

EJERCICIOS RESUELTOS

① Expresa en grados o radianes los siguientes ángulos:

$$60^\circ \times \frac{2\pi \text{ rad}}{360^\circ} = \frac{\pi}{3} \text{ rad}$$

$$\frac{5\pi}{4} \text{ rad} \times \frac{360^\circ}{2\pi \text{ rad}} = 225^\circ$$

$$80^\circ \times \frac{2\pi \text{ rad}}{360^\circ} = \frac{4\pi}{9} \text{ rad}$$

$$\frac{11\pi}{6} \text{ rad} \times \frac{360^\circ}{2\pi \text{ rad}} = 330^\circ$$

② Halla el valor de las siguientes expresiones:

a) $\cos 30 - \operatorname{sen} 60 + \operatorname{tg} 45$

c) $\operatorname{tg} 30 + \operatorname{tg} 60 - \operatorname{sen} 30 \cdot \cos 30$

b) $\operatorname{tg} 60 + \operatorname{sen} 45 - \cos^2 30$

$$a) \cos 30 - \operatorname{sen} 60 + \operatorname{tg} 45 = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 1 = 1$$

$$b) \operatorname{tg} 60 + \operatorname{sen} 45 - \cos^2 30 = \sqrt{3} + \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{3}}{2}\right)^2 =$$

$$= \sqrt{3} + \frac{\sqrt{2}}{2} - \frac{3}{4} = \frac{4\sqrt{3} + 2\sqrt{2} - 3}{4}$$

$$c) \operatorname{tg} 30 + \operatorname{tg} 60 - \operatorname{sen} 30 \cdot \cos 30 = \frac{\sqrt{3}}{3} + \sqrt{3} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} =$$

$$= \frac{\sqrt{3}}{3} + \sqrt{3} - \frac{\sqrt{3}}{4} = \frac{4\sqrt{3} + 12\sqrt{3} - 3\sqrt{3}}{12} = \frac{13\sqrt{3}}{12}$$

③ Sabiendo que α es un ángulo del primer cuadrante, calcula las razones trigonométricas de α si:

a) $\operatorname{sen} \alpha = \frac{1}{4}$

b) $\operatorname{tg} \alpha = 0'49$

Como $0^\circ < \alpha < 90^\circ$ (1º cuadrante) se tiene $\begin{cases} \operatorname{sen} \alpha > 0 \\ \cos \alpha > 0 \\ \operatorname{tg} \alpha > 0 \end{cases}$

a) $\operatorname{sen} \alpha = \frac{1}{4}$

$$\cos^2 \alpha + \operatorname{sen}^2 \alpha = 1 \Rightarrow \cos^2 \alpha + \left(\frac{1}{4}\right)^2 = 1 \Rightarrow \cos \alpha = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$$

$$\operatorname{tg} \alpha = \frac{\operatorname{sen} \alpha}{\cos \alpha} = \frac{\frac{1}{4}}{\frac{\sqrt{15}}{4}} = \frac{1}{\sqrt{15}}$$

$$\sec \alpha = \frac{1}{\cos \alpha} = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15} \quad \cotg \alpha = \frac{1}{\operatorname{tg} \alpha} = \sqrt{15}$$

$$\operatorname{cosec} \alpha = \frac{1}{\operatorname{sen} \alpha} = 4$$

b) $\operatorname{tg} \alpha = 0'49$

$$\operatorname{tg}^2 \alpha = \frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha} = \frac{1 - \cos^2 \alpha}{\cos^2 \alpha} = \frac{1}{\cos^2 \alpha} - 1 \Rightarrow$$

$$\Rightarrow \operatorname{tg}^2 \alpha + 1 = \frac{1}{\cos^2 \alpha} \Rightarrow \cos \alpha = \sqrt{\frac{1}{1 + \operatorname{tg}^2 \alpha}} = \sqrt{\frac{1}{1 + 0'49^2}} = 0'898$$

$$\operatorname{sen} \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - 0'898^2} = 0'44$$

④ Sabiendo que $\cos 50 = 0'6428$ calcular las razones trigonométricas seno, coseno y tangente de los ángulos:

a) 130° b) 230° c) -50° d) 310

Primero hallamos $\operatorname{sen} 50$:

$$\cos^2 \alpha + \operatorname{sen}^2 \alpha = 1 \Rightarrow 0'6428^2 + \operatorname{sen}^2 50 = 1 \Rightarrow \operatorname{sen} 50 = 0'766$$

a) $130 = 180 - 50$

$$\cos 130 = \cos (180 - 50) = -\cos 50 = -0'6428$$

$$\operatorname{sen} 130 = \operatorname{sen} (180 - 50) = \operatorname{sen} 50 = 0'766$$

$$\operatorname{tg} 130 = \frac{\operatorname{sen} 130}{\cos 130} = -1'1917$$

b) $230 = 180 + 50$

$$\cos 230 = \cos (180 + 50) = -\cos 50 = -0'6428$$

$$\operatorname{sen} 230 = \operatorname{sen} (180 + 50) = -\operatorname{sen} 50 = -0'766$$

$$\operatorname{tg} 230 = \frac{\operatorname{sen} 230}{\cos 230} = 1'1917$$

c y d) $310 = 360 - 50$

$$\cos 310 = \cos (360 - 50) = \cos (-50) = \cos 50 = 0'6428$$

$$\operatorname{sen} 310 = \operatorname{sen} (360 - 50) = \operatorname{sen} (-50) = -\operatorname{sen} 50 = -0'766$$

$$\operatorname{tg} 310 = \operatorname{tg} (-50) = -1'1917$$

⑤ Calcula las razones trigonométricas de estos ángulos mayores de 360°

a) 1215° b) 2550 c) 3480

$$a) \quad \begin{array}{r} 1215 \\ 135 \overline{) 360} \end{array} \Rightarrow 1215 = 135 + 3 \cdot 360$$

