operatorname{sen} x \cdot \cos^2 x$$

$$f'(x) = \cos x \cdot \cos^2 x + \operatorname{sen} x \cdot 2 \cos x \cdot (-\operatorname{sen} x) = \cos^3 x - 2 \operatorname{sen}^2 x \cdot \cos x$$

$$q) f(x) = \frac{\operatorname{sen}^2 x}{1 + \cos^2 x}$$

$$f'(x) = \frac{2 \operatorname{sen} x \cdot \cos x \cdot (1 + \cos^2 x) - \operatorname{sen}^2 x \cdot 2 \cos x \cdot (-\operatorname{sen} x)}{(1 + \cos^2 x)^2} =$$

$$= \frac{2 \operatorname{sen} x \cos x + 2 \operatorname{sen} x \cdot \cos^3 x + 2 \operatorname{sen}^3 x \cdot \cos x}{(1 + \cos^2 x)^2} = \frac{2 \operatorname{sen} x \cos x (1 + \cos^2 x + \operatorname{sen}^2 x)}{(1 + \cos^2 x)^2}$$

$$= \frac{(2 \operatorname{sen} x \cos x) \cdot 2}{(1 + \cos^2 x)^2} = \frac{2 \operatorname{sen}(2x)}{(1 + \cos^2 x)^2}$$

$$r) f(x) = \operatorname{sen}^2(x^2)$$

$$f'(x) = 2 \operatorname{sen}(x^2) \cdot \cos(x^2) \cdot 2x = 2x \operatorname{sen}(2x^2)$$

$$s) f(x) = \cos^3(2x+1)$$

$$f'(x) = 3 \cos^2(2x+1) \cdot (-\operatorname{sen}(2x+1)) \cdot 2 = -6 \cos^2(2x+1) \cdot \operatorname{sen}(2x+1)$$

$$t) f(x) = \log_2\left(\frac{1}{x}\right)$$

$$f'(x) = \frac{1}{\frac{1}{x} \cdot \ln 2} \cdot \frac{-1}{x^2} = \frac{-1}{x \cdot \ln 2}$$

$$u) f(x) = \sqrt[3]{\operatorname{sen}(x^2)}$$

$$f'(x) = \frac{1}{\sqrt[3]{\operatorname{sen}^2(x^2)}} \cdot \cos(x^2) \cdot 2x = \frac{2x \cos(x^2)}{\sqrt[3]{\operatorname{sen}^2(x^2)}}$$

$$v) f(x) = \sqrt{\frac{1+2x}{1-2x}}$$

$$f'(x) = \frac{1}{2 \sqrt{\frac{1+2x}{1-2x}}} \cdot \frac{2(1-2x) - (1+2x) \cdot (-2)}{(1-2x)^2} = \frac{2(1-2x+1+2x)}{2(1-2x)^2 \sqrt{\frac{1+2x}{1-2x}}} =$$

$$= \frac{2}{(1-2x) \cdot \sqrt{\frac{(1-2x)^2 \cdot (1+2x)}{(1-2x)}}} = \frac{2}{(1-2x) \sqrt{1-4x^2}}$$

$$w) f(x) = \sqrt{x + \sqrt{x}}$$

$$f'(x) = \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right) = \frac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x + \sqrt{x}}}$$

$$x) f(x) = \ln \sqrt{\frac{x}{x+1}}$$

$$\begin{aligned} f'(x) &= \frac{1}{\sqrt{\frac{x}{x+1}}} \cdot \frac{1}{2\sqrt{\frac{x}{x+1}}} \cdot \frac{1(x+1) - x \cdot 1}{(x+1)^2} = \frac{1}{2 \cdot \frac{x}{x+1}} \cdot \frac{1}{(x+1)^2} = \\ &= \frac{1}{2x(x+1)} \end{aligned}$$

$$y) f(x) = \ln(\sec \sqrt{e^x})$$

$$f'(x) = \frac{1}{\sec(\sqrt{e^x})} \cdot \cos(\sqrt{e^x}) \cdot \frac{1}{2\sqrt{e^x}} \cdot e^x = \frac{\sqrt{e^x}}{2 \cdot \tan(\sqrt{e^x})}$$

$$z) f(x) = \ln \sqrt{\frac{x-1}{x+1}}$$

$$\begin{aligned} f'(x) &= \frac{1}{\sqrt{\frac{x-1}{x+1}}} \cdot \frac{1}{2\sqrt{\frac{x-1}{x+1}}} \cdot \frac{1(x+1) - (x-1) \cdot 1}{(x+1)^2} = \frac{1}{\frac{2(x-1)}{x+1}} \cdot \frac{2}{(x+1)^2} = \\ &= \frac{1}{(x-1)(x+1)} = \frac{1}{x^2 - 1} \end{aligned}$$

⑫ Calcular las derivadas de las siguientes funciones:

$$a_1) f(x) = e^{4x} \quad a_2) f(x) = \log_5(7x+2) \quad a_3) f(x) = 5 \cdot e^{x^2+3x}$$

$$b_1) f(x) = (x^3+1)(x^3-1) \quad b_2) f(x) = x^3 \cdot e^x \quad b_3) f(x) = x^2 \cdot \ln(x^2)$$

$$c_1) f(x) = e^x \cdot \operatorname{tg} x \quad c_2) f(x) = \operatorname{sen} x \cdot \cos(2x) \quad c_3) f(x) = \frac{1}{x-1}$$

$$d_1) f(x) = \frac{-x}{x+1} \quad d_2) f(x) = \frac{3}{x^2-1} \quad d_3) f(x) = \frac{x^2-3}{x^2+3}$$

$$e_1) f(x) = \frac{\ln(x^2+5)}{x} \quad e_2) f(x) = 5x^3 - 3x^2 + 4 \quad e_3) f(x) = (x^2+1)^3$$

$$f_1) f(x) = \sqrt[3]{x^2+3} \quad f_2) f(x) = x^2 \cdot \ln(x) \quad f_3) f(x) = \frac{x^2}{\ln x}$$

$$g_1) f(x) = \operatorname{sen}(3x) \quad g_2) f(x) = (\operatorname{sen} x)^5 \quad g_3) f(x) = e^x \cdot \operatorname{sen} x$$

$$h_1) f(x) = \cos(3x+1) \quad h_2) f(x) = e^{7x^2} \quad h_3) f(x) = e^{\operatorname{sen}(2x)}$$

$$i_1) f(x) = e^x + \ln x \quad i_2) f(x) = \frac{\log x^2}{x} \quad i_3) f(x) = \operatorname{sen}(3x+1) \cdot \cos(3x+1)$$

$$j_1) f(x) = \operatorname{sen}(x^2-5x+7) \quad j_2) f(x) = x \cdot e^{2x+1} \quad j_3) f(x) = \frac{2}{(x+3)^3}$$

$$k_1) f(x) = \frac{x^3}{e} + \frac{x^2+7x}{5} \quad k_2) f(x) = \frac{1}{\sqrt{x-4}} \quad k_3) f(x) = \frac{e^x + e^{-x}}{2}$$

$$l_1) f(x) = \frac{x^3}{(x-1)^2} \quad l_2) f(x) = \sqrt{\frac{x^3}{x^2-4}} \quad l_3) f(x) = \operatorname{tg}^3(x^2)$$

$$\begin{aligned}
m_1) f(x) &= \sqrt{\operatorname{sen} x} & m_2) f(x) &= 7^{x+1} \cdot e^{-x} & m_3) f(x) &= \ln x + e^{\sqrt{x}} \\
n_1) f(x) &= e^{2x} \cdot \operatorname{tg} x & n_2) f(x) &= \log \left(\frac{x^2}{3-x} \right) & n_3) f(x) &= \sqrt{\ln x} \\
o_1) f(x) &= (x^2 - 7x)^3 & o_2) f(x) &= \left(\frac{x}{1+x^2} \right)^2 & o_3) f(x) &= \operatorname{arctg} \left(\frac{x}{1+x} \right) \\
p_1) f(x) &= \frac{x \cdot e^{-x}}{3^x} & p_2) f(x) &= \operatorname{arcsen}(x^2) & p_3) f(x) &= \ln(\operatorname{tg}(2x)) \\
q_1) f(x) &= \ln \sqrt{\frac{x}{x-1}} & q_2) f(x) &= \ln \left(\frac{\sqrt{x}}{1+\operatorname{sen}^2 x} \right) & q_3) f(x) &= \operatorname{arctg} \left(\sqrt{\frac{x}{x+2}} \right) \\
r_1) f(x) &= \frac{\cos x + \cos(x+1)}{\operatorname{sen} x - \operatorname{sen}(x+1)} & r_2) f(x) &= \ln \sqrt{\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}} & r_3) f(x) &= \ln e^{\operatorname{sen} x} \\
s_1) f(x) &= \operatorname{arctg} \left(\sqrt{\frac{1-\cos x}{1+\cos x}} \right) & s_2) f(x) &= \ln \left(\frac{x}{x+1} \right)^x & s_3) f(x) &= \sqrt{e^x + e^{-x}} \\
t_1) f(x) &= \ln \left(\sqrt{\frac{x^3 \cdot \cos x}{\operatorname{sen} x}} \right) & t_2) f(x) &= \frac{e^{x+3} - 1}{x^2 - 9} & t_3) f(x) &= \operatorname{arcsen}(\sqrt{1-\operatorname{sen}^2 x}) \\
u_1) f(x) &= \operatorname{arcsen} \left(\frac{1}{1+x^2} \right) & u_2) f(x) &= \frac{\ln \sqrt{x+1}}{x} & u_3) f(x) &= e^{\ln(\operatorname{sen}^2 x)} \\
v_1) f(x) &= \operatorname{tg}(1+\operatorname{sen} x) & v_2) f(x) &= \operatorname{tg}^3(e^{x^2}) & v_3) f(x) &= 27 + (x-1)^4 \\
w_1) f(x) &= \operatorname{sen}^2 x + \cos^2 x & w_2) f(x) &= \frac{e^x + 1}{e^x - 1} & w_3) f(x) &= \frac{2 + \cos x}{2 - \cos x} \\
x_1) f(x) &= \operatorname{arcsen} \left(\frac{x}{\sqrt{1+x^2}} \right) & x_2) f(x) &= \arccos \left(\frac{1}{\sqrt{1+x^2}} \right) & x_3) f(x) &= e^{5x}
\end{aligned}$$

$$y_1) f(x) = \sqrt{\sec(x)} \quad y_2) f(x) = \operatorname{arccotg}\left(\frac{1}{x}\right) \quad y_3) f(x) = \operatorname{arcsec}(x)$$

$$z_1) f(x) = \frac{\cos(2x) + \operatorname{sen}(2x)}{\cos(2x) - \operatorname{sen}(2x)} \quad z_2) f(x) = \ln \sqrt{\frac{x^2 \operatorname{sen} x}{\cos x}} \quad z_3) f(x) = \ln(1+x)^x$$

$$a_1) f(x) = e^{4x} \rightarrow f'(x) = e^{4x} \cdot 4$$

$$a_2) f(x) = \log_5(7x+2) \rightarrow f'(x) = \frac{1}{(7x+2) \cdot \ln(5)} \cdot 7 = \frac{7}{(7x+2) \ln(5)}$$

$$a_3) f(x) = 5 \cdot e^{x^2+3x} \rightarrow f'(x) = 5 \cdot e^{x^2+3x} \cdot (2x+3)$$

$$b_1) f(x) = (x^3+1) \cdot (x^3-1) = x^6-1 \rightarrow f'(x) = 6x^5$$

$$b_2) f(x) = x^3 \cdot e^x \rightarrow f'(x) = 3x^2 \cdot e^x + x^3 \cdot e^x = e^x(3x^2+x^3)$$

$$b_3) f(x) = x^2 \cdot \ln(x^2) \rightarrow f'(x) = 2x \cdot \ln(x^2) + \cancel{x^2} \cdot \frac{1}{\cancel{x^2}} \cdot 2x = 2x(\ln(x^2)+1)$$

$$c_1) f(x) = e^x \cdot \operatorname{tg} x \rightarrow f'(x) = e^x \cdot \operatorname{tg} x + e^x(1+\operatorname{tg}^2 x) = e^x(1+\operatorname{tg} x + \operatorname{tg}^2 x)$$

$$c_2) f(x) = \operatorname{sen} x \cdot \cos(2x)$$

$$f'(x) = \cos x \cdot \cos(2x) + \operatorname{sen}(x) \cdot (-\operatorname{sen}(2x)) \cdot 2 = \cos x \cdot \cos(2x) - 2\operatorname{sen}(x) \cdot \operatorname{sen}(2x)$$

$$c_3) f(x) = \frac{1}{x-1}$$

$$f'(x) = \frac{0 \cdot (x-1) - 1 \cdot 1}{(x-1)^2} = -\frac{1}{(x-1)^2}$$

$$d_1) f(x) = \frac{-x}{x+1}$$

$$f'(x) = \frac{-1(x+1) + x}{(x+1)^2} = \frac{-1}{(x+1)^2}$$

$$d_2) f(x) = \frac{3}{x^2-1} \longrightarrow f'(x) = \frac{-3 \cdot 2x}{(x^2-1)^2} = \frac{-6x}{(x^2-1)^2}$$

$$d_3) f(x) = \frac{x^2-3}{x^2+3} \longrightarrow f'(x) = \frac{2x(x^2+3) - (x^2-3) \cdot 2x}{(x^2+3)^2} = \frac{2x \cdot 6}{(x^2+3)^2} = \frac{12x}{(x^2+3)^2}$$

$$e_1) f(x) = \frac{\ln(x^2+5)}{x}$$

$$f'(x) = \frac{\frac{1}{x^2+5} \cdot 2x \cdot x - \ln(x^2+5) \cdot 1}{x^2} = \frac{2x^2 - (x^2+5) \cdot \ln(x^2+5)}{x^2(x^2+5)}$$

$$e_2) f(x) = 5x^3 - 3x^2 + 4 \longrightarrow f'(x) = 15x^2 - 6x$$

$$e_3) f(x) = (x^2+1)^3 \longrightarrow f'(x) = 3(x^2+1)^2 \cdot 2x = 6x(x^2+1)^2$$

$$f_1) f(x) = \sqrt[3]{x^2+3} \longrightarrow f'(x) = \frac{1}{3\sqrt[3]{(x^2+3)^2}} \cdot 2x = \frac{2x}{3\sqrt[3]{(x^2+3)^2}}$$

$$f_2) f(x) = x^2 \cdot \ln x \longrightarrow f'(x) = 2x \cdot \ln x + x^2 \cdot \frac{1}{x} = x(2\ln x + 1)$$

$$f_3) f(x) = \frac{x^2}{\ln x}$$

$$f'(x) = \frac{2x \cdot \ln x - x^2 \cdot 1/x}{(\ln x)^2} = \frac{2x \ln x - x}{(\ln x)^2}$$

$$g_1) f(x) = \operatorname{sen}(3x) \longrightarrow f'(x) = \cos(3x) \cdot 3 = 3\cos(3x)$$

$$g_2) f(x) = (\operatorname{sen}(x))^5 \longrightarrow f'(x) = 5(\operatorname{sen} x)^4 \cdot \cos x$$

$$g_3) f(x) = e^x \cdot \operatorname{sen} x \longrightarrow f'(x) = e^x \cdot \operatorname{sen} x + e^x \cdot \cos x$$

$$h_1) f(x) = \cos(3x+1) \longrightarrow f'(x) = -\operatorname{sen}(3x+1) \cdot 3 = -3\operatorname{sen}(3x+1)$$

$$h_2) f(x) = e^{7x^2} \longrightarrow f'(x) = e^{7x^2} \cdot 14x$$

$$h_3) f(x) = e^{\operatorname{sen}(2x)} \longrightarrow f'(x) = e^{\operatorname{sen}(2x)} \cdot \cos(2x) \cdot 2$$

$$i_1) f(x) = e^x + \ln x \longrightarrow f'(x) = e^x + \frac{1}{x}$$

$$i_2) f(x) = \frac{\log x^2}{x} \longrightarrow f'(x) = \frac{\frac{1}{x^2 \ln 10} \cdot 2x \cdot x - \log x^2}{x^2} = \frac{\frac{2}{\ln 10} - \log x^2}{x^2}$$

$$i_3) f(x) = \operatorname{sen}(3x+1) \cdot \cos(3x+1)$$

$$\begin{aligned} f'(x) &= \cos(3x+1) \cdot 3 \cdot \cos(3x+1) + \operatorname{sen}(3x+1) \cdot (-\operatorname{sen}(3x+1)) \cdot 3 = \\ &= 3 \left(\cos^2(3x+1) - \operatorname{sen}^2(3x+1) \right) = 3 \cos(6x+2) \end{aligned}$$

$$j_1) f(x) = \operatorname{sen}(x^2-5x+7)$$

$$f'(x) = \cos(x^2-5x+7) \cdot (2x-5) = (2x-5)\cos(x^2-5x+7)$$

$$j_2) f(x) = x \cdot e^{2x+1} \rightarrow f'(x) = 1 \cdot e^{2x+1} + x \cdot e^{2x+1} \cdot 2 = e^{2x+1} (1+2x)$$

$$j_3) f(x) = \frac{2}{(x+3)^3} \rightarrow f'(x) = \frac{-2 \cdot 3(x+3)^2 \cdot 1}{(x+3)^{6+4}} = \frac{-6}{(x+3)^4}$$

$$k_1) f(x) = \frac{x^3}{2} + \frac{x^2+7x}{5} \rightarrow f'(x) = \frac{3x^2}{2} + \frac{2x+7}{5}$$

$$k_2) f(x) = \frac{1}{\sqrt{x-4}} \rightarrow f'(x) = \frac{-1 \cdot \frac{1}{2\sqrt{x-4}}}{(\sqrt{x-4})^2} = \frac{-1}{2\sqrt{(x-4)^3}}$$

$$k_3) f(x) = \frac{e^x + e^{-x}}{2} \rightarrow f'(x) = \frac{e^x + e^{-x} \cdot (-1)}{2} = \frac{e^x - e^{-x}}{2}$$

$$l_1) f(x) = \frac{x^3}{(x-1)^2}$$

$$f'(x) = \frac{3x^2(x-1)^2 - x^3 \cdot 2(x-1)}{(x-1)^{4+3}} = \frac{3x^3 - 3x^2 - 2x^3}{(x-1)^3} = \frac{x^3 - 3x^2}{(x-1)^3}$$

$$l_2) f(x) = \sqrt{\frac{x^3}{x^2-4}}$$

$$f'(x) = \frac{1}{2\sqrt{\frac{x^3}{x^2-4}}} \cdot \frac{3x^2(x^2-4) - x^3 \cdot 2x}{(x^2-4)^2} = \frac{3x^4 - 12x^2 - 2x^4}{2(x^2-4)^2 \cdot \sqrt{\frac{x^3}{x^2-4}}} =$$

$$= \frac{x^4 - 12x^2}{2(x^2-4)^2 \cdot x \cdot \sqrt{\frac{x}{x^2-4}}} = \frac{x(x^3 - 12x)}{2x(x^2-4)^2 \cdot \sqrt{\frac{x}{x^2-4}}} = \frac{x^3 - 12x}{2(x^2-4)^2 \cdot \sqrt{\frac{x}{x^2-4}}}$$

$$l_3) f(x) = \operatorname{tg}^3(x^2)$$

$$f'(x) = 3 \cdot \operatorname{tg}^2(x^2) \cdot (1 + \operatorname{tg}^2(x^2)) \cdot 2x = 6x \operatorname{tg}^2(x^2) \cdot (1 + \operatorname{tg}^2(x^2))$$

$$m_1) f(x) = \sqrt{\sec x} \longrightarrow f'(x) = \frac{1 \cdot \cos x}{2\sqrt{\sec x}}$$

$$m_2) f(x) = 7^{x+1} \cdot e^{-x} \longrightarrow f'(x) = 7^{x+1} \cdot \ln 7 \cdot e^{-x} + 7^{x+1} \cdot e^{-x} \cdot (-1) = \\ = 7^{x+1} \cdot e^{-x} (\ln 7 - 1)$$

$$m_3) f(x) = \ln x + e^{\sqrt{x}} \longrightarrow f'(x) = \frac{1}{x} + e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$n_1) f(x) = e^{2x} \cdot \operatorname{tg} x \longrightarrow f'(x) = e^{2x} \cdot 2 \cdot \operatorname{tg} x + e^{2x} \cdot (1 + \operatorname{tg}^2 x) = \\ = e^{2x} (2 \operatorname{tg} x + 1 + \operatorname{tg}^2 x)$$

$$n_2) f(x) = \log\left(\frac{x^2}{3-x}\right)$$

$$f'(x) = \frac{1}{\left(\frac{x^2}{3-x}\right) \cdot \ln 10} \cdot \frac{2x(3-x) + x^2}{(3-x)^2} = \frac{6x - x^2}{x^2 \cdot (3-x) \cdot \ln 10} = \\ = \frac{\cancel{x} (6-x)}{x^{\cancel{2}} \cdot (3-x) \cdot \ln 10} = \frac{6-x}{x(3-x) \ln 10}$$

$$n_3) f(x) = \sqrt{\ln x}$$

$$f'(x) = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$$

$$o_1) f(x) = (x^2 - 7x)^3 \longrightarrow f'(x) = 3(x^2 - 7x)^2 \cdot (2x - 7)$$

$$o_2) f(x) = \left(\frac{x}{1+x^2} \right)^2$$

$$f'(x) = 2 \cdot \left(\frac{x}{1+x^2} \right) \cdot \frac{1(1+x^2) - x \cdot 2x}{(1+x^2)^2} = \frac{2x}{1+x^2} \cdot \frac{1-x^2}{(1+x^2)^2} =$$

$$= \frac{2x - 2x^3}{(1+x^2)^3}$$

$$o_3) f(x) = \arctg\left(\frac{x}{1+x}\right)$$

$$f'(x) = \frac{1}{1 + \left(\frac{x}{1+x}\right)^2} \cdot \frac{1(1+x) - x}{(1+x)^2} = \frac{1}{\frac{(1+x)^2 + x^2}{(1+x)^2}} \cdot \frac{1}{(1+x)^2} =$$

$$= \frac{1}{2x^2 + 2x + 1}$$

$$p_1) f(x) = \frac{x \cdot e^{-x}}{3^x}$$

$$f'(x) = \frac{(1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1)) \cdot 3^x - x \cdot e^{-x} \cdot 3^x \cdot \ln 3}{(3^x)^2} =$$

$$= \frac{e^{-x} (1 - x \cdot e^{-x} - x \cdot e^{-x} \cdot \ln 3)}{3^x}$$

$$P_2) f(x) = \operatorname{arcsen}(x^2)$$

$$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

$$P_3) f(x) = \ln(\operatorname{tg}(2x))$$

$$f'(x) = \frac{1}{\operatorname{tg}(2x)} \cdot \frac{1}{\cos^2(2x)} \cdot 2 = \frac{2}{\operatorname{sen}(2x) \cdot \cos(2x)}$$

$$q_1) f(x) = \ln \sqrt{\frac{x}{x-1}}$$

$$f'(x) = \frac{1}{\sqrt{\frac{x}{x-1}}} \cdot \frac{1}{2\sqrt{\frac{x}{x-1}}} \cdot \frac{1(x-1)-x}{(x-1)^2} = \frac{-1}{2 \cdot \left(\frac{x}{x-1}\right) \cdot (x-1)^2} = \frac{-1}{2x^2-2x}$$