$$\operatorname{sen}(1215) = \operatorname{sen}(135) = \frac{\sqrt{2}}{2}$$

$$\cos(1215) = \cos(135) = -\frac{\sqrt{2}}{2}$$

$$\operatorname{tg}(1215) = \operatorname{tg}(135) = -1$$

$$b) \quad \begin{array}{r} 2550 \\ 30 \overline{) 360} \end{array} \Rightarrow 2550 = 30 + 7 \cdot 360$$

$$\operatorname{sen}(2550) = \operatorname{sen}(30) = \frac{1}{2}$$

$$\cos(2550) = \cos(30) = \frac{\sqrt{3}}{2}$$

$$\operatorname{tg}(2550) = \operatorname{tg}(30) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$c) \quad \begin{array}{r} 3480 \\ 240 \overline{) 360} \end{array} \Rightarrow 3480 = 240 + 9 \cdot 360$$

$$\operatorname{sen}(3480) = \operatorname{sen} 240 = -\frac{\sqrt{3}}{2}$$

$$\cos(3480) = \cos 240 = -\frac{1}{2}$$

$$\operatorname{tg}(3480) = \sqrt{3}$$

⑥ Sabiendo que $\operatorname{sen} \alpha = 1/5$ con $0 < \alpha < 90$, calcula
a) $\operatorname{sen}(90 - \alpha)$ b) $\operatorname{sen}(180 - \alpha)$ c) $\operatorname{sen}(-\alpha)$

$$a) \operatorname{sen}(90 - \alpha) = \cos \alpha = \sqrt{1 - \operatorname{sen}^2 \alpha} = \sqrt{1 - (1/5)^2} = \sqrt{\frac{24}{25}} = \frac{2\sqrt{6}}{5}$$

$$b) \operatorname{sen}(180 - \alpha) = \operatorname{sen} \alpha = 1/5$$

$$c) \operatorname{sen}(-\alpha) = -\operatorname{sen}(\alpha) = -1/5$$

⑦ Sabiendo que $\operatorname{sen} 18 = 0.309$ y $\cos 18 = 0.951$, halla
a) $\operatorname{sen} 72$ b) $\cos 72$ c) $\operatorname{tg}(-72)$

$$a) \operatorname{sen} 72 = \operatorname{sen}(90 - 18) = \cos 18 = 0.951$$

$$b) \cos 72 = \cos(90 - 18) = \operatorname{sen} 18 = 0.309$$

$$c) \operatorname{tg}(-72) = -\operatorname{tg} 72 = -\frac{\operatorname{sen} 72}{\cos 72} = -\frac{0.951}{0.309} = -3.078$$

⑧ A partir de las razones trigonométricas de 30° y 45° calcula las de los ángulos de 75° y $22'5''$

$$\operatorname{sen} 75 = \operatorname{sen} (30+45) = \operatorname{sen} 30 \cdot \cos 45 + \cos 30 \cdot \operatorname{sen} 45 =$$

$$\operatorname{sen}(\alpha+\beta) = \operatorname{sen} \alpha \cos \beta + \cos \alpha \operatorname{sen} \beta$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\cos 75 = \cos (30+45) = \cos 30 \cdot \cos 45 - \operatorname{sen} 30 \operatorname{sen} 45 =$$

$$\cos(\alpha+\beta) = \cos \alpha \cos \beta - \operatorname{sen} \alpha \operatorname{sen} \beta$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\operatorname{tg} 75 = \operatorname{tg} (30+45) = \frac{\operatorname{tg} 30 + \operatorname{tg} 45}{1 - \operatorname{tg} 30 \cdot \operatorname{tg} 45} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3} \cdot 1} =$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$$

$$= \frac{\frac{\sqrt{3} + 3}{3}}{\frac{3 - \sqrt{3}}{3}} = \frac{\sqrt{3} + 3}{3 - \sqrt{3}}$$

$$\operatorname{sen} 22'5 = \operatorname{sen} \left(\frac{45}{2} \right) = \sqrt{\frac{1 - \cos 45}{2}} = \sqrt{\frac{1 - \sqrt{2}/2}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} =$$

$$\operatorname{sen} \left(\frac{\alpha}{2} \right) = \pm \sqrt{\frac{1 - \cos \alpha}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$\cos 22'5 = \cos \left(\frac{45}{2} \right) = \sqrt{\frac{1 + \cos 45}{2}} = \sqrt{\frac{1 + \sqrt{2}/2}{2}} = \sqrt{\frac{2 + \sqrt{2}}{4}}$$

$$\cos \left(\frac{\alpha}{2} \right) = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\operatorname{tg} 22'5 = \operatorname{tg} \left(\frac{45}{2} \right) = \sqrt{\frac{1 - \cos 45}{1 + \cos 45}} = \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}}$$

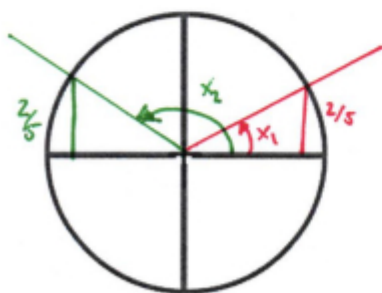
$$\operatorname{tg} \left(\frac{\alpha}{2} \right) = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

⑨ Resolver las siguientes ecuaciones trigonométricas:

a) $5 \operatorname{sen} x = 2$

$$\operatorname{sen} x = \frac{2}{5} \Rightarrow x = \operatorname{arcsen} \left(\frac{2}{5} \right) = 23'578^\circ$$

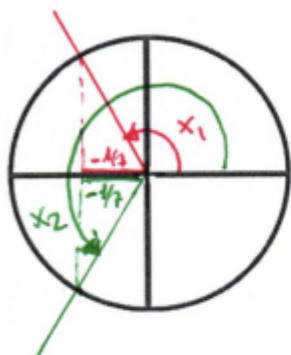
↳ Esto es lo que dice la calculadora pero...