$$q_2) f(x) = \ln\left(\frac{\sqrt{x}}{1+\operatorname{sen}^2 x}\right)$$

$$\operatorname{sen}(2x) = 2\operatorname{sen} x \cdot \cos x$$

$$f'(x) = \frac{1}{\sqrt{x}} \cdot \frac{\frac{1}{2\sqrt{x}} \cdot (1+\operatorname{sen}^2 x) - \sqrt{x} \cdot 2\operatorname{sen} x \cdot \cos x}{(1+\operatorname{sen}^2 x)^2} =$$

$$= \frac{1+\operatorname{sen}^2 x - 2x \cdot \operatorname{sen}(2x)}{2x(1+\operatorname{sen}^2 x)}$$

$$q_3) f(x) = \operatorname{arctg}\left(\sqrt{\frac{x}{x+2}}\right)$$

$$f'(x) = \frac{1}{1+\left(\sqrt{\frac{x}{x+2}}\right)^2} \cdot \frac{1}{2\sqrt{\frac{x}{x+2}}} \cdot \frac{1 \cdot (x+2) - x}{(x+2)^2} =$$

$$= \frac{1}{\frac{x+2+x}{x+2}} \cdot \frac{1}{\cancel{2} \sqrt{\frac{x}{x+2}}} \cdot \frac{\cancel{2}}{(x+2)^2} = \frac{1}{(2x+2) \cdot (x+2) \cdot \sqrt{\frac{x}{x+2}}} =$$

$$= \frac{1}{(2x+2) \sqrt{\frac{(x+2)^2 \cdot x}{(x+2)}}} = \frac{1}{(2x+2) \cdot \sqrt{x^2+2x}}$$

$$r_1) f(x) = \frac{\cos x + \cos(x+1)}{\operatorname{sen} x - \operatorname{sen}(x+1)}$$

$$f'(x) = \frac{(-\operatorname{sen} x - \operatorname{sen}(x+1)) \cdot (\operatorname{sen} x - \operatorname{sen}(x+1)) - (\cos x + \cos(x+1)) \cdot (\cos x - \cos(x+1))}{(\operatorname{sen} x - \operatorname{sen}(x+1))^2}$$

$$= \frac{-\operatorname{sen}^2(x) + \cancel{\operatorname{sen}(x) \cdot \operatorname{sen}(x+1)} - \cancel{\operatorname{sen}(x+1) \operatorname{sen} x} + \operatorname{sen}^2(x+1) - (\cos^2(x) - \cos^2(x+1))}{(\operatorname{sen} x - \operatorname{sen}(x+1))^2} =$$

$$= \frac{-(\operatorname{sen}^2 x + \cos^2 x) + (\operatorname{sen}^2(x+1) + \cos^2(x+1))}{(\operatorname{sen} x - \operatorname{sen}(x+1))^2} = 0$$

$$r_2) f(x) = \ln \sqrt{\frac{1 - \operatorname{sen} x}{1 + \operatorname{sen} x}}$$

$$f'(x) = \frac{1}{\sqrt{\frac{1 - \operatorname{sen} x}{1 + \operatorname{sen} x}}} \cdot \frac{1}{\cancel{2} \sqrt{\frac{1 - \operatorname{sen} x}{1 + \operatorname{sen} x}}} \cdot \frac{-\cos x \cdot (1 + \operatorname{sen} x) - (1 - \operatorname{sen} x) \cdot \cos x}{(1 + \operatorname{sen} x)^2} =$$

$$= \frac{1}{\cancel{2} \cdot \left(\frac{1 - \operatorname{sen} x}{1 + \operatorname{sen} x} \right)} \cdot \frac{\cos x (-1 - \cancel{\operatorname{sen} x} - 1 + \cancel{\operatorname{sen} x})}{(1 + \operatorname{sen} x)^2} = \frac{-2 \cdot \cos x}{\cancel{2} (1 - \operatorname{sen} x)(1 + \operatorname{sen} x)} = \frac{-1}{\cos x}$$

$\underbrace{1 - \operatorname{sen}^2 x}_{\cos^2 x}$

$$r_3) f(x) = \ln(e^{\operatorname{sen} x}) = \operatorname{sen} x \rightarrow f'(x) = \cos x$$

$$s_1) f(x) = \operatorname{arctg}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$$

$$f'(x) = \frac{1}{1 + \left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)^2} \cdot \frac{1}{2\sqrt{\frac{1-\cos x}{1+\cos x}}} \cdot \frac{\operatorname{sen} x (1+\cos x) + (1-\cos x) \cdot \operatorname{sen} x}{(1+\cos x)^2} =$$

$$= \frac{1}{\frac{1+\cos x + 1-\cos x}{1+\cos x}} \cdot \frac{1}{2\sqrt{\frac{1-\cos x}{1+\cos x}}} \cdot \frac{2\operatorname{sen} x}{(1+\cos x)^2} =$$

$$= \frac{\operatorname{sen} x}{2(1+\cos x)\sqrt{\frac{1-\cos x}{1+\cos x}}} = \frac{\operatorname{sen} x}{2 \cdot \sqrt{(1+\cos x)^2 \cdot \frac{1-\cos x}{1+\cos x}}} = \frac{\operatorname{sen} x}{2\sqrt{1-\cos^2 x}} =$$

$$= \frac{\operatorname{sen} x}{2 \cdot \operatorname{sen} x} = \frac{1}{2}$$

$$s_2) f(x) = \ln\left(\frac{x}{x+1}\right)^x = x \cdot \ln\left(\frac{x}{x+1}\right)$$

$$f'(x) = 1 \cdot \ln\left(\frac{x}{x+1}\right) + x \cdot \frac{1}{\frac{x}{x+1}} \cdot \frac{1 \cdot (x+1) - x}{(x+1)^2} = \ln\left(\frac{x}{x+1}\right) + \frac{1}{x+1}$$

$$s_3) f(x) = \sqrt{e^x + e^{-x}}$$

$$f'(x) = \frac{1}{2\sqrt{e^x + e^{-x}}} \cdot (e^x + e^{-x} \cdot (-1)) = \frac{e^x - e^{-x}}{2\sqrt{e^x + e^{-x}}}$$

$$t_1) f(x) = \ln \sqrt{\frac{x^3 \cdot \cos x}{\operatorname{sen} x}}$$

$$f'(x) = \frac{1}{\sqrt{\frac{x^3 \cdot \cos x}{\operatorname{sen} x}}} \cdot \frac{1}{2 \sqrt{\frac{x^3 \cdot \cos x}{\operatorname{sen} x}}} \cdot \frac{(3x^2 \cdot \cos x + x^3 \cdot (-\operatorname{sen} x)) \operatorname{sen} x - x^3 \cdot \cos^2 x}{(\operatorname{sen} x)^2} =$$

$$= \frac{3x^2 \cos x \operatorname{sen} x - x^3 \cdot (\overbrace{\operatorname{sen}^2 x + \cos^2 x}^1)}{2 \cdot \frac{x^3 \cdot \cos x}{\cancel{\operatorname{sen} x}} \cdot (\operatorname{sen} x)^2} = \frac{\cancel{x^2} (3 \cos x \operatorname{sen} x - x)}{2 \cancel{x^3} \cos x \operatorname{sen} x} =$$

$$= \frac{3 \cos x \operatorname{sen} x - x}{2x \cos x \operatorname{sen} x} = \frac{3}{2x} - \frac{1}{\operatorname{sen}(2x)}$$

$$t_2) f(x) = \frac{e^{x+3} - 1}{x^2 - 9}$$

$$f'(x) = \frac{e^{x+3}(x^2 - 9) - (e^{x+3} - 1) \cdot 2x}{(x^2 - 9)^2} = \frac{e^{x+3}(x^2 - 2x - 9) + 2x}{(x^2 - 9)^2}$$

$$t_3) f(x) = \operatorname{arcsen}(\sqrt{1 - \operatorname{sen}^2 x})$$

$$f'(x) = \frac{1}{\sqrt{1 - (\sqrt{1 - \operatorname{sen}^2 x})^2}} \cdot \frac{1}{2 \sqrt{1 - \operatorname{sen}^2 x}} \cdot (-2 \operatorname{sen} x \cdot \cos x) =$$

$$= \frac{1}{\sqrt{1 - \underbrace{\cos^2 x}_{\operatorname{sen}^2 x}}} \cdot \frac{1}{2 \sqrt{\cos^2 x}} \cdot -2 \operatorname{sen} x \cos x = \frac{-2 \operatorname{sen} x \cos x}{2 \operatorname{sen} x \cos x} = -1$$

$$u_1) f(x) = \operatorname{arcsen} \left(\frac{1}{1+x^2} \right)$$

$$f'(x) = \frac{1}{\sqrt{1 - \left(\frac{1}{1+x^2} \right)^2}} \cdot \frac{-1 \cdot 2x}{(1+x^2)^2} = \frac{1}{\sqrt{\frac{(1+x^2)^2 - 1}{(1+x^2)^2}}} \cdot \frac{-2x}{(1+x^2)^2} =$$

$$= \frac{-2x}{(1+x^2)^2 \cdot \frac{1}{(1+x^2)} \cdot \sqrt{1+2x^2+x^4-1}} = \frac{-2x}{(1+x^2) \sqrt{x^2(2+x^2)}} =$$

$$= \frac{-2x}{(1+x^2) \cdot x \cdot \sqrt{2+x^2}} = \frac{-2}{(1+x^2) \sqrt{2+x^2}}$$

$$u_2) f(x) = \frac{\ln \sqrt{x+1}}{x} = \frac{\ln (x+1)^{1/2}}{x} = \frac{\ln (x+1)}{2x}$$

$$f'(x) = \frac{\frac{1}{x+1} \cdot 2x - 2 \ln(x+1)}{4x^2} = \frac{2 \left(\frac{x}{x+1} - \ln(x+1) \right)}{4x^2} = \frac{x - (x+1) \ln(x+1)}{2x^2(x+1)}$$

$$u_3) f(x) = e^{\ln(\operatorname{sen}^2 x)} = \operatorname{sen}^2 x \rightarrow f'(x) = 2 \operatorname{sen} x \cos x = \operatorname{sen}(2x)$$

$$v_1) f(x) = \operatorname{tg}(1 + \operatorname{sen} x) \rightarrow f'(x) = (1 + \operatorname{tg}^2(1 + \operatorname{sen} x)) \cdot \cos x$$

$$v_2) f(x) = \operatorname{tg}^3(e^{x^2}) \rightarrow f'(x) = 3 \cdot \operatorname{tg}^2(e^{x^2}) \cdot (1 + \operatorname{tg}^2(e^{x^2})) \cdot e^{x^2} \cdot 2x =$$

$$= 6x \cdot e^{x^2} \cdot \operatorname{tg}^2(e^{x^2}) (1 + \operatorname{tg}^2(e^{x^2}))$$

$$v_3) f(x) = 27 + (x-1)^4 \rightarrow f'(x) = 4(x-1)^3$$

$$w_1) f(x) = \operatorname{sen}^2 x + \cos^2 x = 1 \longrightarrow f'(x) = 0$$

$$w_2) f(x) = \frac{e^x + 1}{e^x - 1}$$

$$f'(x) = \frac{e^x(e^x - 1) - (e^x + 1) \cdot e^x}{(e^x - 1)^2} = \frac{e^x(\cancel{e^x - 1} - \cancel{e^x - 1})}{(e^x - 1)^2} = \frac{-2e^x}{(e^x - 1)^2}$$

$$w_3) f(x) = \frac{2 + \cos x}{2 - \cos x}$$

$$f'(x) = \frac{-\operatorname{sen} x (2 - \cos x) - (2 + \cos x) \cdot \operatorname{sen} x}{(2 - \cos x)^2} = \frac{\operatorname{sen} x (\cancel{-2 + \cos x} - \cancel{2 - \cos x})}{(2 - \cos x)^2} =$$

$$= \frac{-4 \operatorname{sen} x}{(2 - \cos x)^2}$$

$$x_1) f(x) = \operatorname{arcsen} \left(\frac{x}{\sqrt{1+x^2}} \right)$$

$$f'(x) = \frac{1}{\sqrt{1 - \left(\frac{x}{\sqrt{1+x^2}} \right)^2}} \cdot \frac{1 \cdot \sqrt{1+x^2} - x \cdot \frac{1}{\sqrt{1+x^2}} \cdot 2x}{1+x^2} =$$

$$= \frac{1}{\sqrt{\frac{1+x^2-x^2}{1+x^2}}} \cdot \frac{1+x^2-x^2}{(1+x^2) \cdot \sqrt{1+x^2}} = \frac{1}{1} \cdot \frac{1}{(1+x^2) \cdot \sqrt{1+x^2}} = \frac{1}{1+x^2}$$

$$x_2) f(x) = \arccos\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$f'(x) = \frac{-1}{\sqrt{1 - \left(\frac{1}{\sqrt{1+x^2}}\right)^2}} \cdot \frac{-1 \cdot \frac{1}{\sqrt{1+x^2}} \cdot 2x}{1+x^2} = \frac{-1}{\sqrt{\frac{1+x^2-1}{1+x^2}}} \cdot \frac{-x}{(1+x^2) \cdot \sqrt{1+x^2}} =$$

$$= \frac{x}{\frac{x}{\sqrt{1+x^2}} \cdot (1+x^2) \cdot \sqrt{1+x^2}} = \frac{1}{1+x^2}$$

$$x_3) f(x) = e^{5x} \longrightarrow f'(x) = e^{5x} \cdot 5 = 5 \cdot e^{5x}$$

$$y_1) f(x) = \sqrt{\sec(x)}$$

$$f'(x) = \frac{1}{2\sqrt{\sec(x)}} \cdot \sec(x) \cdot \operatorname{tg}(x) = \frac{\sec(x) \cdot \operatorname{tg}(x)}{2\sqrt{\sec(x)}}$$

$$y_2) f(x) = \operatorname{arccotg}\left(\frac{1}{x}\right)$$

$$f'(x) = \frac{-1}{1 + \left(\frac{1}{x}\right)^2} \cdot -\frac{1}{x^2} = \frac{1}{\frac{x^2+1}{x^2}} \cdot \frac{1}{x^2} = \frac{1}{x^2+1}$$

$$y_3) f(x) = \operatorname{arcsec}(x)$$

$$f'(x) = \frac{1}{x\sqrt{x^2-1}}$$

$$z_1) f(x) = \frac{\cos(2x) + \operatorname{sen}(2x)}{\cos(2x) - \operatorname{sen}(2x)}$$

$$f'(x) = \frac{(-\operatorname{sen}(2x) \cdot 2 + \cos(2x) \cdot 2) \cdot (\cos(2x) - \operatorname{sen}(2x)) - (\cos(2x) + \operatorname{sen}(2x)) \cdot (-\operatorname{sen}(2x) \cdot 2 - \cos(2x))}{(\cos(2x) - \operatorname{sen}(2x))^2}$$

$$= \frac{4\operatorname{sen}^2(2x) + 4\cos^2(2x)}{\cos^2(2x) - 2\operatorname{sen}(2x) \cdot \cos(2x) + \operatorname{sen}^2(4x)} = \frac{4}{1 - \operatorname{sen}(4x)}$$

$$z_2) f(x) = \ln \sqrt{\frac{x^2 \operatorname{sen} x}{\cos x}} = \ln \sqrt{x^2 \cdot \operatorname{tg} x} = \ln (x^2 \cdot \operatorname{tg} x)^{1/2} = \frac{1}{2} \ln (x^2 \cdot \operatorname{tg} x)$$

$$f'(x) = \frac{1}{2} \cdot \frac{1}{x^2 \cdot \operatorname{tg} x} \cdot (2x \operatorname{tg} x + x^2 (1 + \operatorname{tg}^2 x)) = \frac{x}{2x^2 \cdot \operatorname{tg} x} [2 \operatorname{tg} x + x(1 + \operatorname{tg}^2 x)]$$

$$= \frac{1}{2x \operatorname{tg} x} [2 \operatorname{tg} x + x(1 + \operatorname{tg}^2 x)] = \frac{1}{x} + \frac{1 + \operatorname{tg}^2 x}{2 \operatorname{tg} x} =$$

$$= \frac{1}{x} + \frac{\frac{1}{\cos^2 x}}{2 \frac{\operatorname{sen} x}{\cos x}} = \frac{1}{x} + \frac{1}{2 \operatorname{sen} x \cos x} = \frac{1}{x} + \frac{1}{\operatorname{sen}(2x)}$$

$$z_3) f(x) = \ln(1+x)^x = x \cdot \ln(1+x)$$

$$f'(x) = 1 \cdot \ln(1+x) + x \cdot \frac{1}{1+x} = \ln(1+x) + \frac{x}{1+x}$$

⑬ Calcular la primera y la segunda derivada de las funciones:

a) $f(x) = 5x^3 + 2x^2 - 7$ b) $f(x) = e^{x^2}$ c) $f(x) = \ln(x^2 - 1)$

a) $f(x) = 5x^3 + 2x^2 - 7$

$$f'(x) = 15x^2 + 4x$$

$$f''(x) = 30x + 4$$

b) $f(x) = e^{x^2}$

$$f'(x) = e^{x^2} \cdot 2x$$

$$f''(x) = e^{x^2} \cdot 2x \cdot 2x + e^{x^2} \cdot 2 = e^{x^2} (4x^2 + 2)$$

c) $f(x) = \ln(x^2 - 1)$

$$f'(x) = \frac{1}{x^2 - 1} \cdot 2x = \frac{2x}{x^2 - 1}$$

$$f''(x) = \frac{2(x^2 - 1) - 2x \cdot 2x}{(x^2 - 1)^2} = \frac{2x^2 - 2 - 4x^2}{(x^2 - 1)^2} = \frac{-2x^2 - 2}{(x^2 - 1)^2}$$

⑭ Dada la función $f(x) = 2(e^x + 2) - x - \frac{x^2}{2}$, comprueba que se cumple la ecuación $f''(x) = f'(x) + x$

$$f(x) = 2(e^x + 2) - x - \frac{x^2}{2}$$

$$f'(x) = 2e^x - 1 - x$$

$$f''(x) = 2e^x - 1$$

$$f''(x) = f'(x) + x$$

$$2e^x - 1 = 2e^x - 1 - x + x = 2e^x - 1$$

como queríamos demostrar

15) Resuelve la ecuación $f'(x) = 0$, siendo:

a) $f(x) = \frac{x^2+1}{x^2-1}$

b) $f(x) = \frac{x^3}{x^2-1}$

c) $f(x) = \frac{x^2+1}{x}$

a) $f(x) = \frac{x^2+1}{x^2-1}$

$$f'(x) = \frac{2x(x^2-1) - (x^2+1) \cdot 2x}{(x^2-1)^2} = \frac{2x(\cancel{x^2-1} - \cancel{x^2-1})}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$f'(x) = 0 \rightarrow \frac{-4x}{(x^2-1)^2} = 0 \Rightarrow -4x = 0 \Rightarrow \boxed{x=0}$$

b) $f(x) = \frac{x^3}{x^2-1}$

$$f'(x) = \frac{3x^2(x^2-1) - x^3 \cdot 2x}{(x^2-1)^2} = \frac{3x^4 - 3x^2 - 2x^4}{(x^2-1)^2} = \frac{x^4 - 3x^2}{(x^2-1)^2}$$

$$f'(x) = 0 \rightarrow \frac{x^4 - 3x^2}{(x^2-1)^2} = 0 \Rightarrow x^2(x^2-3) = 0$$

$\rightarrow \boxed{x=0}$

$\rightarrow x^2-3=0 \rightarrow \begin{cases} \boxed{x=\sqrt{3}} \\ \boxed{x=-\sqrt{3}} \end{cases}$

c) $f(x) = \frac{x^2+1}{x}$

$$f'(x) = \frac{2x \cdot x - (x^2+1)}{x^2} = \frac{x^2-1}{x^2}$$

$$f'(x) = 0 \rightarrow \frac{x^2-1}{x^2} = 0 \rightarrow x^2-1 = 0 \Rightarrow x^2 = 1$$