$$\Rightarrow x = \operatorname{arcsen} \left(\frac{2}{5} \right) = \begin{cases} x_1 = 23'578^\circ + K \cdot 360^\circ \\ x_2 = 156'422^\circ + K \cdot 360^\circ \end{cases}$$

b) $7 \cos x = -1$

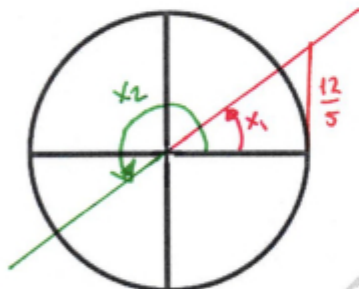
$$\cos x = -\frac{1}{7} \Rightarrow x = \arccos\left(-\frac{1}{7}\right) = 98'213''$$



$$\Rightarrow x = \arccos\left(-\frac{1}{7}\right) = \begin{cases} x_1 = 98'213'' + k \cdot 360^\circ \\ x_2 = 261'787'' + k \cdot 360^\circ \end{cases}$$

c) $5 \operatorname{tg} x = 12$

$$\operatorname{tg} x = \frac{12}{5} \Rightarrow x = \operatorname{arctg}\left(\frac{12}{5}\right) = 67'38''$$



$$x = \operatorname{arctg}\left(\frac{12}{5}\right) = \begin{cases} x_1 = 67'38'' + k \cdot 360^\circ \\ x_2 = 247'38'' + k \cdot 360^\circ \end{cases}$$

Como vemos, en este caso se pueden agrupar todas las soluciones en una única expresión dada por:

$$x = \operatorname{arctg}\left(\frac{12}{5}\right) = 67'38'' + k \cdot 180^\circ$$

$$d) \operatorname{sen} 2x = 1$$

$$2x = \operatorname{arcsen}(1) \Rightarrow 2x = 90^\circ + k \cdot 360^\circ \Rightarrow x = 45^\circ + k \cdot 180^\circ$$

$$e) \cos x + \operatorname{sen}(2x) = 0$$

$$\cos x + 2 \operatorname{sen} x \cdot \cos x = 0$$

$$\cos x (1 + 2 \operatorname{sen} x) = 0 \begin{cases} \rightarrow \cos x = 0 \\ \rightarrow 1 + 2 \operatorname{sen} x = 0 \end{cases}$$

$$\cos x = 0 \Rightarrow x = \arccos(0) = \begin{cases} x_1 = 90^\circ + k \cdot 360^\circ \\ x_2 = 270^\circ + k \cdot 360^\circ \end{cases} \Rightarrow$$

$$\Rightarrow x = 90^\circ + k \cdot 180^\circ$$

$$1 + 2 \operatorname{sen} x = 0 \Rightarrow \operatorname{sen} x = -\frac{1}{2} \Rightarrow x = \operatorname{arcsen}(-\frac{1}{2}) = -30^\circ$$

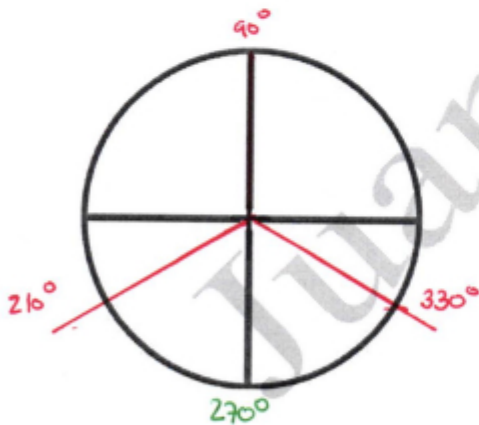
$$x = \operatorname{arcsen}(-\frac{1}{2}) = \begin{cases} x_3 = 210^\circ + k \cdot 360^\circ \\ x_4 = 330^\circ + k \cdot 360^\circ \end{cases}$$

* Tenemos que expresar todas las razones trigonométricas que intervienen en la ecuación en función de un mismo ángulo (como norma general)

También es interesante ver como en muchas ocasiones se pueden agrupar todas las soluciones en única expresión. En la ecuación que acabamos de resolver hemos obtenido:

$$\cos x (1 + 2\sec x) = 0 \rightarrow \begin{cases} \cos x = 0 \rightarrow \begin{cases} x_1 = 90^\circ + k \cdot 360^\circ \\ x_2 = 270^\circ + k \cdot 360^\circ \end{cases} \\ 1 + 2\sec x = 0 \rightarrow \begin{cases} x_3 = 210^\circ + k \cdot 360^\circ \\ x_4 = 330^\circ + k \cdot 360^\circ \end{cases} \end{cases}$$

Si nos fijamos bien, se pueden agrupar todas en una única expresión según:



$$\Rightarrow \begin{cases} x_1 = 90^\circ + k \cdot 120^\circ \\ x_2 = 270^\circ + k \cdot 360^\circ \end{cases}$$

10) Demostrar las siguientes identidades trigonométricas

a) $\cotg x \cdot \sec x = \operatorname{cosec} x$

$$\frac{\cancel{\cos x}}{\operatorname{sen} x} \cdot \frac{1}{\cancel{\cos x}} = \frac{1}{\operatorname{sen} x} \Rightarrow \frac{1}{\operatorname{sen} x} = \frac{1}{\operatorname{sen} x}$$

b) $(\cos \alpha + \operatorname{sen} \alpha)^2 = 1 + \operatorname{sen}(2\alpha)$

$$(\cos^2 \alpha + 2 \cos \alpha \operatorname{sen} \alpha + \operatorname{sen}^2 \alpha) = 1 + 2 \cos \alpha \operatorname{sen} \alpha$$

$$1 + 2 \cos \alpha \operatorname{sen} \alpha = 1 + 2 \cos \alpha \operatorname{sen} \alpha$$

c) $\operatorname{tg} x = \frac{1 - \cos 2x}{\operatorname{sen} 2x}$

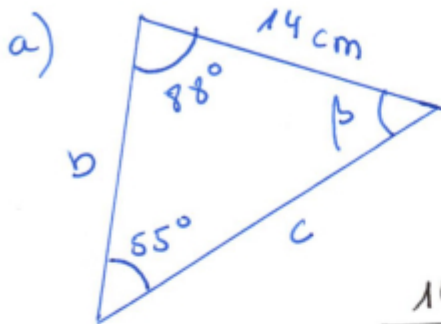
$$\frac{1 - \cos 2x}{\operatorname{sen} 2x} = \frac{1 - (\cos^2 x - \operatorname{sen}^2 x)}{2 \operatorname{sen} x \cos x} = \frac{1 - \cos^2 x + \operatorname{sen}^2 x}{2 \operatorname{sen} x \cos x} =$$

$$= \frac{\operatorname{sen}^2 x + \operatorname{sen}^2 x}{2 \operatorname{sen} x \cos x} = \frac{\cancel{2} \operatorname{sen}^2 x}{\cancel{2} \operatorname{sen} x \cos x} = \frac{\operatorname{sen} x}{\cos x} = \operatorname{tg} x$$

d) $\frac{\operatorname{tga} + \operatorname{tg} b}{\operatorname{tga} - \operatorname{tg} b} = \frac{\operatorname{sen}(a+b)}{\operatorname{sen}(a-b)}$

$$\frac{\frac{\operatorname{sen} a}{\cos a} + \frac{\operatorname{sen} b}{\cos b}}{\frac{\operatorname{sen} a}{\cos a} - \frac{\operatorname{sen} b}{\cos b}} = \frac{\frac{\operatorname{sen} a \cos b + \operatorname{sen} b \cos a}{\cancel{\cos a \cos b}}}{\frac{\operatorname{sen} a \cos b - \operatorname{sen} b \cos a}{\cancel{\cos a \cos b}}} = \frac{\operatorname{sen}(a+b)}{\operatorname{sen}(a-b)}$$

11) Resuelve los siguientes triángulos:



$$\frac{a}{\operatorname{sen} \alpha} = \frac{b}{\operatorname{sen} \beta} = \frac{c}{\operatorname{sen} \gamma}$$

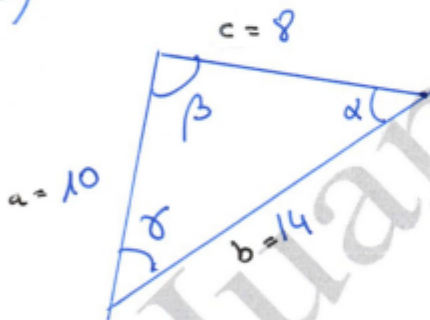
$$55 + 88 + \beta = 180 \Rightarrow \beta = 37^\circ$$

$$\frac{14}{\operatorname{sen} 55} = \frac{b}{\operatorname{sen} 37} = \frac{c}{\operatorname{sen} 88}$$

$$\frac{14}{\operatorname{sen} 55} = \frac{b}{\operatorname{sen} 37} \Rightarrow b = 10'29 \text{ cm}$$

$$\frac{14}{\operatorname{sen} 55} = \frac{c}{\operatorname{sen} 88} \Rightarrow c = 17'08 \text{ cm}$$

b)



$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$10^2 = 14^2 + 8^2 - 2 \cdot 14 \cdot 8 \cdot \cos \alpha$$

$$-160 = -224 \cdot \cos \alpha$$

$$\cos \alpha = \frac{5}{7} \Rightarrow \alpha = 44'41''$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$14^2 = 10^2 + 8^2 - 2 \cdot 10 \cdot 8 \cos \beta$$

$$32 = -160 \cos \beta$$

$$\cos \beta = -\frac{1}{5} \Rightarrow \beta = 101'54''$$

$$\alpha + \beta + \gamma = 180^\circ \Rightarrow \gamma = 180 - 44'41'' - 101'54'' = 34'05''$$

- ⑫ Utiliza las transformaciones en producto para simplificar al máximo la expresión:

$$\frac{\operatorname{sen} 4a + \operatorname{sen} 2a}{\cos 4a + \cos 2a}$$

$$\frac{\operatorname{sen} 4a + \operatorname{sen} 2a}{\cos 4a + \cos 2a} = \frac{\cancel{2} \operatorname{sen} \left(\frac{4a+2a}{2} \right) \cos \left(\frac{4a-2a}{2} \right)}{\cancel{2} \cos \left(\frac{4a+2a}{2} \right) \cdot \cos \left(\frac{4a-2a}{2} \right)} =$$

$$= \frac{\operatorname{sen} \left(\frac{6a}{2} \right)}{\cos \left(\frac{6a}{2} \right)} = \frac{\operatorname{sen} (3a)}{\cos (3a)} = \operatorname{tg} (3a)$$

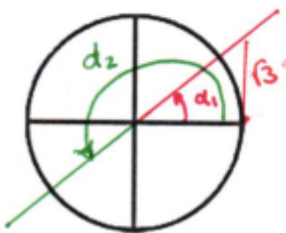
- ⑬ Resuelve las siguientes ecuaciones trigonométricas:

a) $\operatorname{sen}(\alpha + 30) = 2 \cos \alpha$

$$\operatorname{sen} \alpha \cdot \cos 30 + \cos \alpha \operatorname{sen} 30 = 2 \cos \alpha$$

$$\operatorname{sen} \alpha \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cos \alpha = 2 \cos \alpha$$

$$\frac{\sqrt{3}}{2} \operatorname{sen} \alpha = \frac{3}{2} \cos \alpha \Rightarrow \operatorname{tg} \alpha = \frac{3}{\sqrt{3}} = \sqrt{3}$$



$$\alpha = \operatorname{arctg}(\sqrt{3}) \begin{cases} \alpha_1 = 60^\circ + k \cdot 360^\circ \\ \alpha_2 = 240^\circ + k \cdot 360^\circ \end{cases}$$

$$\Rightarrow \boxed{\alpha = 60^\circ + k \cdot 180^\circ}$$

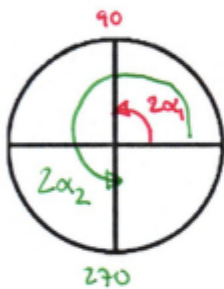
$$b) \operatorname{sen}(3\alpha) - \operatorname{sen} \alpha = 0$$