$\rightarrow \boxed{x=1}$

$\rightarrow \boxed{x=-1}$

16) Calcula la derivada n -ésima de cada una de las siguientes funciones:

a) $f(x) = e^{-x}$

b) $f(x) = 2^x$

c) $f(x) = x \cdot e^x$

a) $f(x) = e^{-x}$

$$f'(x) = e^{-x} \cdot (-1) = -e^{-x}$$

$$f''(x) = -e^{-x} \cdot (-1) = e^{-x}$$

$$f'''(x) = e^{-x} \cdot (-1) = -e^{-x}$$

$$f^{(4)}(x) = -e^{-x} \cdot (-1) = e^{-x}$$

$$f^{(n)}(x) = (-1)^n \cdot e^{-x}$$

b) $f(x) = 2^x$

$$f'(x) = 2^x \cdot \ln 2$$

$$f''(x) = 2^x \ln 2 \cdot \ln 2 = 2^x \cdot (\ln 2)^2$$

$$f'''(x) = 2^x \cdot (\ln 2)^2 \cdot \ln 2 = 2^x \cdot (\ln 2)^3$$

$$f^{(n)}(x) = (\ln 2)^n \cdot 2^x$$

c) $f(x) = x \cdot e^x$

$$f'(x) = 1 \cdot e^x + x \cdot e^x = e^x(x+1)$$

$$f''(x) = e^x(x+1) + e^x \cdot 1 = e^x(x+2)$$

$$f'''(x) = e^x(x+2) + e^x \cdot 1 = e^x(x+3)$$

$$f^{(n)}(x) = e^x(x+n)$$

17) Encuentra la derivada de las siguientes funciones expresadas en forma implícita:

a) $x^2 + y^2 = 4$ b) $x^2 + 2y^2 = 9$ c) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

d) $x^2 + y^2 + 2x - 4y - 25 = 0$ e) $\frac{x^2}{25} - \frac{y^2}{16} = 1$ f) $y + \cos(xy) = 0$

g) $y = \text{tg}(x \cdot y)$ h) $xy + \text{sen } y = 5$ i) $y^6 + y - 2x^2 = 0$

j) $y = xy + \text{sen}(xy)$ k) $y = e^y + x$ l) $y = x^3 - 2xy + y^2$

m) $x = \text{arctg}(x - y)$ n) $x^3 + y^3 = xy$ o) $\text{sen } y = x \cdot y$

p) $y^2 = \frac{x - y}{x + y}$ q) $\frac{y}{x} = 1 + \ln y$ r) $x^2 + y^2 - 2xy = 0$

s) $x = \cos(x \cdot y)$ t) $x^3 + 3y^2 - 2xy = 0$ u) $e^{2y} - \ln x^3 = 3$

v) $x^3 + y^3 + xy = 0$ w) $y^3 - 2xy + 7 = 3x + 1$ x) $(2y^2 + 3)^3 = 5x^3 - 3x$

y) $\sqrt{x} + \sqrt{y} = 5$ z) $y^6 + y^4 - 2x^2 = 0$

a) $x^2 + y^2 = 4$

$$2x + 2y \cdot y' = 0 \Rightarrow \cancel{2y} \cdot y' = \cancel{-2x} \Rightarrow \boxed{y' = -\frac{x}{y}}$$

b) $x^2 + 2y^2 = 9$

$$2x + 4y \cdot y' = 0 \Rightarrow \cancel{4}^2 y \cdot y' = \cancel{-2x} \Rightarrow \boxed{y' = -\frac{x}{2y}}$$

$$c) \frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\frac{2x}{16} + \frac{2y \cdot y'}{9} = 0 \Rightarrow \frac{2y y'}{9} = -\frac{2x}{16} \Rightarrow y' = -\frac{9x}{16y}$$

$$d) x^2 + y^2 + 2x - 4y - 25 = 0$$

$$2x + 2y \cdot y' + 2 - 4y' = 0 \Rightarrow 2y y' - 4y' = -2x - 2$$

$$2y'(y-2) = -2(x+1) \Rightarrow y' = -\frac{x+1}{y-2}$$

$$e) \frac{x^2}{25} - \frac{y^2}{16} = 1$$

$$\frac{2x}{25} - \frac{2y y'}{16} = 0 \Rightarrow \frac{2y y'}{16} = \frac{2x}{25} \Rightarrow y' = \frac{16x}{25y}$$

$$f) y + \cos(xy) = 0$$

$$y' - \sin(xy) \cdot [y + x \cdot y'] = 0 \Rightarrow y' - y \sin(xy) - x \cdot y' \sin(xy) = 0$$

$$\Rightarrow y' - x y' \sin(xy) = y \sin(xy) \Rightarrow y'(1 - x \sin(xy)) = y \sin(xy)$$

$$\Rightarrow y' = \frac{y \sin(xy)}{1 - x \sin(xy)}$$

$$g) y = \lg(xy)$$

$$y' = \frac{1}{\cos^2(xy)} (y + x \cdot y') \Rightarrow y' \cos^2(xy) = y + x y' \Rightarrow$$

$$\Rightarrow y' \cdot \cos^2(xy) - x \cdot y' = y \Rightarrow y' = \frac{y}{\cos^2(xy) - x}$$

$$h) \quad xy + \operatorname{sen}(y) = 5$$

$$y + x \cdot y' + \cos(y) \cdot y' = 0 \Rightarrow xy' + y' \cos(y) = -y \Rightarrow$$

$$\Rightarrow y'(x + \cos(y)) = -y \Rightarrow \boxed{y' = \frac{-y}{x + \cos(y)}}$$

$$i) \quad y^6 + y - 2x^2 = 0$$

$$6y^5 \cdot y' + y' - 4x = 0 \Rightarrow 6y^5 \cdot y' + y' = 4x \Rightarrow$$

$$\Rightarrow y' \cdot (6y^5 + 1) = 4x \Rightarrow \boxed{y' = \frac{4x}{6y^5 + 1}}$$

$$j) \quad y = xy + \operatorname{sen}(xy)$$

$$y' = y + x \cdot y' + \cos(xy) \cdot (y + xy') \Rightarrow$$

$$\Rightarrow y' - x \cdot y' - xy' \cos(xy) = y + y \cos(xy) \Rightarrow$$

$$\Rightarrow y'(1 - x - x \cos(xy)) = y + y \cos(xy) \Rightarrow \boxed{y' = \frac{y + y \cos(xy)}{1 - x - x \cos(xy)}}$$

$$k) \quad y = e^y + x$$

$$y' = e^y \cdot y' + 1 \Rightarrow y' - e^y \cdot y' = 1$$

$$\Rightarrow y'(1 - e^y) = 1 \Rightarrow \boxed{y' = \frac{1}{1 - e^y}}$$

$$e) y = x^3 - 2xy + y^2$$

$$y' = 3x^2 - (2y + 2x \cdot y') + 2y \cdot y' \Rightarrow y' + 2xy' - 2yy' = 3x^2 - 2y$$

$$\Rightarrow y'(1 + 2x - 2y) = 3x^2 - 2y \Rightarrow y' = \frac{3x^2 - 2y}{1 + 2x - 2y}$$

$$m) x = \arctg(x-y)$$

$$1 = \frac{1}{1 + (x-y)^2} \cdot (1 - y') \Rightarrow \cancel{1 + (x-y)^2} = \cancel{1} - y' \Rightarrow y' = -(x-y)^2$$

$$n) x^3 + y^3 = xy$$

$$3x^2 + 3y^2 \cdot y' = y + x \cdot y' \Rightarrow 3y^2 \cdot y' - x \cdot y' = y - 3x^2 \Rightarrow$$

$$\Rightarrow y'(3y^2 - x) = y - 3x^2 \Rightarrow y' = \frac{y - 3x^2}{3y^2 - x}$$

$$o) \operatorname{sen} y = x \cdot y$$

$$\cos(y) \cdot y' = y + x \cdot y' \Rightarrow \cos(y) \cdot y' - x \cdot y' = y \Rightarrow$$

$$\Rightarrow y'(\cos y - x) = y \Rightarrow y' = \frac{y}{\cos y - x}$$

$$p) y^2 = \frac{x-y}{x+y}$$

$$2y \cdot y' = \frac{(1-y')(x+y) - (x-y)(1+y')}{(x+y)^2} \Rightarrow$$

$$\Rightarrow 2y(x+y)^2 \cdot y' = \cancel{x+y} - y'(x+y) - \cancel{x+y} - y'(x-y) \Rightarrow$$

$$\Rightarrow 2y(x+y)^2 \cdot y' + y'(x+y) + y'(x-y) = 2y \Rightarrow$$

$$\Rightarrow y' [2y(x+y)^2 + \cancel{x+y} + \cancel{x-y}] = 2y \Rightarrow$$

$$\Rightarrow \cancel{2y} y' [y(x+y)^2 + x] = \cancel{2y} \Rightarrow$$

$$y' = \frac{y}{y(x+y)^2 + x}$$

$$q) \frac{y}{x} = 1 + \ln y$$

$$\frac{y'x - y}{x^2} = \frac{1}{y} \cdot y' \Rightarrow y'xy - y^2 = x^2 \cdot y' \Rightarrow$$

$$\Rightarrow y'xy - x^2 \cdot y' = y^2 \Rightarrow y'(xy - x^2) = y^2 \Rightarrow y' = \frac{y^2}{xy - x^2}$$

$$r) x^2 + y^2 - 2xy = 0$$

$$2x + 2y \cdot y' - 2y - 2xy' = 0 \Rightarrow 2yy' - 2x \cdot y' = 2y - 2x$$

$$\Rightarrow \cancel{2y'}(\cancel{y-x}) = \cancel{2(y-x)} \Rightarrow y' = 1$$

$$s) x = \cos(xy)$$

$$1 = -\sin(xy) \cdot (y + x \cdot y') \Rightarrow 1 = -y \sin(xy) - x y' \sin(xy)$$

$$1 + y \sin(xy) = -x \cdot y' \cdot \sin(xy) \Rightarrow y' = \frac{1 + y \sin(xy)}{-x \sin(xy)}$$

$$t) \quad x^3 + 3y^2 - 2ay = 0$$

$$3x^2 + 6y \cdot y' - 2ay' = 0 \Rightarrow 3x^2 = 2ay' - 6y \cdot y' \Rightarrow$$

$$\Rightarrow 3x^2 = y'(2a - 6y) \Rightarrow y' = \frac{3x^2}{2a - 6y}$$

$$u) \quad e^{2y} - \ln x^3 = 3$$

$$e^{2y} \cdot 2y' - \frac{1}{x^3} \cdot 3x^2 = 0 \Rightarrow e^{2y} \cdot 2y' = \frac{3}{x} \Rightarrow y' = \frac{3}{2x \cdot e^{2y}}$$

$$v) \quad x^3 + y^3 + xy = 0$$

$$3x^2 + 3y^2 \cdot y' + y + x \cdot y' = 0 \Rightarrow 3y^2 \cdot y' + x \cdot y' = -3x^2 - y$$

$$\Rightarrow y'(3y^2 + x) = -3x^2 - y \Rightarrow y' = \frac{-3x^2 - y}{3y^2 + x}$$

$$w) \quad y^3 - 2xy + 7 = 3x + 1$$

$$3y^2 \cdot y' - 2y - 2xy' = 3 \Rightarrow 3y^2 \cdot y' - 2xy' = 3 + 2y$$

$$\Rightarrow y'(3y^2 - 2x) = 3 + 2y \Rightarrow y' = \frac{3 + 2y}{3y^2 - 2x}$$

$$x) \quad (2y^2 + 3)^3 = 5x^3 - 3x$$

$$3(2y^2 + 3)^2 \cdot 4y \cdot y' = 15x^2 - 3 \Rightarrow y' = \frac{15x^2 - 3}{12y(2y^2 + 3)^2}$$

$$y) \sqrt{x} + \sqrt{y} = 5$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot y' = 0 \Rightarrow \frac{1}{2\sqrt{y}} \cdot y' = -\frac{1}{2\sqrt{x}} \Rightarrow y' = -\sqrt{\frac{y}{x}}$$

$$z) y^6 + y^4 - 2x^2 = 0$$

$$6y^5 \cdot y' + 4y^3 \cdot y' - 4x = 0 \Rightarrow y' (6y^5 + 4y^3) = 4x$$

$$\Rightarrow y' = \frac{4x}{6y^5 + 4y^3}$$

18) Encuentra las derivadas de las siguientes funciones:

$$a) y = x^x$$

$$b) y = x^{\ln x}$$

$$c) y = (\ln x)^{2x}$$

$$d) y = x^{\cos x}$$

$$e) y = (2\sqrt{x})^{\ln x}$$

$$f) y = (\sec x)^{\ln x}$$

$$g) y = (\ln x)^{\lg x}$$

$$h) y = x^{\sec x}$$

$$i) y = (\lg x)^{\arctg(x)}$$

$$j) y = (\sqrt{x})^x$$

$$k) y = x^{\sqrt{x}}$$

$$l) y = (x - \sec x)^{\sqrt{x}}$$

$$m) y = (\sec x)^{2x}$$

$$n) y = \sqrt[x]{\cos x}$$

$$o) y = (\operatorname{arcsec} x)^{\sec x}$$

$$p) y = (1+x^2)^x$$

$$q) y = (\sec x)^{\cos x}$$

$$r) y = \sqrt[x]{x^3}$$

$$s) y = \left(1 + \frac{1}{x}\right)^x$$

$$t) y = (\lg x)^x$$

$$a) y = x^x \Rightarrow \ln y = \ln x^x \Rightarrow \ln y = x \cdot \ln x$$

$$\frac{1}{y} \cdot y' = \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}} \Rightarrow y' = y(\ln x + 1) \Rightarrow$$

$$\Rightarrow y' = x^x (\ln x + 1)$$

$$b) y = x^{\ln x} \Rightarrow \ln y = \ln x^{\ln x} \Rightarrow \ln y = \ln x \cdot \ln x = (\ln x)^2$$

$$\frac{1}{y} \cdot y' = 2(\ln x) \cdot \frac{1}{x} \Rightarrow y' = y \cdot \frac{2 \ln x}{x} \Rightarrow$$

$$\Rightarrow y' = x^{\ln x} \cdot \frac{2 \ln x}{x}$$

$$c) y = (\ln x)^{2x} \Rightarrow \ln y = \ln ((\ln x)^{2x}) \Rightarrow \ln y = 2x \cdot \ln(\ln x)$$

$$\frac{1}{y} \cdot y' = 2 \ln(\ln x) + \cancel{2x} \cdot \frac{1}{\ln x} \cdot \frac{1}{\cancel{x}} \Rightarrow$$

$$\Rightarrow y' = y \cdot \left[2 \ln(\ln x) + \frac{2}{\ln x} \right] = (\ln x)^{2x} \cdot \left[2 \ln(\ln x) + \frac{2}{\ln x} \right]$$

$$d) y = x^{\cos x} \Rightarrow \ln y = \ln x^{\cos x} \Rightarrow \ln y = \cos x \cdot \ln x$$

$$\frac{1}{y} \cdot y' = -\sin x \cdot \ln x + \frac{\cos x}{x} \Rightarrow y' = x^{\cos x} \cdot \left[-\sin x \cdot \ln x + \frac{\cos x}{x} \right]$$

$$e) y = (2\sqrt{x})^{\ln x} \Rightarrow \ln y = \ln (2\sqrt{x})^{\ln x} \Rightarrow \ln y = \ln(x) \cdot \ln(2\sqrt{x})$$

$$\frac{1}{y} \cdot y' = \frac{1}{x} \cdot \ln(2\sqrt{x}) + \ln(x) \cdot \frac{1}{2\sqrt{x}} \cdot \frac{2}{\cancel{2\sqrt{x}}} \Rightarrow$$

$$\Rightarrow y' = (2\sqrt{x})^{\ln x} \cdot \left[\frac{\ln(2\sqrt{x})}{x} + \frac{\ln(x)}{2x} \right]$$

$$f) y = (\operatorname{sen} x)^{\ln x} \Rightarrow \ln y = \ln (\operatorname{sen} x)^{\ln x} \Rightarrow \ln y = \ln x \cdot \ln (\operatorname{sen} x)$$

$$\frac{1}{y} \cdot y' = \frac{1}{x} \cdot \ln (\operatorname{sen} x) + \ln x \cdot \frac{1}{\operatorname{sen} x} \cdot \cos x \Rightarrow$$

$$y' = (\operatorname{sen} x)^{\ln x} \cdot \left[\frac{\ln (\operatorname{sen} x)}{x} + \frac{\ln x}{\operatorname{tg} x} \right]$$

$$g) y = (\ln x)^{\operatorname{tg} x} \Rightarrow \ln y = \ln ((\ln x)^{\operatorname{tg} x}) \Rightarrow \ln y = \operatorname{tg} x \cdot \ln (\ln x)$$

$$\frac{1}{y} \cdot y' = \frac{1}{\cos^2 x} \cdot \ln (\ln x) + \operatorname{tg} x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} \Rightarrow$$

$$y' = (\ln x)^{\operatorname{tg} x} \cdot \left[\frac{\ln (\ln x)}{\cos^2 x} + \frac{\operatorname{tg} x}{x \cdot \ln x} \right]$$

$$h) y = x^{\operatorname{sen} x} \Rightarrow \ln y = \ln x^{\operatorname{sen} x} \Rightarrow \ln y = \operatorname{sen} x \cdot \ln x$$

$$\frac{1}{y} \cdot y' = \cos x \cdot \ln x + \operatorname{sen} x \cdot \frac{1}{x} \Rightarrow y' = x^{\operatorname{sen} x} \cdot \left[\cos x \cdot \ln x + \frac{\operatorname{sen} x}{x} \right]$$

$$i) y = (\operatorname{tg} x)^{\operatorname{arctg} x} \Rightarrow \ln y = \ln (\operatorname{tg} x)^{\operatorname{arctg} x} \Rightarrow \ln y = \operatorname{arctg} x \cdot \ln (\operatorname{tg} x)$$

$$\frac{1}{y} \cdot y' = \frac{1}{1+x^2} \cdot \ln (\operatorname{tg} x) + \operatorname{arctg} x \cdot \frac{1}{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x} \Rightarrow$$

$$\Rightarrow y' = (\operatorname{tg} x)^{\operatorname{arctg} x} \cdot \left[\frac{\ln (\operatorname{tg} x)}{1+x^2} + \frac{\operatorname{arctg} x}{\operatorname{sen} x \cdot \cos x} \right]$$

$$j) y = (\sqrt{x})^x \Rightarrow \ln y = \ln(\sqrt{x})^x \Rightarrow \ln y = x \cdot \ln(\sqrt{x})$$

$$\frac{1}{y} \cdot y' = \ln \sqrt{x} + x \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \ln \sqrt{x} + \frac{1}{2} \Rightarrow$$

$$\Rightarrow y' = (\sqrt{x})^x \cdot \left(\ln \sqrt{x} + \frac{1}{2} \right)$$

$$k) y = (x)^{\sqrt{x}} \Rightarrow \ln y = \ln(x)^{\sqrt{x}} \Rightarrow \ln y = \sqrt{x} \cdot \ln x$$

$$\frac{1}{y} \cdot y' = \frac{1}{2\sqrt{x}} \cdot \ln x + \frac{\sqrt{x}}{x} \Rightarrow y' = x^{\sqrt{x}} \cdot \left[\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x} \right]$$

$$l) y = (x - \operatorname{sen} x)^{\sqrt{x}} \Rightarrow \ln y = \ln(x - \operatorname{sen} x)^{\sqrt{x}} \Rightarrow \ln y = \sqrt{x} \cdot \ln(x - \operatorname{sen} x)$$

$$\frac{1}{y} \cdot y' = \frac{1}{2\sqrt{x}} \ln(x - \operatorname{sen} x) + \sqrt{x} \cdot \frac{1}{x - \operatorname{sen} x} \cdot (1 - \cos x) \Rightarrow$$

$$\Rightarrow y' = (x - \operatorname{sen} x)^{\sqrt{x}} \cdot \left[\frac{\ln(x - \operatorname{sen} x)}{2\sqrt{x}} + \frac{\sqrt{x}(1 - \cos x)}{x - \operatorname{sen} x} \right]$$

$$m) y = (\operatorname{sen} x)^{2x} \Rightarrow \ln y = \ln(\operatorname{sen} x)^{2x} \Rightarrow \ln y = 2x \cdot \ln(\operatorname{sen} x)$$

$$\frac{1}{y} \cdot y' = 2 \ln(\operatorname{sen} x) + 2x \cdot \frac{1}{\operatorname{sen} x} \cdot \cos x \Rightarrow$$

$$\Rightarrow y' = 2(\operatorname{sen} x)^{2x} \left[\ln(\operatorname{sen} x) + \frac{x}{\operatorname{tg} x} \right]$$

$$n) y = \sqrt[x]{\cos x} \Rightarrow \ln y = \ln(\cos x)^{1/x} \Rightarrow \ln y = \frac{1}{x} \cdot \ln(\cos x)$$

$$\frac{1}{y} \cdot y' = -\frac{1}{x^2} \cdot \ln(\cos x) + \frac{1}{x} \cdot \frac{1}{\cos x} \cdot \sin x \Rightarrow$$

$$\Rightarrow y' = \frac{\sqrt[x]{\cos x}}{x} \cdot \left(\lg x - \frac{\ln(\cos x)}{x} \right)$$

$$o) y = (\operatorname{arcsen} x)^{\operatorname{sen} x} \Rightarrow \ln y = \ln(\operatorname{arcsen} x)^{\operatorname{sen} x} \Rightarrow \ln y = \operatorname{sen} x \cdot \ln(\operatorname{arcsen} x)$$

$$\frac{1}{y} \cdot y' = \cos x \cdot \ln(\operatorname{arcsen} x) + \operatorname{sen} x \cdot \frac{1}{\operatorname{arcsen} x} \cdot \frac{1}{\sqrt{1-x^2}} \Rightarrow$$

$$\Rightarrow y' = (\operatorname{arcsen} x)^{\operatorname{sen} x} \cdot \left[\cos x \cdot \ln(\operatorname{arcsen} x) + \frac{\operatorname{sen} x}{\operatorname{arcsen} x \cdot \sqrt{1-x^2}} \right]$$

$$p) y = (1+x^2)^x \Rightarrow \ln y = \ln(1+x^2)^x \Rightarrow \ln y = x \cdot \ln(1+x^2)$$

$$\frac{1}{y} \cdot y' = \ln(1+x^2) + x \cdot \frac{1}{1+x^2} \cdot 2x \Rightarrow y' = (1+x^2)^x \cdot \left(\ln(1+x^2) + \frac{2x^2}{1+x^2} \right)$$

$$q) y = (\operatorname{sen} x)^{\cos x} \Rightarrow \ln y = \ln(\operatorname{sen} x)^{\cos x} \Rightarrow \ln y = \cos x \cdot \ln(\operatorname{sen} x)$$

$$\frac{1}{y} \cdot y' = -\operatorname{sen} x \cdot \ln(\operatorname{sen} x) + \cos x \cdot \frac{1}{\operatorname{sen} x} \cdot \cos x \Rightarrow$$

$$\Rightarrow y' = (\operatorname{sen} x)^{\cos x} \cdot \left[-\operatorname{sen} x \cdot \ln(\operatorname{sen} x) + \frac{\cos x}{\operatorname{tg} x} \right]$$

$$r) y = \sqrt[x]{x^3} \Rightarrow \ln y = \ln(x)^{3/x} \Rightarrow \ln y = \frac{3}{x} \cdot \ln x$$

$$\frac{1}{y} \cdot y' = -\frac{3}{x^2} \ln x + \frac{3}{x} \cdot \frac{1}{x} \Rightarrow y' = \sqrt[x]{x^3} \cdot \frac{3}{x^2} \cdot (1 - \ln x)$$

$$s) y = \left(1 + \frac{1}{x}\right)^x \Rightarrow \ln y = \ln\left(1 + \frac{1}{x}\right)^x \Rightarrow \ln y = x \cdot \ln\left(1 + \frac{1}{x}\right)$$

$$\frac{1}{y} \cdot y' = \ln\left(1 + \frac{1}{x}\right) + x \cdot \frac{1}{1 + \frac{1}{x}} \cdot -\frac{1}{x^2} = \ln\left(1 + \frac{1}{x}\right) + x \cdot \frac{1}{\frac{x+1}{x}} \cdot -\frac{1}{x^2}$$

$$\Rightarrow y' = \left(1 + \frac{1}{x}\right)^x \cdot \left[\ln\left(1 + \frac{1}{x}\right) - \frac{1}{x+1}\right]$$

$$t) y = (\lg x)^x \Rightarrow \ln y = \ln(\lg x)^x \Rightarrow \ln y = x \cdot \ln(\lg x)$$

$$\frac{1}{y} \cdot y' = \ln(\lg x) + x \cdot \frac{1}{\lg x} \cdot \frac{1}{\cos^2 x} \Rightarrow$$

$$\Rightarrow y' = (\lg x)^x \cdot \left[\ln(\lg x) + \frac{x}{\operatorname{sen} x \cos x}\right]$$

①9) Estudia la derivabilidad de las siguientes funciones:

a) $f(x) = x^3 + 3x^2 - 5$

b) $f(x) = \sqrt{x}$

c) $f(x) = \ln(x)$

d) $f(x) = \frac{1}{x}$

e) $f(x) = \frac{e^x}{x-3}$

f) $f(x) = \sqrt[3]{(x-2)}$

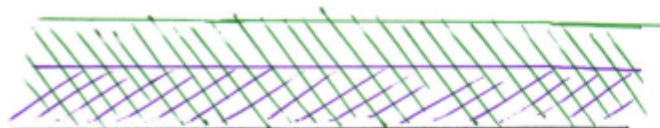
g) $f(x) = \operatorname{sen} x$

h) $f(x) = \frac{1}{\ln(x)}$

i) $f(x) = 3^x$

a) $f(x) = x^3 + 3x^2 - 5 \rightarrow \text{Dom}(f(x)) = \mathbb{R}$ //

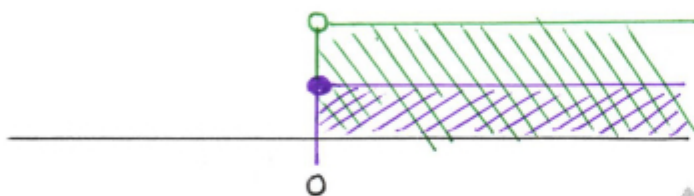
$f'(x) = 3x^2 + 6x \rightarrow \text{Dom}(f'(x)) = \mathbb{R}$ //