$$2 \cos \left(\frac{3\alpha + \alpha}{2} \right) \cdot \operatorname{sen} \left(\frac{3\alpha - \alpha}{2} \right) = 0$$

$$2 \cos(2\alpha) \cdot \operatorname{sen} \alpha = 0$$

$$\cos(2\alpha) \cdot \operatorname{sen} \alpha = 0 \quad \begin{cases} \rightarrow \cos 2\alpha = 0 \\ \rightarrow \operatorname{sen} \alpha = 0 \end{cases}$$

$$\cos 2\alpha = 0 \rightarrow 2\alpha = \arccos 0$$



$$\Rightarrow \begin{cases} 2\alpha = 90^\circ + k \cdot 360^\circ \\ 2\alpha = 270^\circ + k \cdot 360^\circ \end{cases} \Rightarrow 2\alpha = 90^\circ + k \cdot 180^\circ$$

$$\Rightarrow \boxed{\alpha = 45^\circ + k \cdot 90^\circ}$$

$$\operatorname{sen} \alpha = 0 \rightarrow \alpha = \operatorname{arcsen}(0)$$



$$\alpha = \operatorname{arcsen}(0) \quad \begin{cases} \alpha = 0 + k \cdot 360 \\ \alpha = 180 + k \cdot 360 \end{cases} \Rightarrow \boxed{\alpha = 0 + k \cdot 180}$$

$$c) \ 2 \cos^2 \alpha + \cos \alpha - 1 = 0$$

$$\cos \alpha = t$$

$$2t^2 + t - 1 = 0 \begin{cases} t = 1/2 \\ t = -1 \end{cases}$$

$$\cos \alpha = \frac{1}{2} \rightarrow \alpha = \arccos\left(\frac{1}{2}\right) = \begin{cases} \alpha = 60^\circ + k \cdot 360^\circ \\ \alpha = 300^\circ + k \cdot 360^\circ \end{cases}$$

$$\cos \alpha = -1 \rightarrow \alpha = \arccos(-1) = 180^\circ + k \cdot 360^\circ$$

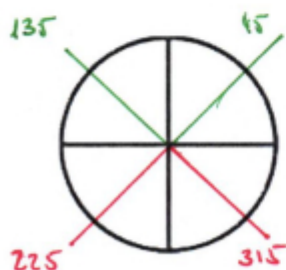
Si nos fijamos, se puede expresar con única expresión:



$$\Rightarrow \boxed{\alpha = 60^\circ + k \cdot 120^\circ}$$

$$d) \ 2 \sin^2 \alpha - 1 = 0$$

$$\sin^2 \alpha = \frac{1}{2} \Rightarrow \sin \alpha = \pm \sqrt{\frac{1}{2}} \begin{cases} \sin \alpha = \frac{\sqrt{2}}{2} \\ \sin \alpha = -\frac{\sqrt{2}}{2} \end{cases}$$



$$\bullet \ \alpha = \arcsin\left(\frac{\sqrt{2}}{2}\right) \rightarrow \begin{cases} \alpha = 45^\circ + k \cdot 360^\circ \\ \alpha = 135^\circ + k \cdot 360^\circ \end{cases}$$

$$\bullet \ \alpha = \arcsin\left(-\frac{\sqrt{2}}{2}\right) \rightarrow \begin{cases} \alpha = 225^\circ + k \cdot 360^\circ \\ \alpha = 315^\circ + k \cdot 360^\circ \end{cases}$$

$$\Rightarrow \boxed{\alpha = 45^\circ + k \cdot 90^\circ}$$

$$e) \operatorname{tg}^2 \alpha - \operatorname{tg} \alpha = 0$$

$$\operatorname{tg} \alpha (1 - \operatorname{tg} \alpha) = 0 \begin{cases} \rightarrow \operatorname{tg} \alpha = 0 \\ \rightarrow 1 - \operatorname{tg} \alpha = 0 \Rightarrow \operatorname{tg} \alpha = 1 \end{cases}$$

$$\operatorname{tg} \alpha = 0 \rightarrow \alpha = \operatorname{arctg}(0) \rightarrow \begin{cases} \alpha = 0^\circ + k \cdot 360^\circ \\ \alpha = 180^\circ + k \cdot 360^\circ \end{cases} \quad \boxed{\alpha = 0^\circ + k \cdot 180^\circ}$$

$$\operatorname{tg} \alpha = 1 \rightarrow \alpha = \operatorname{arctg}(1) \rightarrow \begin{cases} \alpha = 45^\circ + k \cdot 360^\circ \\ \alpha = 225^\circ + k \cdot 360^\circ \end{cases}$$

$$f) \operatorname{sen}^2 \alpha + 3 \cos \alpha = 3$$

$$2 \cdot (1 - \cos^2 \alpha) + 3 \cos \alpha = 3$$

$$2 - 2 \cos^2 \alpha + 3 \cos \alpha - 3 = 0$$

$$-2 \cos^2 \alpha + 3 \cos \alpha - 1 = 0 \quad ; \quad \cos \alpha = t$$

$$-2t^2 + 3t - 1 = 0 \quad \begin{cases} \rightarrow t = 1 \\ \rightarrow t = 1/2 \end{cases}$$

$$\cos \alpha = 1 \rightarrow \alpha = \arccos(1) = 0^\circ + k \cdot 360^\circ$$

$$\cos \alpha = 1/2 \rightarrow \alpha = \arccos(1/2) = \begin{cases} \alpha = 60^\circ + k \cdot 360^\circ \\ \alpha = 300^\circ + k \cdot 360^\circ \end{cases}$$

$$g) \operatorname{sen}(5\alpha) - \operatorname{sen}(3\alpha) = 0$$

$$2 \cos\left(\frac{5\alpha+3\alpha}{2}\right) \operatorname{sen}\left(\frac{5\alpha-3\alpha}{2}\right) = 0$$

$$2 \cos(4\alpha) \cdot \operatorname{sen} \alpha = 0$$

$$\cos(4\alpha) \cdot \operatorname{sen} \alpha = 0 \quad \begin{cases} \rightarrow \cos(4\alpha) = 0 \\ \rightarrow \operatorname{sen}(\alpha) = 0 \end{cases}$$

$$\cos(4\alpha) = 0 \rightarrow 4\alpha = \arccos 0$$



$$\Rightarrow \begin{cases} 4\alpha = 90^\circ + k \cdot 360^\circ \\ 4\alpha = 270^\circ + k \cdot 360^\circ \end{cases} \quad 4\alpha = 90^\circ + k \cdot 180^\circ$$

$$\Rightarrow \boxed{\alpha = 22'5^\circ + k \cdot 45^\circ}$$

$$\operatorname{sen} \alpha = 0 \rightarrow \alpha = \operatorname{arcsen}(0) \quad \begin{cases} \alpha = 0^\circ + k \cdot 360^\circ \\ \alpha = 180^\circ + k \cdot 360^\circ \end{cases} \quad \boxed{\alpha = 0^\circ + k \cdot 180^\circ}$$

$$h) 4 \cos(2\alpha) + 3 \cos \alpha = 1$$

$$4(\cos^2 \alpha - \operatorname{sen}^2 \alpha) + 3 \cos \alpha = 1$$

$$4 \cos^2 \alpha - 4 \operatorname{sen}^2 \alpha + 3 \cos \alpha = 1$$

$$4 \cos^2 \alpha - 4(1 - \cos^2 \alpha) + 3 \cos \alpha = 1$$

$$4 \cos^2 \alpha - 4 + 4 \cos^2 \alpha + 3 \cos \alpha = 1$$

$$8 \cos^2 \alpha + 3 \cos \alpha - 5 = 0 \quad ; \quad \cos \alpha = t$$

$$8t^2 + 3t - 5 = 0 \begin{cases} \rightarrow t = 5/8 \\ \rightarrow t = -1 \end{cases}$$

$$\cos \alpha = \frac{5}{8} \rightarrow \alpha = \arccos\left(\frac{5}{8}\right)$$



$$\Rightarrow \begin{cases} \alpha = 51'318'' + k \cdot 360^\circ \\ \alpha = 308'682'' + k \cdot 360^\circ \end{cases}$$

$$\cos \alpha = -1 \rightarrow \boxed{\alpha = \arccos(-1) = 180^\circ + k \cdot 360^\circ}$$

$$i) \operatorname{tg}(2\alpha) + 2\cos\alpha = 0$$

$$\frac{\operatorname{sen}(2\alpha)}{\cos(2\alpha)} + 2\cos\alpha = 0 \Rightarrow \frac{\operatorname{sen}(2\alpha) + 2\cos\alpha \cdot \cos(2\alpha)}{\cos(2\alpha)} = 0$$

$$\operatorname{sen}(2\alpha) + 2\cos\alpha \cdot \cos(2\alpha) = 0$$

$$2\operatorname{sen}\alpha \cdot \cos\alpha + 2\cos\alpha \cdot (\cos^2\alpha - \operatorname{sen}^2\alpha) = 0$$

$$2\cos\alpha [\operatorname{sen}\alpha + \cos^2\alpha - \operatorname{sen}^2\alpha] = 0$$

$$2\cos\alpha [\operatorname{sen}\alpha + (1 - \operatorname{sen}^2\alpha) - \operatorname{sen}^2\alpha] = 0$$

$$2\cos\alpha \cdot [-2\operatorname{sen}^2\alpha + \operatorname{sen}\alpha + 1] = 0$$

$$\rightarrow 2 \cos \alpha = 0 \rightarrow \cos \alpha = 0$$

$$\rightarrow -2 \operatorname{sen}^2 \alpha + \operatorname{sen} \alpha + 1 = 0$$

$$\cos \alpha = 0 \rightarrow \alpha = \arccos(0) \rightarrow \begin{cases} \alpha = 90^\circ + k \cdot 360^\circ \\ \alpha = 270^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{\alpha = 90^\circ + k \cdot 180^\circ}$$

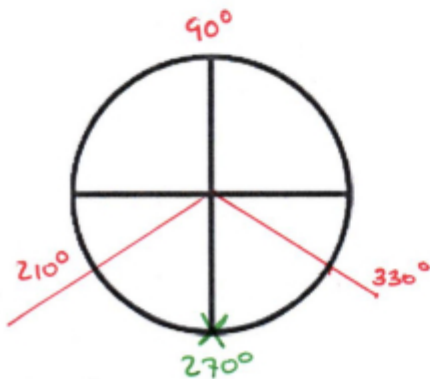
$$-2 \operatorname{sen}^2 \alpha + \operatorname{sen} \alpha + 1 = 0 ; \operatorname{sen} \alpha = t$$

$$-2t^2 + t + 1 = 0 \rightarrow \begin{cases} t = 1 \\ t = -1/2 \end{cases}$$

$$\operatorname{sen} \alpha = 1 \rightarrow \boxed{\alpha = \operatorname{arcsen}(1) = 90^\circ + k \cdot 360^\circ}$$

$$\operatorname{sen} \alpha = -1/2 \rightarrow \boxed{\alpha = \operatorname{arcsen}(-1/2) \rightarrow \begin{cases} \alpha = 210^\circ + k \cdot 360^\circ \\ \alpha = 330^\circ + k \cdot 360^\circ \end{cases}}$$