$\Rightarrow f(x)$ es derivable en $\mathbb{R}$

b) $f(x) = \sqrt{x} \rightarrow \text{Dom}(f(x)) = [0, +\infty)$

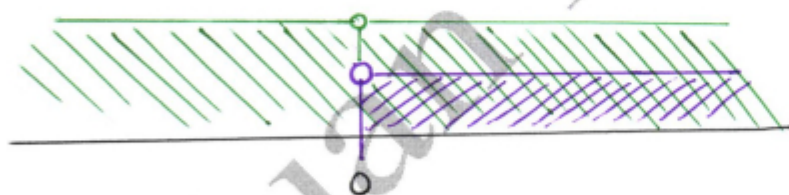
$f'(x) = \frac{1}{2\sqrt{x}} \rightarrow \text{Dom}(f'(x)) = (0, +\infty)$



$\Rightarrow f(x)$ es derivable en $(0, +\infty)$

c) $f(x) = \ln(x) \rightarrow \text{Dom}(f(x)) = (0, +\infty)$

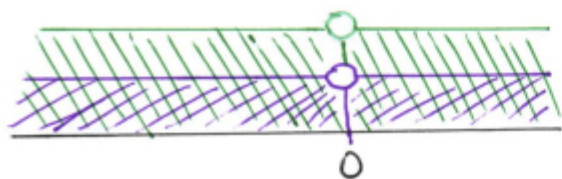
$f'(x) = \frac{1}{x} \rightarrow \text{Dom}(f'(x)) = \mathbb{R} - \{0\}$



$\Rightarrow f(x)$ es derivable en $(0, +\infty)$

d) $f(x) = \frac{1}{x} \rightarrow \text{Dom}(f(x)) = \mathbb{R} - \{0\}$

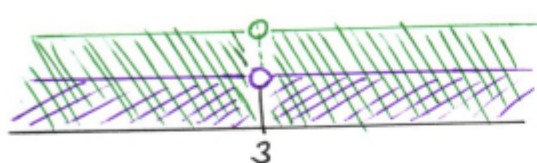
$f'(x) = -\frac{1}{x^2} \rightarrow \text{Dom}(f'(x)) = \mathbb{R} - \{0\}$



$\Rightarrow f(x)$ es derivable en $\mathbb{R} - \{0\}$

$$e) f(x) = \frac{e^x}{x-3} \rightarrow \text{Dom}(f(x)) = \mathbb{R} - \{3\}$$

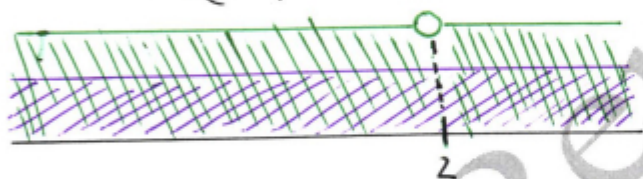
$$f'(x) = \frac{e^x(x-3) - e^x}{(x-3)^2} = \frac{e^x(x-4)}{(x-3)^2} \rightarrow \text{Dom}(f'(x)) = \mathbb{R} - \{3\}$$



$\Rightarrow f(x)$ es derivable en $\mathbb{R} - \{3\}$

$$f) f(x) = \sqrt[3]{x-2} \rightarrow \text{Dom}(f(x)) = \mathbb{R}$$

$$f'(x) = \frac{1}{3\sqrt[3]{(x-2)^2}} \rightarrow \text{Dom}(f'(x)) = \mathbb{R} - \{2\}$$



$\Rightarrow f(x)$ es derivable en $\mathbb{R} - \{2\}$

$$g) f(x) = \sin x \rightarrow \text{Dom}(f(x)) = \mathbb{R} \quad \left\{ \begin{array}{l} f(x) \text{ es derivable en } \mathbb{R} \\ f'(x) = \cos x \rightarrow \text{Dom}(f'(x)) = \mathbb{R} \end{array} \right.$$

$$h) f(x) = \frac{1}{\ln x} \quad \left\{ \begin{array}{l} x > 0 \\ \ln x = 0 \Rightarrow x = e^0 = 1 \end{array} \right\} \text{Dom}(f(x)) = (0, 1) \cup (1, +\infty)$$

$$f'(x) = \frac{-1}{x \cdot (\ln x)^2} \quad \left\{ \begin{array}{l} x > 0 \\ x \cdot (\ln x)^2 = 0 \end{array} \right. \quad \left\{ \begin{array}{l} x = 0 \text{ (No sirve)} \\ \ln x = 0 \Rightarrow x = 1 \end{array} \right.$$

$$\text{Dom}(f'(x)) = (0, 1) \cup (1, +\infty)$$

$\Rightarrow f(x)$ es derivable en $(0, 1) \cup (1, +\infty)$

$$i) \left. \begin{aligned} f(x) &= 3^x \rightarrow \text{Dom}(f(x)) = \mathbb{R} \\ f'(x) &= 3^x \cdot \ln 3 \rightarrow \text{Dom}(f'(x)) = \mathbb{R} \end{aligned} \right\} f(x) \text{ es derivable en } \mathbb{R}$$

(20) Estudia la continuidad y la derivabilidad de las siguientes funciones, haciendo además su representación gráfica:

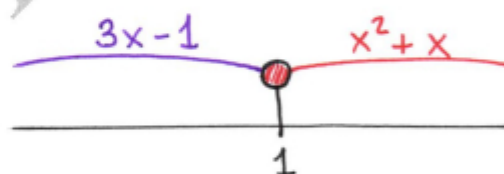
$$a) f(x) = \begin{cases} 3x-1 & \text{si } x < 1 \\ x^2+x & \text{si } x \geq 1 \end{cases}$$

$$b) f(x) = \begin{cases} -x^2 & \text{si } x < 0 \\ x^2 & \text{si } x \geq 0 \end{cases}$$

$$c) f(x) = \begin{cases} 2x-1 & \text{si } x < 3 \\ x^2-4 & \text{si } x \geq 3 \end{cases}$$

$$d) f(x) = \begin{cases} 3x-2 & \text{si } x \leq 2 \\ 3x+1 & \text{si } x > 2 \end{cases}$$

$$a) f(x) = \begin{cases} 3x-1 & \text{si } x < 1 \\ x^2+x & \text{si } x \geq 1 \end{cases}$$



(Si $x < 1$) $\rightarrow f(x) = 3x-1 \rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 1)$

(Si $x > 1$) $\rightarrow f(x) = x^2+x \rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(1, +\infty)$

(Si $x = 1$) Continuidad:

$$f(1) = 1^2 + 1 = 2$$

$$\lim_{x \rightarrow 1} f(x) \rightarrow \left\{ \begin{aligned} \lim_{x \rightarrow 1^-} (3x-1) &= 2 \\ \lim_{x \rightarrow 1^+} (x^2+x) &= 2 \end{aligned} \right\} \Rightarrow \lim_{x \rightarrow 1} f(x) = 2$$

Como $f(1) = \lim_{x \rightarrow 1} f(x) \Rightarrow$ Continua en $x = 1$

Para la derivabilidad:

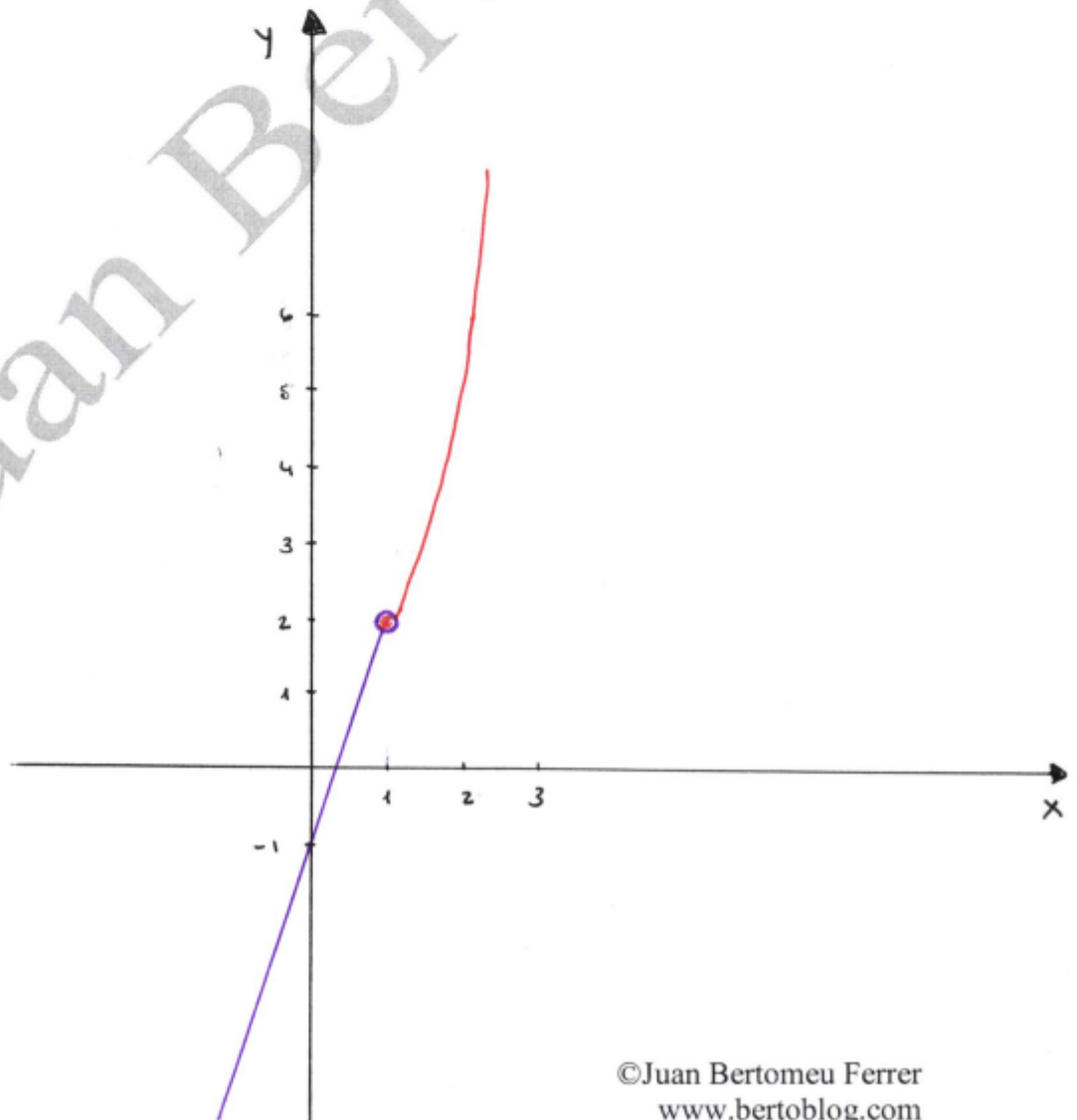
$$f'(x) = \begin{cases} 3 & \text{si } x < 1 \\ 2x+1 & \text{si } x > 1 \end{cases}$$

Si $x = 1$ Derivabilidad.

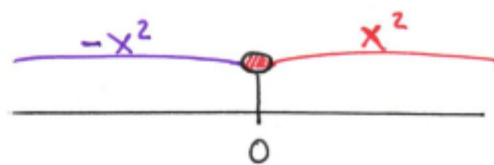
$$\left. \begin{array}{l} f'(1^-) = 3 \\ f'(1^+) = 2 \cdot 1 + 1 = 3 \end{array} \right\} \text{ Como } f'(1^-) = f'(1^+) \Rightarrow \text{Derivable en } x = 1$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

x	$f(x) = 3x - 1$
0	-1
1	2 NO
x	$f(x) = x^2 + x$
1	2 SI
2	6
3	12
...	...



$$b) f(x) = \begin{cases} -x^2 & \text{si } x < 0 \\ x^2 & \text{si } x \geq 0 \end{cases}$$



Si $x < 0 \rightarrow f(x) = -x^2 \rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0 \rightarrow f(x) = x^2 \rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = 0^2 = 0$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} -x^2 = 0 \\ \lim_{x \rightarrow 0^+} x^2 = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 0$$

Como $f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow$ Continua en $x = 0$

Para la derivabilidad:

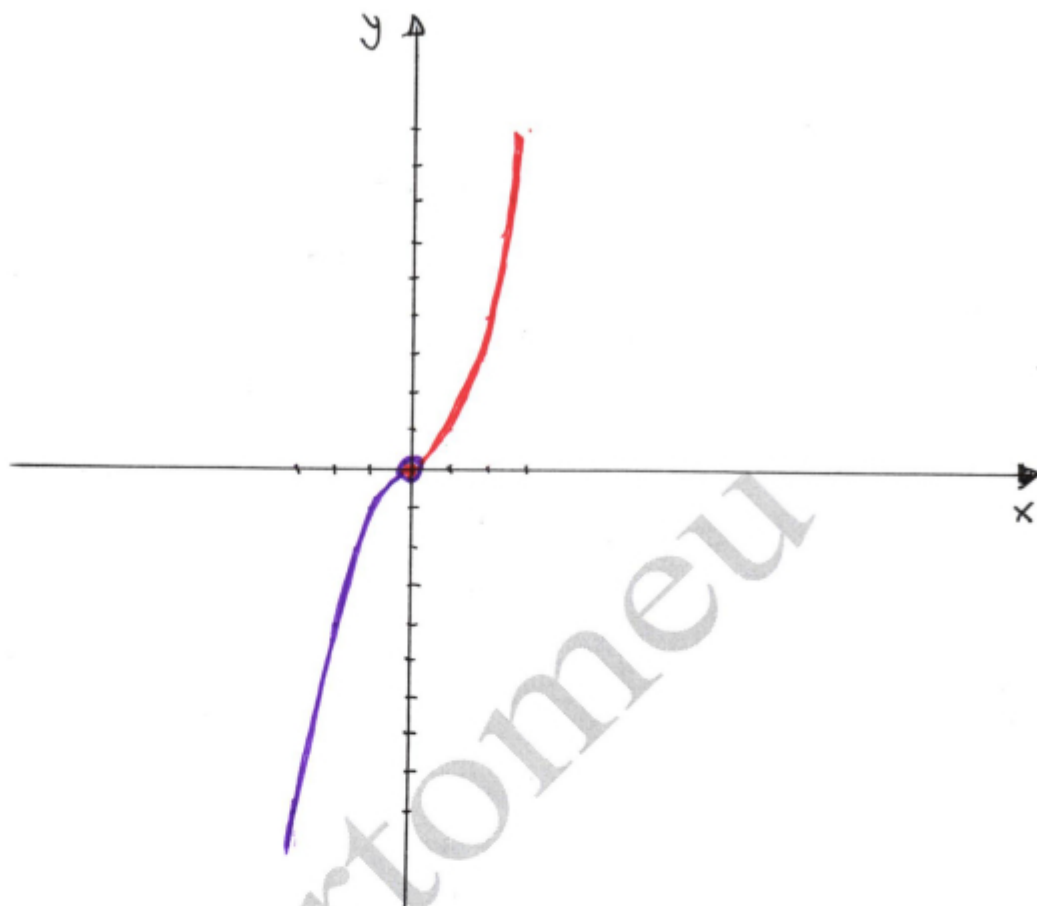
$$f'(x) = \begin{cases} -2x & \text{si } x < 0 \\ 2x & \text{si } x \geq 0 \end{cases}$$

Si $x = 0$ Derivabilidad:

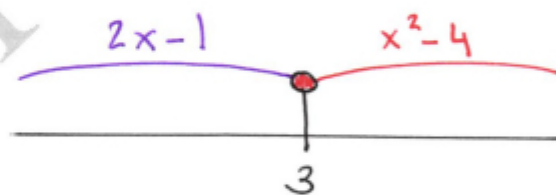
$$\left. \begin{array}{l} f'(0^-) = -2 \cdot 0 = 0 \\ f'(0^+) = 2 \cdot 0 = 0 \end{array} \right\} \text{ Como } f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

x	$f(x) = -x^2$
-3	-9
-2	-4
-1	-1
0	0 No
x	$f(x) = x^2$
0	0 SI
1	1
2	4
3	9
...	...



$$c) f(x) = \begin{cases} 2x-1 & \text{si } x < 3 \\ x^2-4 & \text{si } x \geq 3 \end{cases}$$



(Si $x < 3$) $\rightarrow f(x) = 2x-1 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, 3)$

(Si $x > 3$) $\rightarrow f(x) = x^2-4 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(3, +\infty)$

(Si $x = 3$) Continuidad:

$$f(3) = 3^2 - 4 = 5$$

$$\lim_{x \rightarrow 3} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 3^-} (2x-1) = 5 \\ \lim_{x \rightarrow 3^+} (x^2-4) = 5 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 3} f(x) = 5$$

Como $f(3) = \lim_{x \rightarrow 3} f(x) \Rightarrow$ Continua en $x=3$

Para la derivabilidad:

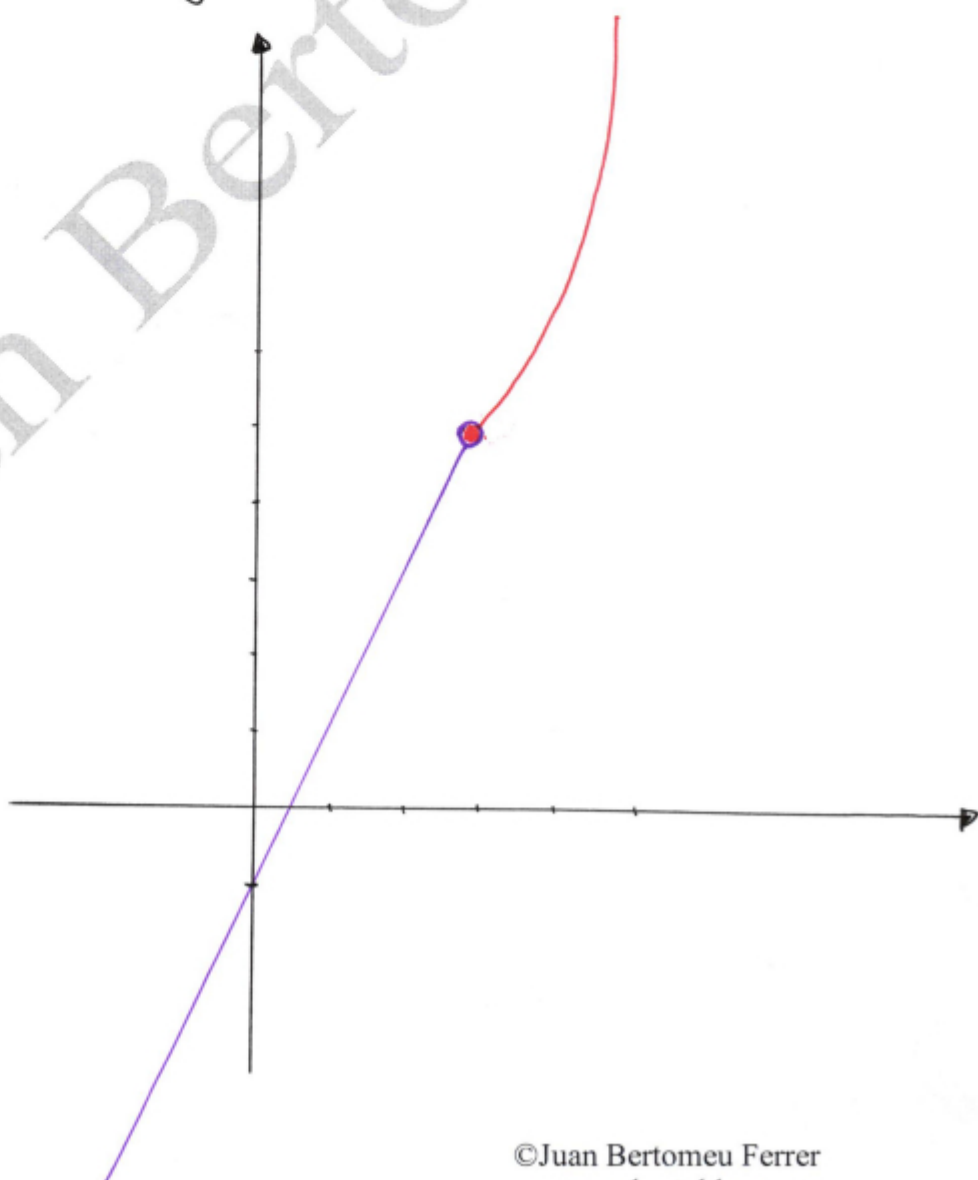
$$f'(x) = \begin{cases} 2 & \text{si } x < 3 \\ 2x & \text{si } x > 3 \end{cases}$$

Si $x=3$ Derivabilidad:

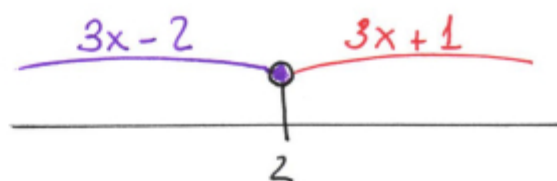
$$\left. \begin{array}{l} f'(3^-) = 2 \\ f'(3^+) = 2 \cdot 3 = 6 \end{array} \right\} f'(3^-) \neq f'(3^+) \Rightarrow \nexists f'(3) \Rightarrow \text{No derivable en } x=3$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{3\}$

x	$f(x) = 2x - 1$
0	-1
3	5 NO
x	$f(x) = x^2 - 4$
3	5 SI
4	12
5	21
...	...



$$d) f(x) = \begin{cases} 3x-2 & \text{si } x \leq 2 \\ 3x+1 & \text{si } x > 2 \end{cases}$$



Si $x < 2$ $\rightarrow f(x) = 3x-2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 2)$

Si $x > 2$ $\rightarrow f(x) = 3x+1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(2, +\infty)$

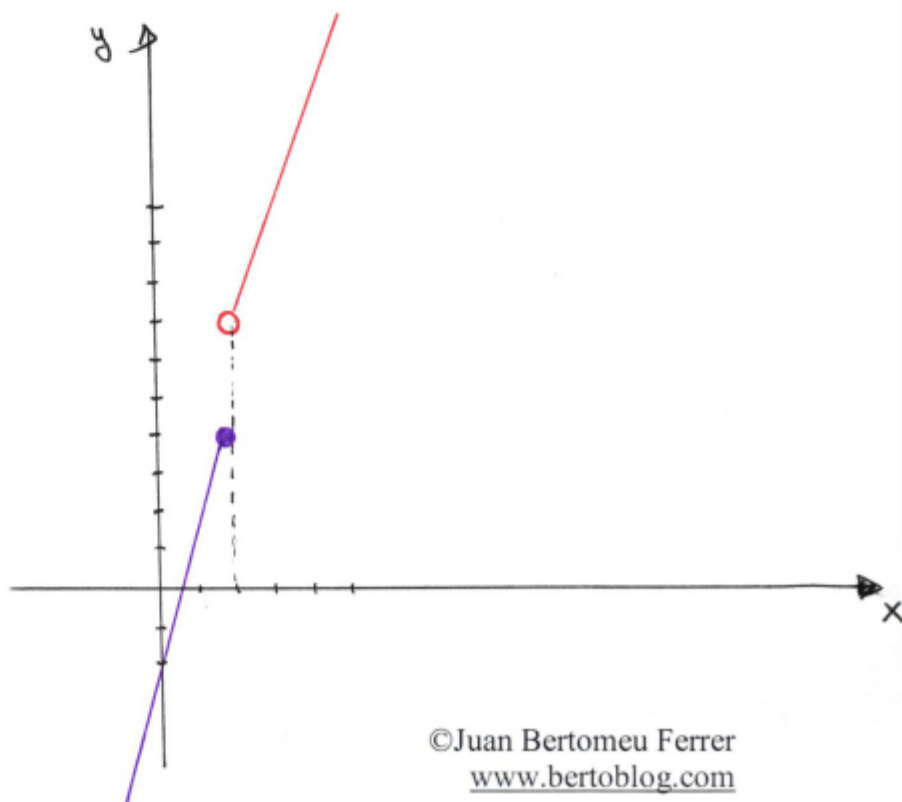
Si $x = 2$ Continuidad:

$$f(2) = 3 \cdot 2 - 2 = 4$$

$$\lim_{x \rightarrow 2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 2^-} (3x-2) = 4 \\ \lim_{x \rightarrow 2^+} (3x+1) = 7 \end{array} \right\} \Rightarrow \nexists \lim_{x \rightarrow 2} f(x)$$