Si nos fijamos bien, podemos agrupar todas estas soluciones en una expresión más sencilla:



$$\Rightarrow \begin{cases} \alpha = 90^\circ + k \cdot 120^\circ \\ \alpha = 270^\circ + k \cdot 360^\circ \end{cases}$$

$$j) \sqrt{2} \cdot \cos\left(\frac{\alpha}{2}\right) - \cos \alpha = 1$$

$$\sqrt{2} \cdot \cos\left(\frac{\alpha}{2}\right) - \cos\left(2 \cdot \frac{\alpha}{2}\right) = 1$$

$$\sqrt{2} \cdot \cos\left(\frac{\alpha}{2}\right) - \left(\cos^2\left(\frac{\alpha}{2}\right) - \operatorname{sen}^2\left(\frac{\alpha}{2}\right)\right) = 1$$

$$\sqrt{2} \cdot \cos\left(\frac{\alpha}{2}\right) - \cos^2\left(\frac{\alpha}{2}\right) + \operatorname{sen}^2\left(\frac{\alpha}{2}\right) = 1$$

$$\sqrt{2} \cdot \cos\left(\frac{\alpha}{2}\right) - \cos^2\left(\frac{\alpha}{2}\right) + \cancel{1} - \cos^2\left(\frac{\alpha}{2}\right) = \cancel{1}$$

$$-2 \cos^2\left(\frac{\alpha}{2}\right) + \sqrt{2} \cdot \cos\left(\frac{\alpha}{2}\right) = 0$$

$$\cos\left(\frac{\alpha}{2}\right) \cdot \left[-2 \cos\left(\frac{\alpha}{2}\right) + \sqrt{2}\right] = 0$$

$$\cos\left(\frac{\alpha}{2}\right) = 0$$

$$-2 \cos\left(\frac{\alpha}{2}\right) + \sqrt{2} = 0$$

$$\cos\left(\frac{\alpha}{2}\right) = 0 \rightarrow \frac{\alpha}{2} = \arccos 0 \rightarrow \left\{ \begin{array}{l} \frac{\alpha}{2} = 90^\circ + K \cdot 360^\circ \\ \frac{\alpha}{2} = 270^\circ + K \cdot 360^\circ \end{array} \right\} \rightarrow$$

$$\Rightarrow \frac{\alpha}{2} = 90^\circ + K \cdot 180^\circ \Rightarrow \boxed{\alpha = 180^\circ + K \cdot 360^\circ}$$

$$-2 \cos\left(\frac{\alpha}{2}\right) + \sqrt{2} = 0 \Rightarrow \cos\left(\frac{\alpha}{2}\right) = \frac{\sqrt{2}}{2} \Rightarrow \frac{\alpha}{2} = \arccos\left(\frac{\sqrt{2}}{2}\right) \rightarrow$$

$$\rightarrow \left\{ \begin{array}{l} \frac{\alpha}{2} = 45^\circ + K \cdot 360^\circ \rightarrow \boxed{\alpha = 90^\circ + K \cdot 720^\circ} \\ \frac{\alpha}{2} = 315^\circ + K \cdot 360^\circ \rightarrow \boxed{\alpha = 630^\circ + K \cdot 720^\circ} \end{array} \right.$$

$$l) \operatorname{sen}(180 - \alpha) = \cos(270 - \alpha) + \cos 180$$

$$\operatorname{sen} \alpha = \cancel{\cos 270} \cos \alpha + \operatorname{sen} 270 \cdot \operatorname{sen} \alpha - 1$$

$$\operatorname{sen} \alpha = -\operatorname{sen} \alpha - 1 \Rightarrow 2\operatorname{sen} \alpha = -1$$

$$\operatorname{sen} \alpha = -\frac{1}{2} \rightarrow \alpha = \operatorname{arcsen}\left(-\frac{1}{2}\right) \rightarrow \begin{cases} \alpha = 210^\circ + k \cdot 360^\circ \\ \alpha = 330^\circ + k \cdot 360^\circ \end{cases}$$

$$m) \operatorname{sen}(45 - \alpha) + \sqrt{2} \operatorname{sen} \alpha = 0$$

$$\operatorname{sen} 45 \cdot \cos \alpha - \cos 45 \operatorname{sen} \alpha + \sqrt{2} \operatorname{sen} \alpha = 0$$

$$\frac{\sqrt{2}}{2} \cdot \cos \alpha - \frac{\sqrt{2}}{2} \operatorname{sen} \alpha + \sqrt{2} \operatorname{sen} \alpha = 0$$

$$\frac{\sqrt{2}}{2} \cdot \cos \alpha + \operatorname{sen} \alpha \left(\sqrt{2} - \frac{\sqrt{2}}{2} \right) = 0$$

$$\frac{\sqrt{2}}{2} \cos \alpha + \frac{\sqrt{2}}{2} \operatorname{sen} \alpha = 0 \rightarrow \cos \alpha + \operatorname{sen} \alpha = 0$$

$$\operatorname{sen} \alpha = -\cos \alpha \Rightarrow \frac{\operatorname{sen} \alpha}{\cos \alpha} = -1 \Rightarrow \operatorname{tg} \alpha = -1$$

$$\alpha = \operatorname{arctg}(-1) \rightarrow \begin{cases} \alpha = 135^\circ + k \cdot 360^\circ \\ \alpha = 315^\circ + k \cdot 360^\circ \end{cases} \rightarrow \boxed{\alpha = 135^\circ + k \cdot 180^\circ}$$

14) Sabiendo que $\text{sen } 54^\circ = 0'81$, halla:

$$\cos(108^\circ); \text{tg}(27^\circ); \text{sen}(24^\circ); \cos(99^\circ); \cos(126^\circ)$$

$$\begin{aligned} \text{a) } \cos(108^\circ) &= \cos(2 \cdot 54^\circ) = \cos^2(54^\circ) - \text{sen}^2(54^\circ) = \\ &= (1 - \text{sen}^2(54^\circ)) - \text{sen}^2(54^\circ) = 1 - 2\text{sen}^2(54^\circ) = 1 - 2 \cdot 0'81^2 = \\ &= -0'3122 \end{aligned}$$

$$\text{b) } \text{tg}(27^\circ) = \text{tg}\left(\frac{54^\circ}{2}\right) = + \sqrt{\frac{1 - \cos 54^\circ}{1 + \cos 54^\circ}} = \sqrt{\frac{1 - 0'59}{1 + 0'59}} = 0'51$$

$$\cos 54^\circ = \sqrt{1 - \text{sen}^2 54^\circ} = \sqrt{1 - 0'81^2} = 0'59$$

$$\begin{aligned} \text{c) } \text{sen}(24^\circ) &= \text{sen}(54^\circ - 30^\circ) = \text{sen } 54^\circ \cdot \cos 30^\circ - \cos 54^\circ \cdot \text{sen } 30^\circ = \\ &= 0'81 \cdot \frac{\sqrt{3}}{2} - 0'59 \cdot \frac{1}{2} = 0'41 \end{aligned}$$

$$\begin{aligned} \text{d) } \cos(99^\circ) &= \cos(54^\circ + 45^\circ) = \cos 54^\circ \cdot \cos 45^\circ - \text{sen } 54^\circ \cdot \text{sen } 45^\circ = \\ &= 0'59 \cdot \frac{\sqrt{2}}{2} - 0'81 \cdot \frac{\sqrt{2}}{2} = -0'16 \end{aligned}$$

$$\text{e) } \cos(126^\circ) = \cos(180^\circ - 54^\circ) = -\cos(54^\circ) = -0'59$$