$f(x)$ presenta en $x=2$ una discontinuidad de SALTO FINITO y por tanto tampoco es derivable en $x=2$

x	$f(x) = 3x-2$
0	-2
2	4 SI
x	$f(x) = 3x+1$
2	7 NO
3	10



21) Estudia la continuidad y la derivabilidad de las siguientes funciones:

$$a) f(x) = \begin{cases} x^2 - 1 & \text{si } x \leq 1 \\ x - 1 & \text{si } x > 1 \end{cases}$$

$$b) f(x) = \begin{cases} e^{-x} & \text{si } x \leq 0 \\ 1 - x & \text{si } x > 0 \end{cases}$$

$$c) f(x) = \begin{cases} \ln(x-1) & \text{si } x < 2 \\ 3x - 6 & \text{si } x \geq 2 \end{cases}$$

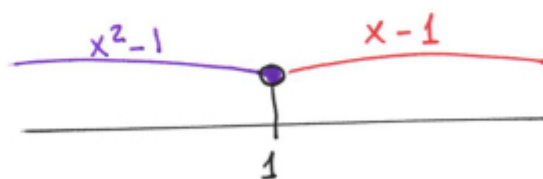
$$d) f(x) = \begin{cases} x^2 - 2x + 1 & \text{si } x < 2 \\ -2x + 5 & \text{si } x \geq 2 \end{cases}$$

$$e) f(x) = \begin{cases} 2x - 5 & \text{si } x < 3 \\ \sqrt{x-2} & \text{si } x \geq 3 \end{cases}$$

$$f) f(x) = \begin{cases} 2^x + 2 & \text{si } x \leq 0 \\ x^2 + x + 3 & \text{si } x > 0 \end{cases}$$

$$g) f(x) = \begin{cases} 0 & \text{si } x < 0 \\ x^2 & \text{si } 0 \leq x < 1 \\ x & \text{si } x \geq 1 \end{cases}$$

$$a) f(x) = \begin{cases} x^2 - 1 & \text{si } x \leq 1 \\ x - 1 & \text{si } x > 1 \end{cases}$$



Si $x < 1$ $\rightarrow f(x) = x^2 - 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, 1)$

(Si $x > 1$) $\rightarrow f(x) = x - 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(1, +\infty)$

(Si $x = 1$) Continuidad:

$$f(1) = 1^2 - 1 = 0$$

$$\lim_{x \rightarrow 1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 1^-} x^2 - 1 = 0 \\ \lim_{x \rightarrow 1^+} x - 1 = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 1} f(x) = 0$$

Como $f(1) = \lim_{x \rightarrow 1} f(x) \Rightarrow$ Continua en $x = 1$

Para la derivabilidad:

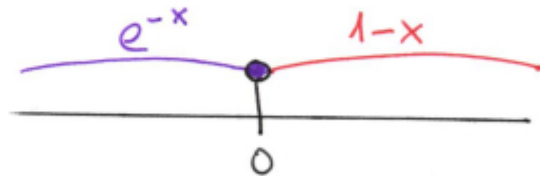
$$f'(x) = \begin{cases} 2x & \text{si } x < 1 \\ 1 & \text{si } x > 1 \end{cases}$$

(Si $x = 1$) Derivabilidad:

$$\left. \begin{array}{l} f'(1^-) = 2 \cdot 1 = 2 \\ f'(1^+) = 1 \end{array} \right\} f'(1^-) \neq f'(1^+) \Rightarrow \nexists f'(1) \Rightarrow \text{No derivable en } x = 1$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{1\}$

$$b) f(x) = \begin{cases} e^{-x} & \text{si } x \leq 0 \\ 1-x & \text{si } x > 0 \end{cases}$$



(Si $x < 0$) $\rightarrow f(x) = e^{-x} \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0 \rightarrow f(x) = 1 - x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = e^{-0} = 1$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} e^{-x} = 1 \\ \lim_{x \rightarrow 0^+} 1 - x = 1 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 1$$

Como $f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow$ Continua en $x = 0$

Para la derivabilidad:

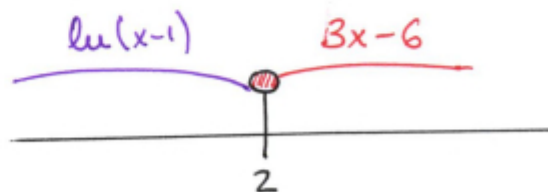
$$f'(x) = \begin{cases} -e^{-x} & \text{si } x \leq 0 \\ -1 & \text{si } x > 0 \end{cases}$$

Si $x = 0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = -e^{-0} = -1 \\ f'(0^+) = -1 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

$$c) f(x) = \begin{cases} \ln(x-1) & \text{si } x < 2 \\ 3x-6 & \text{si } x \geq 2 \end{cases}$$



Si $x < 2 \rightarrow f(x) = \ln(x-1) \Rightarrow$ Continua y derivable en $(1, +\infty)$

$\hookrightarrow$ Continua y derivable en $(1, 2)$

Si $x > 2 \rightarrow f(x) = 3x - 6 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(2, +\infty)$

Si $x = 2$ Continuidad:

$$f(2) = 3 \cdot 2 - 6 = 0$$

$$\lim_{x \rightarrow 2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 2^-} \ln(x-1) = 0 \\ \lim_{x \rightarrow 2^+} 3x - 6 = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 2} f(x) = 0$$

Como $f(2) = \lim_{x \rightarrow 2} f(x) \Rightarrow$ Continua en $x = 2$

Para la derivabilidad:

$$f'(x) = \begin{cases} \frac{1}{x-1} & \text{si } 1 < x < 2 \\ 3 & \text{si } x > 2 \end{cases}$$

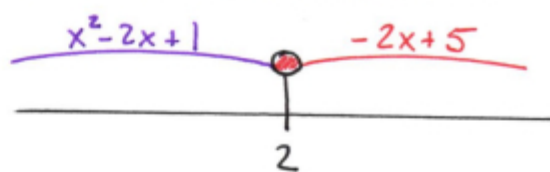
Si $x = 2$ Derivabilidad:

$$\left. \begin{array}{l} f'(2^-) = \frac{1}{2-1} = 1 \\ f'(2^+) = 3 \end{array} \right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow \text{No derivable en } x = 2$$

$\Rightarrow f(x)$ es continua en $(1, +\infty)$ y derivable en $(1, 2) \cup (2, +\infty)$

$\nexists f(x)$ si $x \leq 1$

$$d) f(x) = \begin{cases} x^2 - 2x + 1 & \text{si } x < 2 \\ -2x + 5 & \text{si } x \geq 2 \end{cases}$$



(Si $x < 2$) $\rightarrow f(x) = x^2 - 2x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable $(-\infty, 2)$

(Si $x > 2$) $\rightarrow f(x) = -2x + 5 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(2, +\infty)$

(Si $x = 2$) Continuidad:

$$f(2) = -2 \cdot 2 + 5 = 1$$

$$\lim_{x \rightarrow 2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 2^-} (x^2 - 2x + 1) = 1 \\ \lim_{x \rightarrow 2^+} (-2x + 5) = 1 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 2} f(x) = 1$$

Como $f(2) = \lim_{x \rightarrow 2} f(x) \Rightarrow$ Continua en $x = 2$

Para la derivabilidad:

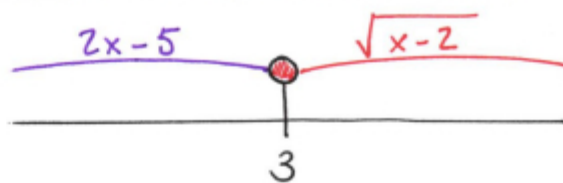
$$f'(x) = \begin{cases} 2x - 2 & \text{si } x < 2 \\ -2 & \text{si } x > 2 \end{cases}$$

(Si $x = 2$) Derivabilidad:

$$\left. \begin{array}{l} f'(2^-) = 2 \cdot 2 - 2 = 2 \\ f'(2^+) = -2 \end{array} \right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow \text{No derivable en } x = 2$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{2\}$

$$e) f(x) = \begin{cases} 2x-5 & \text{si } x < 3 \\ \sqrt{x-2} & \text{si } x \geq 3 \end{cases}$$



Si $x < 3$ $\rightarrow f(x) = 2x - 5 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable $(-\infty, 3)$

Si $x > 3$ $\rightarrow f(x) = \sqrt{x-2} \Rightarrow$ Continua en $[2, +\infty)$ y derivable en $(2, +\infty)$
 $\hookrightarrow$ Continua y derivable en $(3, +\infty)$

Si $x = 3$ Continuidad:

$$f(3) = \sqrt{3-2} = 1$$

$$\lim_{x \rightarrow 3} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 3^-} (2x-5) = 1 \\ \lim_{x \rightarrow 3^+} \sqrt{x-2} = 1 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 3} f(x) = 1$$

Como $f(3) = \lim_{x \rightarrow 3} f(x) \Rightarrow$ Continua en $x = 3$

Para la derivabilidad:

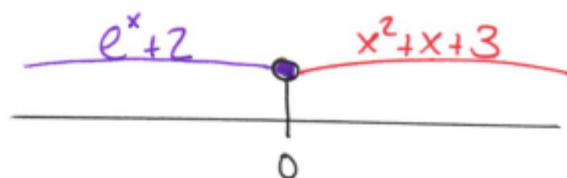
$$f'(x) = \begin{cases} 2 & \text{si } x < 3 \\ \frac{1}{2\sqrt{x-2}} & \text{si } x > 3 \end{cases}$$

Si $x = 3$ Derivabilidad:

$$\left. \begin{array}{l} f'(3^-) = 2 \\ f'(3^+) = \frac{1}{2 \cdot \sqrt{1}} = \frac{1}{2} \end{array} \right\} f'(3^-) \neq f'(3^+) \Rightarrow \nexists f'(3) \Rightarrow \text{No derivable en } x = 3$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{3\}$

$$f) f(x) = \begin{cases} e^x + 2 & \text{si } x \leq 0 \\ x^2 + x + 3 & \text{si } x > 0 \end{cases}$$



(Si $x < 0$) $\rightarrow f(x) = e^x + 2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable $(-\infty, 0)$

(Si $x > 0$) $\rightarrow f(x) = x^2 + x + 3 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

(Si $x = 0$) Continuidad:

$$f(0) = e^0 + 2 = 3$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} e^x + 2 = 3 \\ \lim_{x \rightarrow 0^+} (x^2 + x + 3) = 3 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 3$$

Como $f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow$ Continua en $x = 0$

Para la derivabilidad:

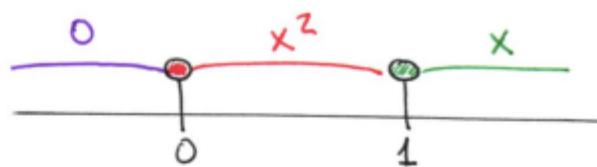
$$f'(x) = \begin{cases} e^x & \text{si } x < 0 \\ 2x + 1 & \text{si } x > 0 \end{cases}$$

(Si $x = 0$) Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = e^0 = 1 \\ f'(0^+) = 2 \cdot 0 + 1 = 1 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

$$g) f(x) = \begin{cases} 0 & \text{si } x < 0 \\ x^2 & \text{si } 0 \leq x < 1 \\ x & \text{si } x \geq 1 \end{cases}$$



Si $x < 0 \rightarrow f(x) = 0 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable $(-\infty, 0)$

Si $0 < x < 1 \rightarrow f(x) = x^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, 1)$

Si $x > 1 \rightarrow f(x) = x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(1, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = 0^2 = 0$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} 0 = 0 \\ \lim_{x \rightarrow 0^+} x^2 = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 0$$

Como $f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow$ Continua en $x = 0$

Si $x = 1$ Continuidad:

$$f(1) = 1$$

$$\lim_{x \rightarrow 1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 1^-} x^2 = 1 \\ \lim_{x \rightarrow 1^+} x = 1 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 1} f(x) = 1$$

Como $f(1) = \lim_{x \rightarrow 1} f(x) \Rightarrow$ Continua en $x = 1$

Para la derivabilidad,

$$f'(x) = \begin{cases} 0 & \text{si } x < 0 \\ 2x & \text{si } 0 < x < 1 \\ 1 & \text{si } x > 1 \end{cases}$$

Si $x=0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = 0 \\ f'(0^+) = 2 \cdot 0 = 0 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x=0$$

Si $x=1$ Derivabilidad:

$$\left. \begin{array}{l} f'(1^-) = 2 \cdot 1 = 2 \\ f'(1^+) = 1 \end{array} \right\} f'(1^-) \neq f'(1^+) \Rightarrow \nexists f'(1) \Rightarrow \text{No derivable en } x=1$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{1\}$

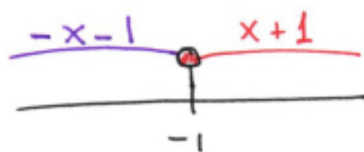
(22) Expresa las siguientes funciones como funciones definidas a trozos para estudiar su derivabilidad:

a) $f(x) = |x+1|$ b) $f(x) = |x^2 - 2x|$ c) $f(x) = 2x \cdot |x-4|$

d) $f(x) = |x+2| + |x-3|$ e) $f(x) = x^3 - 4 \cdot |x| + 3$ f) $f(x) = \frac{|x^2-1|}{1+|x|}$

a) $f(x) = |x+1|$

$$x+1 = 0 \Rightarrow x = -1$$



$$f(x) = \begin{cases} -x-1 & \text{si } x < -1 \\ x+1 & \text{si } x > -1 \end{cases}$$

(Si $x < -1$) $\rightarrow f(x) = -x - 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, -1)$

(Si $x > -1$) $\rightarrow f(x) = x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-1, +\infty)$

(Si $x = -1$) Continuidad:

$$f(-1) = -1 + 1 = 0$$

$$\lim_{x \rightarrow -1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -1^-} -x - 1 = 0 \\ \lim_{x \rightarrow -1^+} x + 1 = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow -1} f(x) = 0$$

Como $f(-1) = \lim_{x \rightarrow -1} f(x) \Rightarrow$ Continua en $x = -1$

Para la derivabilidad:

$$f'(x) = \begin{cases} -1 & \text{si } x < -1 \\ 1 & \text{si } x > -1 \end{cases}$$

(Si $x = -1$) Derivabilidad:

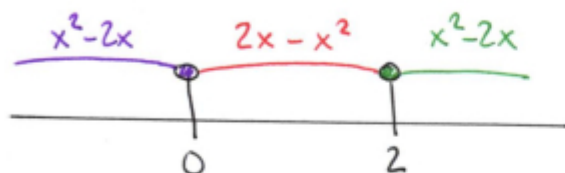
$$\left. \begin{array}{l} f'(-1^-) = -1 \\ f'(-1^+) = 1 \end{array} \right\} f'(-1^-) \neq f'(-1^+) \Rightarrow \nexists f'(-1) \Rightarrow \text{no derivable en } x = -1$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{-1\}$

$$b) f(x) = |x^2 - 2x|$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0 \begin{cases} x=0 \\ x=2 \end{cases}$$



$$f(x) = \begin{cases} x^2 - 2x & \text{si } x \leq 0 \\ 2x - x^2 & \text{si } 0 < x < 2 \\ x^2 - 2x & \text{si } x \geq 2 \end{cases}$$

Si $x < 0$ $\rightarrow f(x) = x^2 - 2x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $0 < x < 2$ $\rightarrow f(x) = 2x - x^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, 2)$

Si $x > 2$ $\rightarrow f(x) = x^2 - 2x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(2, +\infty)$

Si $x = 0$ Continuidad:

$$\begin{aligned} f(0) &= 0^2 - 2 \cdot 0 = 0 \\ \lim_{x \rightarrow 0} f(x) &\rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (x^2 - 2x) = 0 \\ \lim_{x \rightarrow 0^+} (2x - x^2) = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 0 \end{aligned}$$

Como $f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow$ Continua en $x = 0$

Si $x = 2$ Continuidad:

$$\begin{aligned} f(2) &= 2^2 - 2 \cdot 2 = 0 \\ \lim_{x \rightarrow 2} f(x) &\rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 2^-} (2x - x^2) = 0 \\ \lim_{x \rightarrow 2^+} (x^2 - 2x) = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 2} f(x) = 0 \end{aligned}$$

Como $f(2) = \lim_{x \rightarrow 2} f(x) \Rightarrow$ Continua en $x = 2$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2x - 2 & \text{si } x < 0 \\ 2 - 2x & \text{si } 0 < x < 2 \\ 2x - 2 & \text{si } x > 2 \end{cases}$$

Si $x = 0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = 2 \cdot 0 - 2 = -2 \\ f'(0^+) = 2 - 2 \cdot 0 = 2 \end{array} \right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow \text{No derivable en } x = 0$$

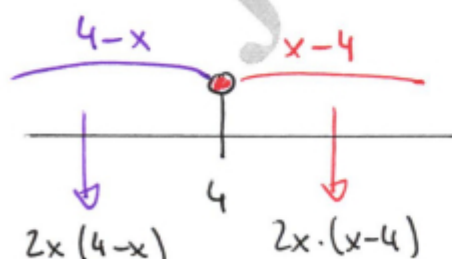
Si $x = 2$ Derivabilidad:

$$\left. \begin{array}{l} f'(2^-) = 2 - 2 \cdot 2 = -2 \\ f'(2^+) = 2 \cdot 2 - 2 = 2 \end{array} \right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow \text{No derivable en } x = 2$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{0, 2\}$

c) $f(x) = 2x \cdot |x - 4|$

$$x - 4 = 0 \Rightarrow x = 4$$



$$\Rightarrow f(x) = \begin{cases} 8x - 2x^2 & \text{si } x < 4 \\ 2x^2 - 8x & \text{si } x > 4 \end{cases}$$

Si $x < 4 \rightarrow f(x) = 8x - 2x^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 4)$

Si $x > 4 \rightarrow f(x) = 2x^2 - 8x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(4, +\infty)$

Si $x = 4$ Continuidad:

$$f(4) = 2 \cdot 4^2 - 8 \cdot 4 = 0$$

$$\lim_{x \rightarrow 4} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 4^-} (8x - 2x^2) = 0 \\ \lim_{x \rightarrow 4^+} (2x^2 - 8x) = 0 \end{array} \right\} \lim_{x \rightarrow 4} f(x) = 0$$

Como $f(4) = \lim_{x \rightarrow 4} f(x) \Rightarrow$ Continua en $x = 4$

Para la derivabilidad:

$$f'(x) = \begin{cases} 8 - 4x & \text{si } x < 4 \\ 4x - 8 & \text{si } x > 4 \end{cases}$$

Si $x = 4$ Derivabilidad:

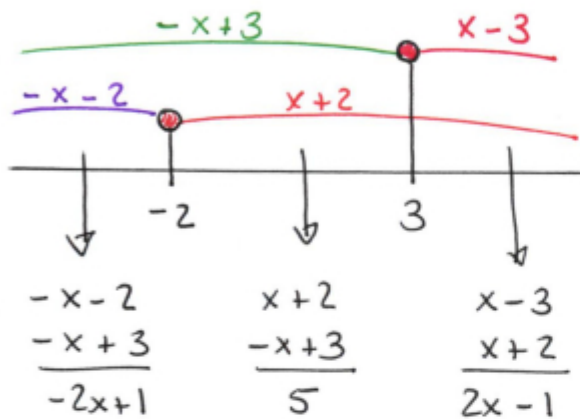
$$\left. \begin{array}{l} f'(4^-) = 8 - 4 \cdot 4 = -8 \\ f'(4^+) = 4 \cdot 4 - 8 = 8 \end{array} \right\} f'(4^-) \neq f'(4^+) \Rightarrow \nexists f'(4) \Rightarrow \text{No derivable en } x = 4$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{4\}$

$$d) f(x) = |x+2| + |x-3|$$

$$x+2=0 \rightarrow x=-2$$

$$x-3=0 \rightarrow x=3$$



$$\Rightarrow f(x) = \begin{cases} -2x+1 & \text{si } x < -2 \\ 5 & \text{si } -2 \leq x < 3 \\ 2x-1 & \text{si } x \geq 3 \end{cases}$$

Si $x < -2 \Rightarrow f(x) = -2x+1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, -2)$

Si $-2 < x < 3 \rightarrow f(x) = 5 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-2, 3)$

Si $x > 3 \rightarrow f(x) = 2x-1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(3, +\infty)$

Si $x = -2$ Continuidad:

$$f(-2) = 5$$

$$\lim_{x \rightarrow -2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -2^-} (-2x+1) = 5 \\ \lim_{x \rightarrow -2^+} 5 = 5 \end{array} \right\} \Rightarrow \lim_{x \rightarrow -2} f(x) = 5$$

Como $f(-2) = \lim_{x \rightarrow -2} f(x) \Rightarrow$ Continua en $x = -2$

Si $x = 3$ Continuidad:

$$f(3) = 2 \cdot 3 - 1 = 5$$

$$\lim_{x \rightarrow 3} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 3^-} 5 = 5 \\ \lim_{x \rightarrow 3^+} (2x - 1) = 5 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 3} f(x) = 5$$

Como $f(3) = \lim_{x \rightarrow 3} f(x) \Rightarrow$ Continua en $x = 3$

Para la derivabilidad:

$$f'(x) = \begin{cases} -2 & \text{si } x < -2 \\ 0 & \text{si } -2 < x < 3 \\ 2 & \text{si } x > 3 \end{cases}$$

Si $x = -2$ Derivabilidad:

$$\left. \begin{array}{l} f'(-2^-) = -2 \\ f'(-2^+) = 0 \end{array} \right\} f'(-2^-) \neq f'(-2^+) \Rightarrow \nexists f'(-2) \Rightarrow$$

No derivable
en $x = -2$

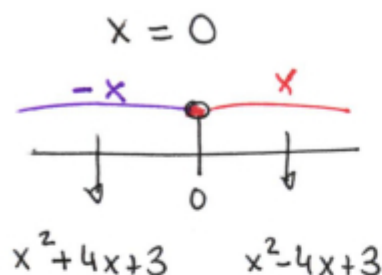
Si $x = 3$ Derivabilidad:

$$\left. \begin{array}{l} f'(3^-) = 0 \\ f'(3^+) = 2 \end{array} \right\} f'(3^-) \neq f'(3^+) \Rightarrow \nexists f'(3) \Rightarrow$$

No derivable en
 $x = 3$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{-2, 3\}$

$$e) f(x) = x^2 - 4|x| + 3$$



$$\Rightarrow f(x) = \begin{cases} x^2 + 4x + 3 & \text{si } x < 0 \\ x^2 - 4x + 3 & \text{si } x > 0 \end{cases}$$

Si $x < 0 \rightarrow f(x) = x^2 + 4x + 3 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0 \rightarrow f(x) = x^2 - 4x + 3 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = 0^2 - 4 \cdot 0 + 3 = 3$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \begin{cases} \lim_{x \rightarrow 0^-} (x^2 + 4x + 3) = 3 \\ \lim_{x \rightarrow 0^+} (x^2 - 4x + 3) = 3 \end{cases} \Rightarrow \lim_{x \rightarrow 0} f(x) = 3$$

$$\text{Como } f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow \text{Continua en } x = 0$$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2x + 4 & \text{si } x < 0 \\ 2x - 4 & \text{si } x > 0 \end{cases}$$

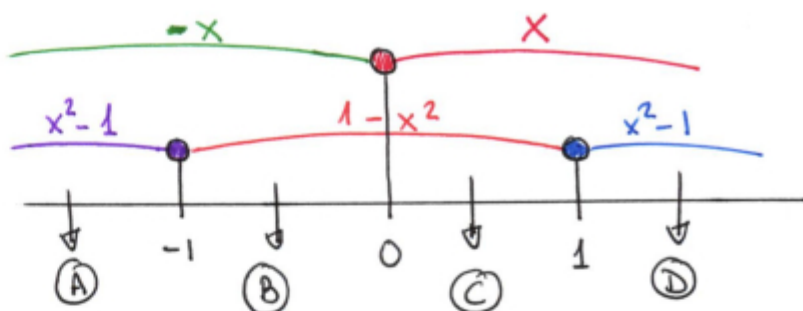
Si $x = 0$ Derivabilidad:

$$\left. \begin{aligned} f'(0^-) &= 2 \cdot 0 + 4 = 4 \\ f'(0^+) &= 2 \cdot 0 - 4 = -4 \end{aligned} \right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow \text{No derivable en } x = 0$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{0\}$

$$f) f(x) = \frac{|x^2 - 1|}{1 + |x|}$$

$$x=0; x^2-1=0 \begin{cases} x=-1 \\ x=+1 \end{cases}$$



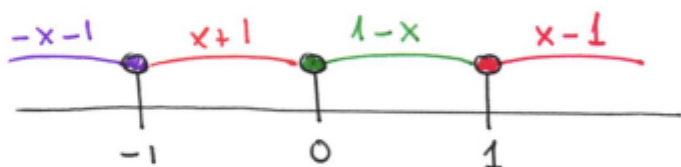
$$\textcircled{A} \quad \frac{x^2 - 1}{1 - x} = \frac{(x+1)\cancel{(x-1)}}{\cancel{(1-x)}} = (-1) \cdot (x+1) = -x - 1$$

$$\textcircled{B} \quad \frac{1 - x^2}{1 - x} = \frac{\cancel{(1-x)}(1+x)}{\cancel{(1-x)}} = x + 1$$

$$\textcircled{D} \quad \frac{x^2 - 1}{1 + x} = \frac{\cancel{(x+1)}(x-1)}{\cancel{(1+x)}} = x - 1$$

$$\textcircled{C} \quad \frac{1 - x^2}{1 + x} = \frac{\cancel{(1-x)}(1+x)}{\cancel{(1+x)}} = 1 - x$$

$$f(x) = \begin{cases} -x - 1 & \text{si } x \leq -1 \\ x + 1 & \text{si } -1 < x < 0 \\ 1 - x & \text{si } 0 \leq x < 1 \\ x - 1 & \text{si } x \geq 1 \end{cases}$$



$\textcircled{\text{Si } x < -1} \rightarrow f(x) = -x - 1 \Rightarrow \text{Continua y derivable en } \mathbb{R}$

$\hookrightarrow \text{Continua y derivable en } (-\infty, -1)$

$\textcircled{\text{Si } -1 < x < 0} \rightarrow f(x) = x + 1 \Rightarrow \text{Continua y derivable en } \mathbb{R}$

$\hookrightarrow \text{Continua y derivable en } (-1, 0)$

$\textcircled{\text{Si } 0 < x < 1} \rightarrow f(x) = 1 - x \Rightarrow \text{Continua y derivable en } \mathbb{R}$

$\hookrightarrow \text{Continua y derivable en } (0, 1)$

$\textcircled{\text{Si } x > 1} \rightarrow f(x) = x - 1 \Rightarrow \text{Continua y derivable en } \mathbb{R}$

$\hookrightarrow \text{Continua y derivable en } (1, +\infty)$

Si $x = -1$ Continuidad:

$$f(-1) = -(-1) - 1 = 0$$

$$\lim_{x \rightarrow -1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -1^-} (-x-1) = 0 \\ \lim_{x \rightarrow -1^+} (x+1) = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow -1} f(x) = 0$$

$$\text{Como } f(-1) = \lim_{x \rightarrow -1} f(x) \Rightarrow \text{Continua en } x = -1$$

Si $x = 0$ Continuidad:

$$f(0) = 1 - 0 = 1$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (x+1) = 1 \\ \lim_{x \rightarrow 0^+} (1-x) = 1 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 1$$

$$\text{Como } f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow \text{Continua en } x = 0$$

Si $x = 1$ Continuidad:

$$f(1) = 1 - 1 = 0$$

$$\lim_{x \rightarrow 1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 1^-} (1-x) = 0 \\ \lim_{x \rightarrow 1^+} (x-1) = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 1} f(x) = 0$$

$$\text{Como } f(1) = \lim_{x \rightarrow 1} f(x) \Rightarrow \text{Continua en } x = 1$$

Para la derivabilidad:

$$f'(x) = \begin{cases} -1 & \text{si } x < -1 \\ 1 & \text{si } -1 < x < 0 \\ -1 & \text{si } 0 < x < 1 \\ 1 & \text{si } x > 1 \end{cases}$$

Si $x = -1$ Derivabilidad:

$$\left. \begin{array}{l} f'(-1^-) = -1 \\ f'(-1^+) = 1 \end{array} \right\} f'(-1^-) \neq f'(-1^+) \Rightarrow \nexists f'(-1) \Rightarrow \text{No derivable en } x = -1$$

Si $x = 0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = 1 \\ f'(0^+) = -1 \end{array} \right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow \text{No derivable en } x = 0$$

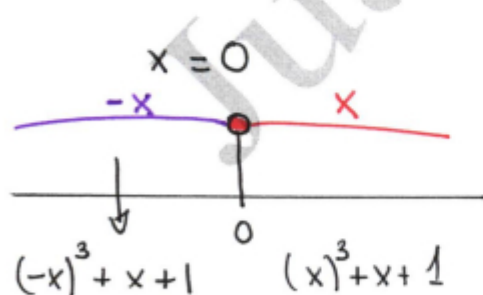
Si $x = 1$ Derivabilidad:

$$\left. \begin{array}{l} f'(1^-) = -1 \\ f'(1^+) = 1 \end{array} \right\} f'(1^-) \neq f'(1^+) \Rightarrow \nexists f'(1) \Rightarrow \text{No derivable en } x = 1$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{-1, 0, 1\}$

(23) Estudia la continuidad de las funciones $f(x) = |x|^3 + x + 1$ y $g(x) = |x|^3 + |x| + 1$. Estudia también la derivabilidad

$$f(x) = |x|^3 + x + 1$$



$$\Rightarrow f(x) = \begin{cases} -x^3 + x + 1 & \text{si } x < 0 \\ x^3 + x + 1 & \text{si } x > 0 \end{cases}$$

Si $x < 0 \rightarrow f(x) = -x^3 + x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0 \rightarrow f(x) = x^3 + x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = 0^3 + 0 + 1 = 1$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (-x^3 + x + 1) = 1 \\ \lim_{x \rightarrow 0^+} (x^3 + x + 1) = 1 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 1$$

Como $f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow$ Continua en $x = 0$

Para la derivabilidad:

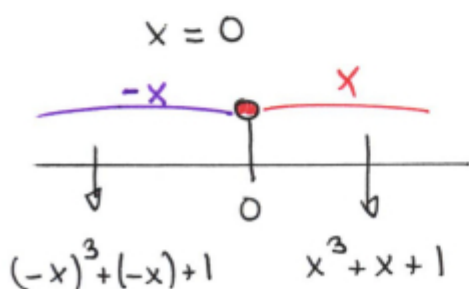
$$f'(x) = \begin{cases} -3x^2 + 1 & \text{si } x < 0 \\ 3x^2 + 1 & \text{si } x > 0 \end{cases}$$

Si $x = 0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = -3 \cdot 0 + 1 = 1 \\ f'(0^+) = 3 \cdot 0 + 1 = 1 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

$$g(x) = |x|^3 + |x| + 1$$



$$\Rightarrow g(x) = \begin{cases} -x^3 - x + 1 & \text{si } x < 0 \\ x^3 + x + 1 & \text{si } x \geq 0 \end{cases}$$

Si $x < 0$ $\rightarrow g(x) = -x^3 - x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0$ $\rightarrow g(x) = x^3 + x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$g(0) = 0^3 + 0 + 1 = 1$$

$$\lim_{x \rightarrow 0} g(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (-x^3 - x + 1) = 1 \\ \lim_{x \rightarrow 0^+} (x^3 + x + 1) = 1 \end{array} \right\} \lim_{x \rightarrow 0} g(x) = 1$$

Como $g(0) = \lim_{x \rightarrow 0} g(x) \Rightarrow$ Continua en $x = 0$

Para la derivabilidad:

$$g'(x) = \begin{cases} -3x^2 - 1 & \text{si } x < 0 \\ 3x^2 + 1 & \text{si } x > 0 \end{cases}$$

Si $x = 0$ Derivabilidad:

$$\left. \begin{array}{l} g'(0^-) = -1 \\ g'(0^+) = +1 \end{array} \right\} g'(0^-) \neq g'(0^+) \Rightarrow \nexists g'(0) \Rightarrow \text{No derivable en } x = 0$$

$\Rightarrow g(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{0\}$

24) Estudia la continuidad y la derivabilidad de las siguientes funciones:

$$a) f(x) = \begin{cases} 1-x^2 & \text{si } x \leq 0 \\ \cos x & \text{si } x > 0 \end{cases}$$

$$b) f(x) = \begin{cases} 2\sec x + 1 & \text{si } x < 0 \\ -x^3 + 2x + 1 & \text{si } x > 0 \end{cases}$$

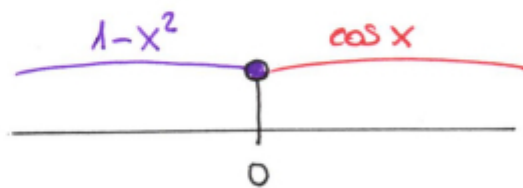
$$c) f(x) = \begin{cases} \frac{x^2+2x-3}{1-x^2} & \text{si } -2 \leq x < 1 \\ 1-3\sqrt{2x-1} & \text{si } 1 \leq x \leq 5 \end{cases}$$

$$d) f(x) = \begin{cases} x(x-2) & \text{si } x \leq 2 \\ \sqrt[3]{x-2} & \text{si } x > 2 \end{cases}$$

$$e) f(x) = \begin{cases} 2x^2+5x+6 & \text{si } x \leq -1 \\ 2 - \frac{1}{x} & \text{si } -1 < x < 2 \\ \frac{5}{4} + 2^{-x} & \text{si } x \geq 2 \end{cases}$$

$$f) f(x) = \begin{cases} -1 & \text{si } x < -4 \\ x+2 & \text{si } -4 \leq x < 2 \\ \frac{8}{x} & \text{si } x \geq 2 \end{cases}$$

$$a) f(x) = \begin{cases} 1-x^2 & \text{si } x \leq 0 \\ \cos x & \text{si } x > 0 \end{cases}$$



si $x < 0$ $\rightarrow f(x) = 1-x^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

si $x > 0$ $\rightarrow f(x) = \cos x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = 1 - 0^2 = 1$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (1 - x^2) = 1 \\ \lim_{x \rightarrow 0^+} (\cos x) = 1 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 1$$

Como $f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow$ Continua en $x = 0$

Para la derivabilidad:

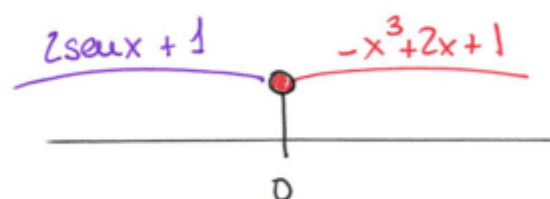
$$f'(x) = \begin{cases} -2x & \text{si } x < 0 \\ -\operatorname{sen} x & \text{si } x > 0 \end{cases}$$

Si $x = 0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = -2 \cdot 0 = 0 \\ f'(0^+) = -\operatorname{sen}(0) = 0 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$.

$$b) f(x) = \begin{cases} 2\operatorname{sen} x + 1 & \text{si } x < 0 \\ -x^3 + 2x + 1 & \text{si } x \geq 0 \end{cases}$$



Si $x < 0 \rightarrow f(x) = 2\operatorname{sen} x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0 \rightarrow f(x) = -x^3 + 2x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x=0$ Continuidad:

$$f(0) = -0^3 + 2 \cdot 0 + 1 = 1$$

$$\lim_{x \rightarrow 0} f(x) \Rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (2\sec x + 1) = 1 \\ \lim_{x \rightarrow 0^+} (-x^3 + 2x + 1) = 1 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 1$$

Como $f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow$ Continua en $x=0$

Para la derivabilidad:

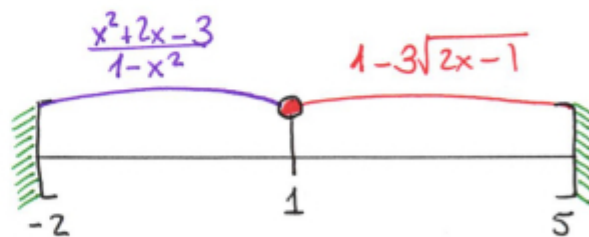
$$f'(x) = \begin{cases} 2\cos x & \text{si } x < 0 \\ -3x^2 + 2 & \text{si } x > 0 \end{cases}$$

Si $x=0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = 2\cos 0 = 2 \\ f'(0^+) = -3 \cdot 0 + 2 = 2 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x=0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

$$c) f(x) = \begin{cases} \frac{x^2+2x-3}{1-x^2} & \text{si } -2 \leq x < 1 \\ 1-3\sqrt{2x-1} & \text{si } 1 \leq x \leq 5 \end{cases}$$



Si $-2 \leq x < 1 \rightarrow f(x) = \frac{x^2+2x-3}{1-x^2} \Rightarrow$ Continua y derivable en $\mathbb{R} - \{-1, 1\}$

$\hookrightarrow$ Continua y derivable en $[-2, -1) \cup (-1, 1)$

SI QUE
AFECTA

NO
AFECTA

En $x = -1$ hay una discontinuidad:

$$\lim_{x \rightarrow -1} \frac{x^2 + 2x - 3}{1 - x^2} = \left[\frac{-4}{0} \right] \rightarrow \begin{cases} \lim_{x \rightarrow -1^-} \frac{x^2 + 2x - 3}{1 - x^2} = +\infty \\ \lim_{x \rightarrow -1^+} \frac{x^2 + 2x - 3}{1 - x^2} = -\infty \end{cases} \neq \lim_{x \rightarrow -1} \frac{x^2 + 2x - 3}{1 - x^2}$$

En $x = -1$ hay una discontinuidad de salto infinito y por tanto $f(x)$ no es derivable en $x = -1$

Si $1 < x \leq 5$ $\rightarrow f(x) = 1 - 3\sqrt{2x-1} \Rightarrow$ Continua en $[\frac{1}{2}, +\infty)$ y derivable en $(\frac{1}{2}, +\infty)$

$\hookrightarrow$ Continua y derivable en $(1, 5]$

Si $x = 1$ Continuidad:

$$f(1) = 1 - 3\sqrt{1} = -2$$

$$\lim_{x \rightarrow 1} f(x) \rightarrow \begin{cases} \lim_{x \rightarrow 1^-} \frac{x^2 + 2x - 3}{1 - x^2} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 1^-} \frac{(x-1)(x+3)}{(1-x)(1+x)} = \\ = \lim_{x \rightarrow 1^-} (-1) \cdot \left(\frac{x+3}{1+x} \right) = -2 \\ \lim_{x \rightarrow 1^+} (1 - 3\sqrt{2x-1}) = -2 \end{cases} \Rightarrow \lim_{x \rightarrow 1} f(x) = -2$$

Como $f(1) = \lim_{x \rightarrow 1} f(x) \Rightarrow$ Continua en $x = 1$

Para la derivabilidad:

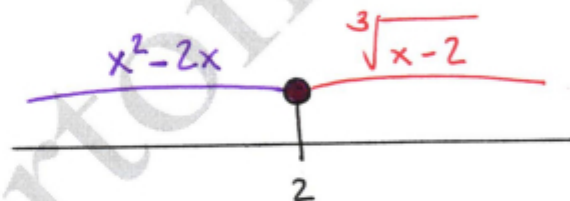
$$f'(x) = \begin{cases} \frac{2}{(1+x)^2} & \text{si } -2 \leq x < 1 \text{ con } x \neq -1 \\ \frac{-3}{\sqrt{2x-1}} & \text{si } 1 < x \leq 5 \end{cases}$$

Si $x = 1$ Derivabilidad:

$$\left. \begin{aligned} f'(1^-) &= \frac{2}{(1+1)^2} = \frac{1}{2} \\ f'(1^+) &= \frac{-3}{\sqrt{1}} = -3 \end{aligned} \right\} f'(1^-) \neq f'(1^+) \Rightarrow \nexists f'(1) \Rightarrow \text{No derivable en } x = 1$$

$$\Rightarrow \begin{cases} f(x) \text{ es continua en } [-2, -1) \cup (-1, 5] \\ f(x) \text{ es derivable en } [-2, -1) \cup (-1, 1) \cup (1, 5] \end{cases}$$

$$d) f(x) = \begin{cases} x(x-2) & \text{si } x \leq 2 \\ \sqrt[3]{x-2} & \text{si } x > 2 \end{cases}$$



Si $x < 2 \rightarrow f(x) = x^2 - 2x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 2)$

Si $x > 2 \rightarrow f(x) = \sqrt[3]{x-2} \Rightarrow$ Continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{2\}$

NO AFECTA

$\hookrightarrow$ Continua y derivable en $(2, +\infty)$

Si $x = 2$ Continuidad:

$$f(2) = 2^2 - 2 \cdot 2 = 0$$

$$\lim_{x \rightarrow 2} f(x) \rightarrow \left\{ \begin{aligned} \lim_{x \rightarrow 2^-} (x^2 - 2x) &= 0 \\ \lim_{x \rightarrow 2^+} (\sqrt[3]{x-2}) &= 0 \end{aligned} \right\} \Rightarrow \lim_{x \rightarrow 2} f(x) = 0$$

Como $f(2) = \lim_{x \rightarrow 2} f(x) \Rightarrow$ Continua en $x = 2$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2x-2 & \text{si } x < 2 \\ \frac{1}{3\sqrt[3]{(x-2)^2}} & \text{si } x > 2 \end{cases}$$

Si $x = 2$ Derivabilidad:

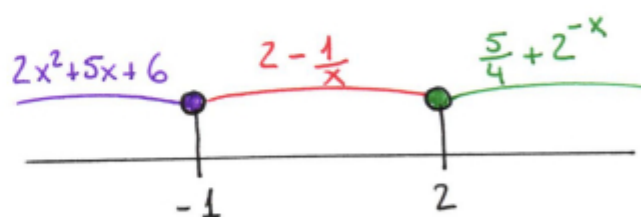
$$f'(2^-) = 2 \cdot 2 - 2 = 2$$

$$\begin{aligned} f'(2^+) &= \lim_{h \rightarrow 0^+} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0^+} \frac{\sqrt[3]{2+h-2} - 0}{h} = \\ &= \lim_{h \rightarrow 0^+} \frac{\sqrt[3]{h}}{h} \cdot \frac{\sqrt[3]{h^2}}{\sqrt[3]{h^2}} = \lim_{h \rightarrow 0^+} \frac{1}{h \sqrt[3]{h^2}} = \lim_{h \rightarrow 0^+} \frac{1}{\sqrt[3]{h^3}} = +\infty \end{aligned}$$

$$\Rightarrow f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow \text{No derivable en } x = 2$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{2\}$

$$e) f(x) = \begin{cases} 2x^2 + 5x + 6 & \text{si } x \leq -1 \\ 2 - \frac{1}{x} & \text{si } -1 < x < 2 \\ \frac{5}{4} + 2^{-x} & \text{si } x \geq 2 \end{cases}$$



Si $x < -1 \rightarrow f(x) = 2x^2 + 5x + 6 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable $(-\infty, -1)$

(Si $-1 < x < 2$) $\rightarrow f(x) = 2 - \frac{1}{x} \Rightarrow$ Continua y derivable en $\mathbb{R} - \{0\}$

SI QUE AFECTA

$\hookrightarrow$ Continua y derivable en $(-1, 0) \cup (0, 2)$

En $x=0$ hay una discontinuidad:

$$\lim_{x \rightarrow 0} 2 - \frac{1}{x} = \lim_{x \rightarrow 0} \frac{2x-1}{x} = \left[\frac{-1}{0} \right] \rightarrow \begin{cases} \lim_{x \rightarrow 0^-} \frac{2x-1}{x} = +\infty \\ \lim_{x \rightarrow 0^+} \frac{2x-1}{x} = -\infty \end{cases}$$

En $x=0$ hay una discontinuidad de salto infinito y por lo tanto, $f(x)$ no será derivable en $x=0$

(Si $x > 2$) $\rightarrow f(x) = \frac{5}{4} + 2^{-x} \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(2, +\infty)$

(Si $x = -1$) Continuidad:

$$f(-1) = 2 \cdot (-1)^2 + 5 \cdot (-1) + 6 = 3$$

$$\lim_{x \rightarrow -1} f(x) \rightarrow \begin{cases} \lim_{x \rightarrow -1^-} (2x^2 + 5x + 6) = 3 \\ \lim_{x \rightarrow -1^+} (2 - \frac{1}{x}) = 3 \end{cases} \Rightarrow \lim_{x \rightarrow -1} f(x) = 3$$

Como $f(-1) = \lim_{x \rightarrow -1} f(x) \Rightarrow$ Continua en $x = -1$

(Si $x = 2$) Continuidad:

$$f(2) = \frac{5}{4} + 2^{-2} = \frac{3}{2}$$

$$\lim_{x \rightarrow 2} f(x) \rightarrow \begin{cases} \lim_{x \rightarrow 2^-} (2 - \frac{1}{x}) = \frac{3}{2} \\ \lim_{x \rightarrow 2^+} (\frac{5}{4} + 2^{-x}) = \frac{3}{2} \end{cases} \Rightarrow \lim_{x \rightarrow 2} f(x) = \frac{3}{2}$$

Como $f(2) = \lim_{x \rightarrow 2} f(x) \Rightarrow$ Continua en $x = 2$

Para la derivabilidad:

$$f'(x) = \begin{cases} 4x+5 & \text{si } x < -1 \\ \frac{1}{x^2} & \text{si } -1 < x < 2 \text{ con } x \neq 0 \\ -2^{-x} \cdot \ln 2 & \text{si } x > 2 \end{cases}$$

(Si $x = -1$) Derivabilidad:

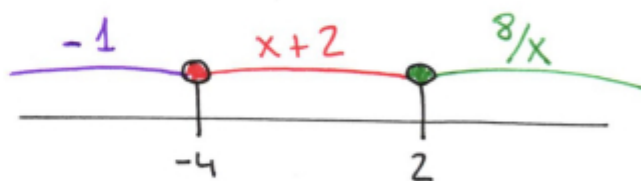
$$\left. \begin{aligned} f'(-1^-) &= 4 \cdot (-1) + 5 = 1 \\ f'(-1^+) &= \frac{1}{(-1)^2} = 1 \end{aligned} \right\} f'(-1^-) = f'(-1^+) \Rightarrow \text{Derivable en } x = -1$$

(Si $x = 2$) Derivabilidad:

$$\left. \begin{aligned} f'(2^-) &= \frac{1}{2^2} = \frac{1}{4} \\ f'(2^+) &= -\frac{1}{4} \cdot \ln 2 \end{aligned} \right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow \text{No derivable en } x = 2$$

$\Rightarrow f(x)$ es continua en $\mathbb{R} - \{0\}$ y derivable en $\mathbb{R} - \{0, 2\}$

$$f) f(x) = \begin{cases} -1 & \text{si } x < -4 \\ x+2 & \text{si } -4 \leq x < 2 \\ \frac{8}{x} & \text{si } x \geq 2 \end{cases}$$



(Si $x < -4$) $\rightarrow f(x) = -1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, -4)$

(Si $-4 < x < 2$) $\rightarrow f(x) = x+2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-4, 2)$

(Si $x > 2$) $\rightarrow f(x) = \frac{8}{x} \Rightarrow$ Continua y derivable en $\mathbb{R} - \{0\}$

$\hookrightarrow$ Continua y derivable en $(2, +\infty)$

$\hookrightarrow$ NO AFECTA

(Si $x = -4$) Continuidad:

$$f(-4) = -4 + 2 = -2$$

$$\lim_{x \rightarrow -4} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -4^-} -1 = -1 \\ \lim_{x \rightarrow -4^+} (x+2) = -2 \end{array} \right\} \Rightarrow \nexists \lim_{x \rightarrow -4} f(x)$$

$f(x)$ presenta en $x = -4$ una discontinuidad de salto finito y por tanto no será derivable en $x = -4$

(Si $x = 2$) Continuidad:

$$f(2) = \frac{8}{2} = 4$$

$$\lim_{x \rightarrow 2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 2^-} (x+2) = 4 \\ \lim_{x \rightarrow 2^+} \frac{8}{x} = 4 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 2} f(x) = 4$$

Como $f(2) = \lim_{x \rightarrow 2} f(x) \Rightarrow$ Continua en $x = 2$

Para la derivabilidad:

$$f'(x) = \begin{cases} 0 & \text{si } x < -4 \\ 1 & \text{si } -4 < x < 2 \\ -\frac{8}{x^2} & \text{si } x > 2 \end{cases}$$

Si $x=2$ Derivabilidad:

$$\left. \begin{array}{l} f'(2^-) = 1 \\ f'(2^+) = -\frac{8}{2^2} = -2 \end{array} \right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow \text{No derivable en } x=2$$

$\Rightarrow f(x)$ es continua en $\mathbb{R} - \{-4\}$ y derivable en $\mathbb{R} - \{-4, 2\}$

25) Calcular el valor de los parámetros para que las siguientes funciones sean derivables en $\mathbb{R}$:

$$a) f(x) = \begin{cases} ax^2 + 3x & \text{si } x \leq 2 \\ x^2 - bx - 4 & \text{si } x > 2 \end{cases}$$

$$b) f(x) = \begin{cases} x^3 - x & \text{si } x \leq 0 \\ ax + b & \text{si } x > 0 \end{cases}$$

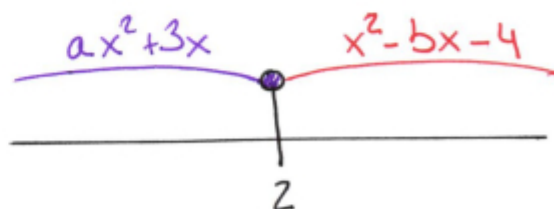
$$c) f(x) = \begin{cases} x^2 - 5x + a & \text{si } x \leq 1 \\ -x^2 + bx & \text{si } x > 1 \end{cases}$$

$$d) f(x) = \begin{cases} ax^2 + bx - 3 & \text{si } x < 1 \\ 2bx - 4 & \text{si } x \geq 1 \end{cases}$$

$$e) f(x) = \begin{cases} (x-1)^3 & \text{si } x \leq 0 \\ ax + b & \text{si } x > 0 \end{cases}$$

$$f) f(x) = \begin{cases} ax^2 + 3x & \text{si } x < -2 \\ x^2 - bx - 4 & \text{si } x \geq -2 \end{cases}$$

$$a) f(x) = \begin{cases} ax^2 + 3x & \text{si } x \leq 2 \\ x^2 - bx - 4 & \text{si } x > 2 \end{cases}$$



Si $x < 2$ $\rightarrow f(x) = ax^2 + 3x \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, 2)$

Si $x > 2$ $\rightarrow f(x) = x^2 - bx - 4 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(2, +\infty)$

Si $x = 2$ Continuidad:

$$f(2) = a \cdot 2^2 + 3 \cdot 2 = 4a + 6$$

$$\lim_{x \rightarrow 2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 2^-} (ax^2 + 3x) = 4a + 6 \\ \lim_{x \rightarrow 2^+} (x^2 - bx - 4) = -2b \end{array} \right\} \quad 4a + 6 = -2b$$

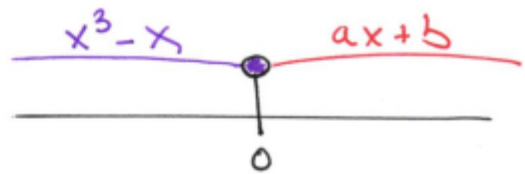
Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} 2ax + 3 & \text{si } x \leq 2 \\ 2x - b & \text{si } x > 2 \end{array} \right. \quad \left\{ \begin{array}{l} f'(2^-) = 4a + 3 \\ f'(2^+) = 4 - b \end{array} \right\} \quad 4a + 3 = 4 - b$$

$f(x)$ será continua y derivable en $x = 2$ (y por tanto en $\mathbb{R}$) cuando se verifique:

$$\left. \begin{array}{l} 4a + 2b = -6 \\ 4a + b = 1 \end{array} \right\} \text{E}_2 - \text{E}_1, \quad \boxed{b = -7} \Rightarrow \boxed{a = 2}$$

$$b) f(x) = \begin{cases} x^3 - x & \text{si } x \leq 0 \\ ax + b & \text{si } x > 0 \end{cases}$$



Si $x < 0 \rightarrow f(x) = x^3 - x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable $(-\infty, 0)$

Si $x > 0 \rightarrow f(x) = ax + b \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = 0^3 - 0 = 0$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \begin{cases} \lim_{x \rightarrow 0^-} (x^3 - x) = 0 \\ \lim_{x \rightarrow 0^+} (ax + b) = b \end{cases} \quad b = 0$$

Para la derivabilidad:

$$f'(x) = \begin{cases} 3x^2 - 1 & \text{si } x \leq 0 \\ a & \text{si } x > 0 \end{cases} \quad \left. \begin{array}{l} f'(0^-) = -1 \\ f'(0^+) = a \end{array} \right\} a = -1$$

$f(x)$ será continua y derivable en $x = 0$ (y por tanto en $\mathbb{R}$) cuando se tenga $a = -1$ y $b = 0$.

$$c) f(x) = \begin{cases} x^2 - 5x + a & \text{si } x \leq 1 \\ -x^2 + bx & \text{si } x > 1 \end{cases}$$



Si $x < 1$ $\rightarrow f(x) = x^2 - 5x + a \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, 1)$

Si $x > 1$ $\rightarrow f(x) = -x^2 + bx \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(1, +\infty)$

Si $x = 1$ Continuidad:

$$f(1) = 1^2 - 5 \cdot 1 + a = a - 4$$

$$\lim_{x \rightarrow 1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 1^-} (x^2 - 5x + a) = a - 4 \\ \lim_{x \rightarrow 1^+} (-x^2 + bx) = b - 1 \end{array} \right\} \quad a - 4 = b - 1$$

Para la derivabilidad:

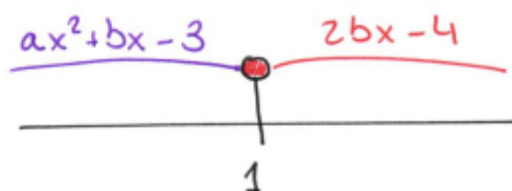
$$f'(x) = \begin{cases} 2x - 5 & \text{si } x \leq 1 \\ -2x + b & \text{si } x > 1 \end{cases} \quad \left. \begin{array}{l} f'(1^-) = -3 \\ f'(1^+) = b - 2 \end{array} \right\} \quad -3 = b - 2$$

$f(x)$ será continua y derivable en $x = 1$ (y por tanto en $\mathbb{R}$)

aunado se verifique:

$$\left. \begin{array}{l} a - b = 3 \\ b = -1 \end{array} \right\} \Rightarrow \boxed{b = -1} \text{ y } \boxed{a = 2}$$

$$d) f(x) = \begin{cases} ax^2 + bx - 3 & \text{si } x < 1 \\ 2bx - 4 & \text{si } x \geq 1 \end{cases}$$



Si $x < 1$ $\rightarrow f(x) = ax^2 + bx - 3 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 1)$

Si $x > 1$ $\rightarrow f(x) = 2bx - 4 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(1, +\infty)$

Si $x = 1$ Continuidad:

$$f(1) = 2b - 4$$

$$\lim_{x \rightarrow 1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 1^-} (ax^2 + bx - 3) = a + b - 3 \\ \lim_{x \rightarrow 1^+} (2bx - 4) = 2b - 4 \end{array} \right\} \quad a + b - 3 = 2b - 4$$

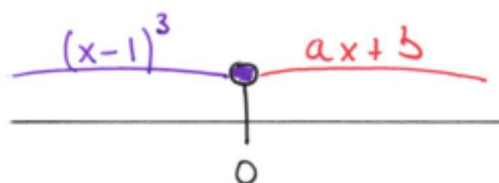
Para la derivabilidad:

$$f'(x) = \begin{cases} 2ax + b & \text{si } x < 1 \\ 2b & \text{si } x \geq 1 \end{cases} \quad \left. \begin{array}{l} f'(1^-) = 2a + b \\ f'(1^+) = 2b \end{array} \right\} 2a + b = 2b$$

$f(x)$ será continua y derivable en $x=1$ (y por tanto en $\mathbb{R}$) cuando se verifique:

$$\left. \begin{array}{l} a - b = -1 \\ 2a - b = 0 \end{array} \right\} E_2 - E_1 \rightarrow \boxed{a = 1} \Rightarrow \boxed{b = 2}$$

$$e) f(x) = \begin{cases} (x-1)^3 & \text{si } x \leq 0 \\ ax+b & \text{si } x > 0 \end{cases}$$



Si $x < 0$ $\rightarrow f(x) = (x-1)^3 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0$ $\rightarrow f(x) = ax+b \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = (0-1)^3 = -1$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (x-1)^3 = -1 \\ \lim_{x \rightarrow 0^+} (ax+b) = b \end{array} \right\} \quad b = -1$$

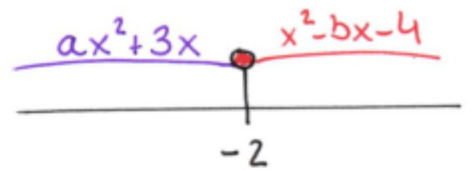
Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} 3(x-1)^2 & \text{si } x \leq 0 \\ a & \text{si } x > 0 \end{array} \right. \quad \left. \begin{array}{l} f'(0^-) = 3 \\ f'(0^+) = a \end{array} \right\} \quad a = 3$$

$f(x)$ será continua y derivable en $x=0$ (y por tanto en $\mathbb{R}$)

cuando se tenga $\boxed{a=3}$ y $\boxed{b=-1}$

$$f) f(x) = \begin{cases} ax^2 + 3x & \text{si } x < -2 \\ x^2 - bx - 4 & \text{si } x \geq -2 \end{cases}$$



Si $x < -2$ $\rightarrow f(x) = ax^2 + 3x \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, -2)$

Si $x > -2$ $\rightarrow f(x) = x^2 - bx - 4 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-2, +\infty)$

Si $x = -2$ Continuidad:

$$f(-2) = (-2)^2 - b \cdot (-2) - 4 = 2b$$

$$\lim_{x \rightarrow -2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -2^-} (ax^2 + 3x) = 4a - 6 \\ \lim_{x \rightarrow -2^+} (x^2 - bx - 4) = 2b \end{array} \right\} \quad 4a - 6 = 2b$$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2ax + 3 & \text{si } x < -2 \\ 2x - b & \text{si } x \geq -2 \end{cases} \quad \left. \begin{array}{l} f'(-2^-) = -4a + 3 \\ f'(-2^+) = -4 - b \end{array} \right\} \quad -4a + 3 = -4 - b$$

$f(x)$ será continua y derivable en $x = -2$ (y por tanto en $\mathbb{R}$) cuando se verifique:

$$\left. \begin{array}{l} 4a - 2b = 6 \\ -4a + b = -7 \end{array} \right\} E_1 + E_2 \rightarrow -b = -1 \Rightarrow \boxed{b = 1} \Rightarrow \boxed{a = 2}$$

26) Determina, si es posible, el valor de los parámetros para que las siguientes funciones sean continuas y derivables en $\mathbb{R}$:

$$a) f(x) = \begin{cases} \cos x & \text{si } x \leq 0 \\ x^2 + a & \text{si } x > 0 \end{cases}$$

$$b) f(x) = \begin{cases} x^2 + ax & \text{si } x \leq -3 \\ \frac{x}{x+4} & \text{si } x > -3 \end{cases}$$

$$c) f(x) = \begin{cases} 2x + e^{1-x} & \text{si } x < 1 \\ 1 + \sqrt{x+m} & \text{si } x \geq 1 \end{cases}$$

$$d) f(x) = \begin{cases} ax^2 + bx + 1 & \text{si } x < 0 \\ a^2 - \sin x & \text{si } x \geq 0 \end{cases}$$

$$e) f(x) = \begin{cases} 1 + a \cdot \sqrt{x+1} \cdot \ln(x+1) & \text{si } x > 0 \\ (x-1)^2 & \text{si } x \leq 0 \end{cases}$$

$$f) f(x) = \begin{cases} (x+b)^2 & \text{si } x < -1 \\ \frac{ax}{\sqrt{x+2}} & \text{si } x \geq -1 \end{cases}$$

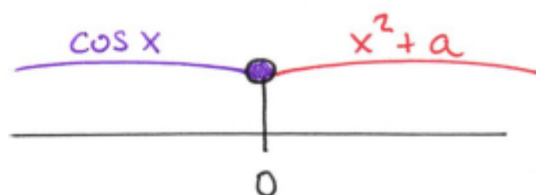
$$g) f(x) = \begin{cases} \ln(e + \sin x) & \text{si } x \leq 0 \\ x^3 + ax + b & \text{si } x > 0 \end{cases}$$

$$h) f(x) = \begin{cases} e^{a(x-\pi)} & \text{si } x < \pi \\ 2a + b \cdot \sin(x-\pi) & \text{si } x \geq \pi \end{cases}$$

$$i) f(x) = \begin{cases} 2e^x + x + a & \text{si } x \leq 0 \\ x^2 + b(x+1) & \text{si } 0 < x \leq 2 \\ x^4 - 3a & \text{si } x > 2 \end{cases}$$

$$j) f(x) = \begin{cases} -x^3 + 7x + c & \text{si } x \leq -2 \\ ax^2 + 3x + 11 & \text{si } -2 < x \leq \frac{3}{2} \\ 18\sqrt{2x+1} + b & \text{si } x > \frac{3}{2} \end{cases}$$

$$a) f(x) = \begin{cases} \cos x & \text{si } x \leq 0 \\ x^2 + a & \text{si } x > 0 \end{cases}$$



Si $x < 0 \rightarrow f(x) = \cos x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0 \rightarrow f(x) = x^2 + a \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = \cos 0 = 1$$

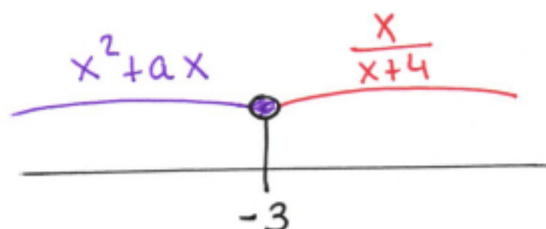
$$\lim_{x \rightarrow 0} f(x) \rightarrow \begin{cases} \lim_{x \rightarrow 0^-} (\cos x) = 1 \\ \lim_{x \rightarrow 0^+} (x^2 + a) = a \end{cases} \quad \boxed{a = 1}$$

Para la derivabilidad:

$$f'(x) = \begin{cases} -\sin x & \text{si } x \leq 0 \\ 2x & \text{si } x > 0 \end{cases} \quad \left. \begin{array}{l} f'(0^-) = -\sin 0 = 0 \\ f'(0^+) = 2 \cdot 0 = 0 \end{array} \right\} f'(0^-) = f'(0^+)$$

Si $\boxed{a = 1}$ $f(x)$ es continua y derivable en $\mathbb{R}$

$$b) f(x) = \begin{cases} x^2 + ax & \text{si } x \leq -3 \\ \frac{x}{x+4} & \text{si } x > -3 \end{cases}$$



Si $x < -3$ $\rightarrow f(x) = x^2 + ax \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, -3)$

Si $x > -3$ $\rightarrow f(x) = \frac{x}{x+4} \Rightarrow$ Continua y derivable en $\mathbb{R} - \{-4\}$

NO AFECTA

$\hookrightarrow$ Continua y derivable en $(-3, +\infty)$

Si $x = -3$ Continuidad:

$$f(-3) = (-3)^2 + a \cdot (-3) = 9 - 3a$$

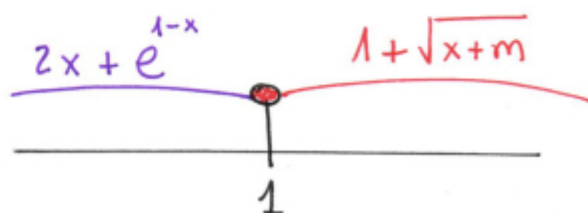
$$\lim_{x \rightarrow -3} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -3^-} (x^2 + ax) = 9 - 3a \\ \lim_{x \rightarrow -3^+} \frac{x}{x+4} = -3 \end{array} \right\} \quad \begin{array}{l} 9 - 3a = -3 \Rightarrow \\ \Rightarrow a = 4 \end{array}$$

Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} 2x + 4 & \text{si } x < -3 \\ \frac{4}{(x+4)^2} & \text{si } x > -3 \end{array} \right. \quad \left. \begin{array}{l} f'(-3^-) = -2 \\ f'(-3^+) = 4 \end{array} \right\} \quad f'(-3^-) \neq f'(-3^+)$$

Por lo tanto, para $a=4$ la función será continua en $\mathbb{R}$ pero no derivable en $x = -3$ (derivable en $\mathbb{R} - \{-3\}$)

$$c) f(x) = \left\{ \begin{array}{ll} 2x + e^{1-x} & \text{si } x < 1 \\ 1 + \sqrt{x+m} & \text{si } x > 1 \end{array} \right.$$



Si $x < 1 \rightarrow f(x) = 2x + e^{1-x} \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 1)$

Si $x > 1 \rightarrow f(x) = 1 + \sqrt{x+m} \Rightarrow$ Continua en $[-m, +\infty)$ y derivable en $(-m, +\infty)$

$m > -1 \rightarrow$ Continua y derivable en $(1, +\infty)$

Si $x = 1$ Continuidad:

$$f(1) = 1 + \sqrt{1+m}$$

$$\lim_{x \rightarrow 1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 1^-} (2x + e^{1-x}) = 3 \\ \lim_{x \rightarrow 1^+} (1 + \sqrt{x+m}) = 1 + \sqrt{1+m} \end{array} \right\} \quad 3 = 1 + \sqrt{1+m}$$

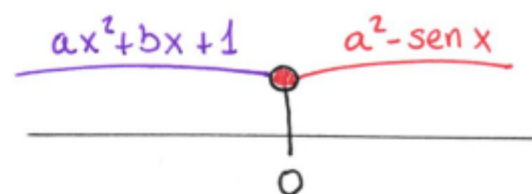
$$2 = \sqrt{1+m} \Rightarrow 4 = 1+m \Rightarrow m = 3$$

Veamos la derivabilidad para ese valor:

$$f'(x) = \left\{ \begin{array}{ll} 2 - e^{1-x} & \text{si } x < 1 \\ \frac{1}{2\sqrt{x+3}} & \text{si } x > 1 \end{array} \right. \quad \left\{ \begin{array}{l} f'(1^-) = 1 \\ f'(1^+) = \frac{1}{4} \end{array} \right\} \quad f'(1^-) \neq f'(1^+)$$

Por lo tanto, para $m = 3$ la función será continua en $x = 1$ (y por tanto en $\mathbb{R}$) pero no será derivable en $x = 1$.

$$d) f(x) = \begin{cases} ax^2 + bx + 1 & \text{si } x < 0 \\ a^2 - \operatorname{sen} x & \text{si } x \geq 0 \end{cases}$$



(Si $x < 0$) $\rightarrow f(x) = ax^2 + bx + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

(Si $x > 0$) $\rightarrow f(x) = a^2 - \operatorname{sen} x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

(Si $x = 0$) Continuidad:

$$f(0) = a^2 - \operatorname{sen}(0) = a^2$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (ax^2 + bx + 1) = 1 \\ \lim_{x \rightarrow 0^+} (a^2 - \operatorname{sen}(x)) = a^2 \end{array} \right\} \quad a^2 = 1$$

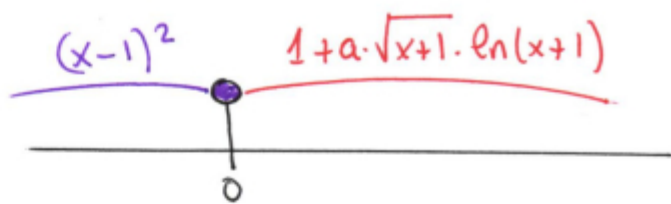
Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} 2ax + b & \text{si } x < 0 \\ -\cos x & \text{si } x \geq 0 \end{array} \right\} \quad \left. \begin{array}{l} f'(0^-) = b \\ f'(0^+) = -1 \end{array} \right\} \quad b = -1$$

Para que sea continua y derivable en $\mathbb{R}$ se tendrá que cumplir:

$$a^2 = 1 \quad \begin{cases} \rightarrow a = -1 \\ \rightarrow a = +1 \end{cases} \quad \text{y } b = -1$$

$$e) f(x) = \begin{cases} 1 + a \cdot \sqrt{x+1} \cdot \ln(x+1) & \text{si } x > 0 \\ (x-1)^2 & \text{si } x \leq 0 \end{cases}$$



Si $x < 0$ $\rightarrow f(x) = (x-1)^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0$ $\rightarrow f(x) = 1 + a \cdot \sqrt{x+1} \cdot \ln(x+1) \Rightarrow$ Continua y derivable en $(-1, +\infty)$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = (0-1)^2 = 1$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (x-1)^2 = 1 \\ \lim_{x \rightarrow 0^+} (1 + a \cdot \sqrt{x+1} \cdot \ln(x+1)) = 1 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 1$$

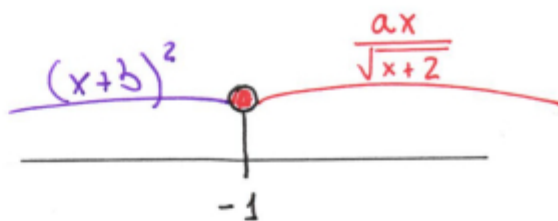
Como $f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow$ Continua en $x=0$ ($\forall a \in \mathbb{R}$)

Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{l} 2(x-1) \text{ si } x \leq 0 \\ \frac{a}{\sqrt{x+1}} \left(\frac{1}{2} \ln(x+1) + 1 \right) \text{ si } x > 0 \end{array} \right. \quad \left. \begin{array}{l} f'(0^-) = -2 \\ f'(0^+) = \frac{a}{\sqrt{1}} \cdot 1 \end{array} \right\} a = -2$$

$\Rightarrow f(x)$ es continua en $\mathbb{R} \forall a \in \mathbb{R}$ y será derivable en $\mathbb{R}$ cuando sea $a = -2$

$$f(x) = \begin{cases} (x+b)^2 & \text{si } x < -1 \\ \frac{ax}{\sqrt{x+2}} & \text{si } x > -1 \end{cases}$$



Si $x < -1 \rightarrow f(x) = (x+b)^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, -1)$

Si $x > -1 \rightarrow f(x) = \frac{ax}{\sqrt{x+2}} \Rightarrow$ Continua y derivable en $(-2, +\infty)$

$\hookrightarrow$ Continua y derivable en $(-1, +\infty)$

Si $x = -1$ Continuidad:

$$f(-1) = \frac{a \cdot (-1)}{\sqrt{1}} = -a$$

$$\lim_{x \rightarrow -1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -1^-} (x+b)^2 = (b-1)^2 \\ \lim_{x \rightarrow -1^+} \frac{ax}{\sqrt{x+2}} = -a \end{array} \right\} \quad (b-1)^2 = -a$$

Para la derivabilidad:

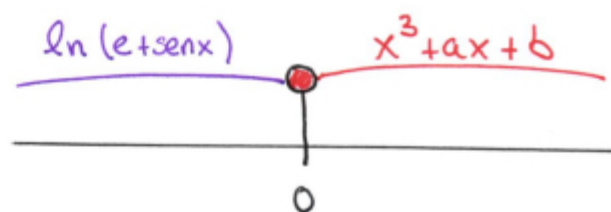
$$f'(x) = \left\{ \begin{array}{l} 2(x+b) \text{ si } x < -1 \\ \frac{ax+4a}{2\sqrt{(x+2)^3}} \text{ si } x > -1 \end{array} \right\} \quad \left\{ \begin{array}{l} f'(-1^-) = 2(b-1) \\ f'(-1^+) = \frac{3a}{2} \end{array} \right\} \quad \left\{ \begin{array}{l} 2(b-1) = \frac{3a}{2} \end{array} \right.$$

Para que sea continua y derivable en $\mathbb{R}$ se tendrá que cumplir:

$$\left\{ \begin{array}{l} (b-1)^2 = -a \\ 2(b-1) = \frac{3a}{2} \end{array} \right\} \quad \left(\frac{3a}{4} \right)^2 = -a \Rightarrow \frac{9a^2}{16} + a = 0 \Rightarrow a \left(\frac{9a}{16} + 1 \right) = 0$$

$$\begin{cases} a = 0 \Rightarrow b = 1 \\ a = -\frac{16}{9} \Rightarrow b = -\frac{1}{3} \end{cases}$$

$$g) f(x) = \begin{cases} \ln(e + \operatorname{sen} x) & \text{si } x < 0 \\ x^3 + ax + b & \text{si } x \geq 0 \end{cases}$$



Si $x < 0$ $\rightarrow f(x) = \ln(e + \operatorname{sen} x) \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0$ $\rightarrow f(x) = x^3 + ax + b \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = 0^3 + 0a + b = b$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} \ln(e + \operatorname{sen} x) = 1 \\ \lim_{x \rightarrow 0^+} (x^3 + ax + b) = b \end{array} \right\} b = 1$$

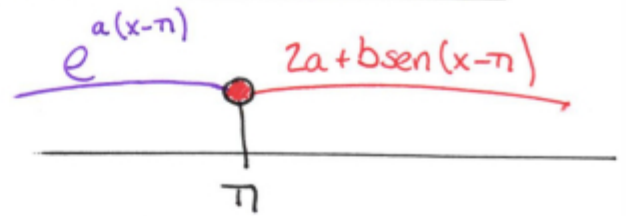
Para la derivabilidad:

$$f'(x) = \begin{cases} \frac{\cos x}{e + \operatorname{sen} x} & \text{si } x < 0 \\ 3x^2 + a & \text{si } x \geq 0 \end{cases} \quad \left. \begin{array}{l} f'(0^-) = 1/e \\ f'(0^+) = a \end{array} \right\} a = \frac{1}{e}$$

$f(x)$ será continua y derivable en $x = 0$ (y por tanto en $\mathbb{R}$)

cundo se tenga $a = \frac{1}{e}$ y $b = 1$

$$h) f(x) = \begin{cases} e^{a(x-\pi)} & \text{si } x < \pi \\ 2a + b \sin(x-\pi) & \text{si } x > \pi \end{cases}$$



Si $x < \pi$ $\rightarrow f(x) = e^{a(x-\pi)} \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, \pi)$

Si $x > \pi$ $\rightarrow f(x) = 2a + b \sin(x-\pi) \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(\pi, +\infty)$

Si $x = \pi$ Continuidad:

$$f(\pi) = 2a + b \cdot \sin 0 = 2a$$

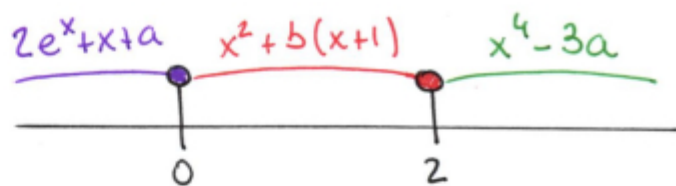
$$\lim_{x \rightarrow \pi} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow \pi^-} e^{a(x-\pi)} = e^0 = 1 \\ \lim_{x \rightarrow \pi^+} (2a + b \sin(x-\pi)) = 2a \end{array} \right\} 2a = 1$$

Para la derivabilidad:

$$f'(x) = \begin{cases} a \cdot e^{a(x-\pi)} & \text{si } x < \pi \\ b \cdot \cos(x-\pi) & \text{si } x > \pi \end{cases} \quad \left. \begin{array}{l} f'(\pi^-) = a \\ f'(\pi^+) = b \end{array} \right\} a = b$$

$f(x)$ será continua y derivable en $x = \pi$ (y por tanto en $\mathbb{R}$)
cuando se tenga $a = b = \frac{1}{2}$

$$i) f(x) = \begin{cases} 2e^x + x + a & \text{si } x \leq 0 \\ x^2 + b(x+1) & \text{si } 0 < x \leq 2 \\ x^4 - 3a & \text{si } x > 2 \end{cases}$$



Si $x < 0 \rightarrow f(x) = 2e^x + x + a \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $0 < x < 2 \rightarrow f(x) = x^2 + b(x+1) \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, 2)$

Si $x > 2 \rightarrow f(x) = x^4 - 3a \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(2, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = 2 \cdot e^0 + 0 + a = a + 2$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (2e^x + x + a) = a + 2 \\ \lim_{x \rightarrow 0^+} (x^2 + b(x+1)) = b \end{array} \right\} \quad a + 2 = b$$

Si $x = 2$ Continuidad:

$$f(2) = 2^2 + b \cdot (2+1) = 3b + 4$$

$$\lim_{x \rightarrow 2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 2^-} (x^2 + b(x+1)) = 3b + 4 \\ \lim_{x \rightarrow 2^+} (x^4 - 3a) = 16 - 3a \end{array} \right\} \quad 3b + 4 = 16 - 3a$$

La función será continua en $x=0$ y en $x=2$ (y por tanto en $\mathbb{R}$) cuando se verifique:

$$\left. \begin{array}{l} a - b = -2 \\ 3a + 3b = 12 \end{array} \right\} \left. \begin{array}{l} a - b = -2 \\ a + b = 4 \end{array} \right\} 2a = 2 \Rightarrow a = 1 \Rightarrow b = 3$$

y para esos valores, estudiemos la derivabilidad:

$$f'(x) = \begin{cases} 2e^x + 1 & \text{si } x < 0 \\ 2x + 3 & \text{si } 0 < x < 2 \\ 4x^3 & \text{si } x > 2 \end{cases}$$

Si $x = 0$ Derivabilidad:

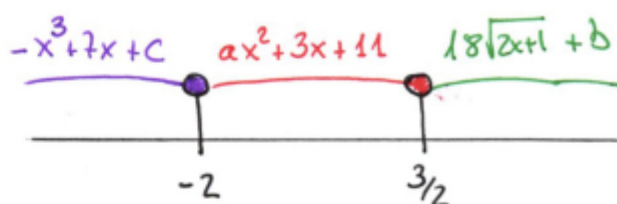
$$\left. \begin{array}{l} f'(0^-) = 2 \cdot e^0 + 1 = 3 \\ f'(0^+) = 2 \cdot 0 + 3 = 3 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

Si $x = 2$ Derivabilidad:

$$\left. \begin{array}{l} f'(2^-) = 2 \cdot 2 + 3 = 7 \\ f'(2^+) = 4 \cdot 2^3 = 32 \end{array} \right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow \text{No derivable en } x = 2$$

$\Rightarrow$ Para $a = 1$ y $b = 3$ $f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{2\}$

$$j) f(x) = \begin{cases} -x^3 + 7x + c & \text{si } x \leq -2 \\ ax^2 + 3x + 11 & \text{si } -2 < x \leq 3/2 \\ 18\sqrt{2x+1} + b & \text{si } x > 3/2 \end{cases}$$



Si $x < -2 \rightarrow f(x) = -x^3 + 7x + c \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, -2)$

Si $-2 < x < 3/2 \rightarrow f(x) = ax^2 + 3x + 11 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-2, 3/2)$

Si $x > 3/2 \rightarrow f(x) = 18\sqrt{2x+1} + b \Rightarrow$ Continua en $[-1/2, +\infty)$ y derivable en $(-1/2, +\infty)$
 $\hookrightarrow$ Continua y derivable en $(3/2, +\infty)$

Si $x = -2$ Continuidad:

$$f(-2) = -(-2)^3 + 7 \cdot (-2) + c = c - 6$$

$$\lim_{x \rightarrow -2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -2^-} (-x^3 + 7x + c) = c - 6 \\ \lim_{x \rightarrow -2^+} (ax^2 + 3x + 11) = 4a + 5 \end{array} \right\} \quad c - 6 = 4a + 5$$

Si $x = 3/2$ Continuidad:

$$f(3/2) = a \cdot \left(\frac{3}{2}\right)^2 + 3 \cdot \frac{3}{2} + 11 = \frac{9a}{4} + \frac{31}{2}$$

$$\lim_{x \rightarrow 3/2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 3/2^-} (ax^2 + 3x + 11) = \frac{9a}{4} + \frac{31}{2} \\ \lim_{x \rightarrow 3/2^+} 18\sqrt{2x+1} + b = 36 + b \end{array} \right\} \quad \frac{9a}{4} + \frac{31}{2} = 36 + b$$

Para la derivabilidad:

$$f'(x) = \begin{cases} -3x^2 + 7 & \text{si } x \leq -2 \\ 2ax + 3 & \text{si } -2 < x \leq 3/2 \\ \frac{18}{\sqrt{2x+1}} & \text{si } x > 3/2 \end{cases}$$

Si $x = -2$ Derivabilidad:

$$\left. \begin{aligned} f'(-2^-) &= -3 \cdot (-2)^2 + 7 = -5 \\ f'(-2^+) &= 2a \cdot (-2) + 3 = 3 - 4a \end{aligned} \right\} -5 = 3 - 4a$$

Si $x = 3/2$ Derivabilidad:

$$\left. \begin{aligned} f'(3/2^-) &= 2a \cdot \frac{3}{2} + 3 = 3a + 3 \\ f'(3/2^+) &= \frac{18}{\sqrt{2 \cdot \frac{3}{2} + 1}} = 9 \end{aligned} \right\} 3a + 3 = 9$$

$f(x)$ será continua y derivable en $\mathbb{R}$ cuando se verifique que:

$$\left. \begin{aligned} -4a + c &= 11 \\ 9a - 4b &= 82 \\ 3 - 4a &= -5 \\ 3a + 3 &= 9 \end{aligned} \right\} \begin{aligned} &\rightarrow -8 + c = 11 \Rightarrow c = 19 \\ &\rightarrow 18 - 4b = 82 \Rightarrow b = -16 \\ &\rightarrow 4a = 8 \Rightarrow a = 2 \\ &\rightarrow 3a = 6 \Rightarrow a = 2 \end{aligned}$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$ si $a = 2$, $b = -16$, y $c = 19$

27) Determina el valor de los parámetros para que las siguientes funciones sean continuas en $\mathbb{R}$. Para los valores obtenidos, estudia la derivabilidad:

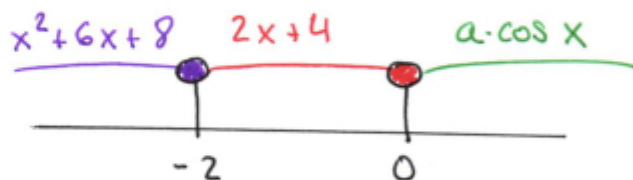
$$a) f(x) = \begin{cases} x^2 + 6x + 8 & \text{si } x \leq -2 \\ 2x + 4 & \text{si } -2 < x \leq 0 \\ a \cdot \cos x & \text{si } x > 0 \end{cases}$$

$$b) f(x) = \begin{cases} \sin x & \text{si } x \leq 0 \\ x - ax^2 & \text{si } x > 0 \end{cases}$$

$$c) f(x) = \begin{cases} 3x + 2 & \text{si } x < 0 \\ x^2 + 2a \cos x & \text{si } 0 \leq x < \pi \\ ax^2 + b & \text{si } x \geq \pi \end{cases}$$

$$d) f(x) = \begin{cases} 2x + a & \text{si } x < -1 \\ ax + b & \text{si } -1 \leq x < 0 \\ 3x^2 + 2 & \text{si } x \geq 0 \end{cases}$$

$$a) f(x) = \begin{cases} x^2 + 6x + 8 & \text{si } x \leq -2 \\ 2x + 4 & \text{si } -2 < x \leq 0 \\ a \cdot \cos x & \text{si } x > 0 \end{cases}$$



Si $x < -2 \rightarrow f(x) = x^2 + 6x + 8 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-\infty, -2)$

Si $-2 < x < 0 \rightarrow f(x) = 2x + 4 \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(-2, 0)$

Si $x > 0 \rightarrow f(x) = a \cdot \cos x \Rightarrow$ Continua y derivable en $\mathbb{R}$
 $\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = -2$ Continuidad:

$$f(-2) = (-2)^2 + 6 \cdot (-2) + 8 = 0$$

$$\lim_{x \rightarrow -2} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -2^-} (x^2 + 6x + 8) = 0 \\ \lim_{x \rightarrow -2^+} (2x + 4) = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow -2} f(x) = 0$$

Como $f(-2) = \lim_{x \rightarrow -2} f(x) \Rightarrow$ Continua en $x = -2$

Si $x = 0$ Continuidad:

$$f(0) = 2 \cdot 0 + 4 = 4$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (2x + 4) = 4 \\ \lim_{x \rightarrow 0^+} a \cdot \cos x = a \end{array} \right\} a = 4$$

Si $a = 4 \Rightarrow \lim_{x \rightarrow 0} f(x) = 4 = f(0) \Rightarrow$ Continua en $x = 0$, y por tanto en $\mathbb{R}$

Y para $a = 4$, veamos la derivabilidad:

$$f'(x) = \begin{cases} 2x + 6 & \text{si } x < -2 \\ 2 & \text{si } -2 < x < 0 \\ -4 \operatorname{sen} x & \text{si } x > 0 \end{cases}$$

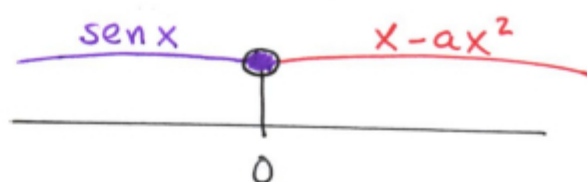
Si $x = -2$ Derivabilidad:

$$\left. \begin{array}{l} f'(-2^-) = -4 + 6 = 2 \\ f'(-2^+) = 2 \end{array} \right\} f'(-2^-) = f'(-2^+) \Rightarrow \text{Derivable en } x = -2$$

Si $x = 0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = 2 \\ f'(0^+) = -4 \cdot \operatorname{sen} 0 = 0 \end{array} \right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow \text{No derivable en } x = 0$$

$$b) f(x) = \begin{cases} \sin x & \text{si } x \leq 0 \\ x - ax^2 & \text{si } x > 0 \end{cases}$$



Si $x < 0$ $\rightarrow f(x) = \sin x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x > 0$ $\rightarrow f(x) = x - ax^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$$f(0) = \sin(0) = 0$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} \sin x = 0 \\ \lim_{x \rightarrow 0^+} x - ax^2 = 0 \end{array} \right\} \Rightarrow \lim_{x \rightarrow 0} f(x) = 0$$

Como $f(0) = \lim_{x \rightarrow 0} f(x)$, $f(x)$ es continua en $x=0$ (y en $\mathbb{R}$) $\forall a \in \mathbb{R}$

Veamos la derivabilidad:

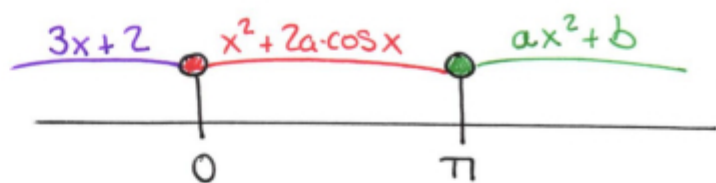
$$f'(x) = \begin{cases} \cos x & \text{si } x \leq 0 \\ 1 - 2ax & \text{si } x > 0 \end{cases}$$

Si $x = 0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = \cos 0 = 1 \\ f'(0^+) = 1 - 2a \cdot 0 = 1 \end{array} \right\} \Rightarrow f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$ $\forall a \in \mathbb{R}$

$$c) f(x) = \begin{cases} 3x+2 & \text{si } x < 0 \\ x^2+2a\cos x & \text{si } 0 \leq x < \pi \\ ax^2+b & \text{si } x \geq \pi \end{cases}$$



(Si $x < 0$) $\rightarrow f(x) = 3x+2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

(Si $0 < x < \pi$) $\rightarrow f(x) = x^2+2a\cos x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, \pi)$

(Si $x \geq \pi$) $\rightarrow f(x) = ax^2+b \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(\pi, +\infty)$

(Si $x = 0$) Continuidad:

$$f(0) = 0^2 + 2a \cdot \cos 0 = 2a$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (3x+2) = 2 \\ \lim_{x \rightarrow 0^+} (x^2+2a\cos x) = 2a \end{array} \right\} \quad 2 = 2a \Rightarrow a = 1$$

(Si $x = \pi$) Continuidad:

$$f(\pi) = a\pi^2 + b$$

$$\lim_{x \rightarrow \pi} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow \pi^-} (x^2+2a\cos x) = \pi^2 - 2a \\ \lim_{x \rightarrow \pi^+} (ax^2+b) = a\pi^2 + b \end{array} \right\} \quad \pi^2 - 2a = a\pi^2 + b$$

$f(x)$ será continua en $\mathbb{R}$ si:

$$\left. \begin{array}{l} a = 1 \\ b = \pi^2 - 2a - a\pi^2 \end{array} \right\} \Rightarrow a = 1 \text{ y } b = -2$$

Y para esos valores, estudiemos la derivabilidad:

$$f'(x) = \begin{cases} 3 & \text{si } x < 0 \\ 2x - 2\sin x & \text{si } 0 < x < \pi \\ 2x & \text{si } x > \pi \end{cases}$$

Si $x = 0$ Derivabilidad:

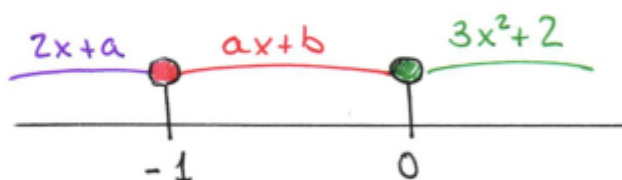
$$\left. \begin{aligned} f'(0^-) &= 3 \\ f'(0^+) &= 2 \cdot 0 - 2\sin 0 = 0 \end{aligned} \right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow \text{No derivable en } x = 0$$

Si $x = \pi$ Derivabilidad:

$$\left. \begin{aligned} f'(\pi^-) &= 2\pi - 2 \cdot \sin \pi = 2\pi \\ f'(\pi^+) &= 2\pi \end{aligned} \right\} f'(\pi^-) = f'(\pi^+) \Rightarrow \text{Derivable en } x = \pi$$

$\Rightarrow$ Para $a=1$ y $b=2$, $f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{0\}$

$$d) f(x) = \begin{cases} 2x+a & \text{si } x < -1 \\ ax+b & \text{si } -1 \leq x < 0 \\ 3x^2+2 & \text{si } x \geq 0 \end{cases}$$



Si $x < -1 \rightarrow f(x) = 2x+a \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua en $(-\infty, -1)$ y derivable

Si $-1 < x < 0 \rightarrow f(x) = ax+b \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-1, 0)$

Si $x > 0 \rightarrow f(x) = 3x^2+2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x = -1$ Continuidad:

$$f(-1) = a \cdot (-1) + b = b - a$$

$$\lim_{x \rightarrow -1} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow -1^-} (2x + a) = a - 2 \\ \lim_{x \rightarrow -1^+} (ax + b) = b - a \end{array} \right\} \quad a - 2 = b - a$$

Si $x = 0$ Continuidad:

$$f(0) = 3 \cdot 0^2 + 2 = 2$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim_{x \rightarrow 0^-} (ax + b) = b \\ \lim_{x \rightarrow 0^+} (3x^2 + 2) = 2 \end{array} \right\} \quad b = 2$$

$f(x)$ será continua en $\mathbb{R}$ si:

$$\left. \begin{array}{l} 2a - b = 2 \\ b = 2 \end{array} \right\} \Rightarrow a = 2 \text{ y } b = 2$$

y para esos valores, estudiemos la derivabilidad:

$$f'(x) = \begin{cases} 2 & \text{si } x < -1 \\ 2 & \text{si } -1 < x < 0 \\ 6x & \text{si } x > 0 \end{cases}$$

donde vemos que $f'(-1^-) = f'(-1^+) = 2 \Rightarrow$ Derivable en $x = -1$

y donde igualmente vemos que $\left\{ \begin{array}{l} f'(0^-) = 2 \\ f'(0^+) = 0 \end{array} \right\} \Rightarrow \nexists f'(0)$

$\Rightarrow$ Para $a = 2$ y $b = 2$, $f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{0\}$