15) Demuestra que:

$$a) \operatorname{sen}(2\alpha) - \operatorname{tg} \alpha \cdot \cos(2\alpha) = \operatorname{tg} \alpha$$

$$2 \operatorname{sen} \alpha \cos \alpha - \frac{\operatorname{sen} \alpha}{\cos \alpha} (\cos^2 \alpha - \operatorname{sen}^2 \alpha) = \frac{\operatorname{sen} \alpha}{\cos \alpha}$$

$$\frac{2 \operatorname{sen} \alpha \cos^2 \alpha - \operatorname{sen} \alpha \cos^2 \alpha + \operatorname{sen}^3 \alpha}{\cancel{\cos \alpha}} = \frac{\cancel{\operatorname{sen} \alpha}}{\cancel{\cos \alpha}}$$

$$\operatorname{sen} \alpha \cos^2 \alpha + \operatorname{sen} \alpha \operatorname{sen}^2 \alpha = \operatorname{sen} \alpha$$

$$\operatorname{sen} \alpha (\cancel{\cos^2 \alpha + \operatorname{sen}^2 \alpha}) = \operatorname{sen} \alpha \overset{1}{\Rightarrow} \boxed{\operatorname{sen} \alpha = \operatorname{sen} \alpha}$$

$$b) \frac{2 \cos(45^\circ + \alpha) \cos(45^\circ - \alpha)}{\cos 2\alpha} = 1$$

$$\frac{2 \cos\left(\frac{90+2\alpha}{2}\right) \cos\left(\frac{90-2\alpha}{2}\right)}{\cos(2\alpha)} = 1$$

$$\frac{\cancel{\cos 90} + \cos(2\alpha)}{\cos(2\alpha)} = \frac{\cos(2\alpha)}{\cos(2\alpha)} = 1$$

$$\text{Si } x = 0^\circ \rightarrow 2 \cdot \operatorname{sen} 0^\circ + \cos y = 1 \Rightarrow$$

$$\Rightarrow \cos y = 1 \Rightarrow y = \arccos(1) = 0^\circ + k_2 \cdot 360^\circ$$

$$\text{Si } x = 90^\circ \rightarrow 2 \cdot \operatorname{sen} 90^\circ + \cos y = 1 \Rightarrow$$

$$\Rightarrow \cos y = -1 \Rightarrow y = \arccos(-1) = 180^\circ + k_2 \cdot 360^\circ$$

Las soluciones por tanto:

$$\begin{cases} (x, y) = (0^\circ + k_1 \cdot 360^\circ, 0^\circ + k_2 \cdot 360^\circ) \\ (x, y) = (90^\circ + k_1 \cdot 360^\circ, 180^\circ + k_2 \cdot 360^\circ) \end{cases}$$

$$c) \left. \begin{array}{l} x + y = 90^\circ \\ \operatorname{sen} x + \operatorname{sen} y = 1 \end{array} \right\} y = 90 - x$$

$$\operatorname{sen} x + \operatorname{sen}(90 - x) = 1$$

$$\operatorname{sen} x + \cos x = 1 \rightarrow \left(\sqrt{1 - \cos^2 x} \right)^2 = (1 - \cos x)^2 \Rightarrow$$

$$\Rightarrow 1 - 2\cos x + \cos^2 x = 1 - \cos^2 x$$

$$\Rightarrow 2\cos^2 x - 2\cos x = 0 \Rightarrow 2\cos x (\cos x - 1) = 0 \begin{cases} \nearrow \cos x = 0 \\ \searrow \cos x = 1 \end{cases}$$

$$\cos x = 0 \rightarrow x = \arccos(0) \rightarrow \begin{cases} x = 90^\circ + k \cdot 360^\circ \\ x = 270^\circ + k \cdot 360^\circ \end{cases}$$

$$\cos x = 1 \rightarrow x = \arccos(1) \rightarrow x = 0^\circ + k \cdot 360^\circ$$

Comprobamos soluciones:

$$\left. \begin{array}{l} \sin(90) + \cos(90) = 1 \quad \checkmark \\ \sin(270) + \cos(270) = -1 \quad \times \\ \sin(0) + \cos(0) = 1 \quad \checkmark \end{array} \right\} \Rightarrow \begin{cases} x = 90^\circ + k \cdot 360^\circ \\ x = 0^\circ + k \cdot 360^\circ \end{cases}$$

$$y = 90 - x$$

$$\text{Si } x = 0^\circ + k \cdot 360^\circ \rightarrow y = 90 - (0 + k \cdot 360) = 90^\circ - 360^\circ k$$

$$\text{Si } x = 90^\circ + k \cdot 360^\circ \rightarrow y = 90 - (90 + k \cdot 360) = 0^\circ - 360^\circ k$$

Las soluciones por tanto:

$$\begin{cases} (x, y) = (0^\circ + k \cdot 360^\circ, 90^\circ - k \cdot 360^\circ) \\ (x, y) = (90^\circ + k \cdot 360^\circ, 0^\circ - k \cdot 360^\circ) \end{cases}$$

$$d) \left. \begin{array}{l} \operatorname{tg} x + \operatorname{tg} y = 1 + \sqrt{3} \\ \cotg x \cdot \operatorname{tg} y = \sqrt{3} \end{array} \right\} \frac{1}{\operatorname{tg} x} \cdot \operatorname{tg} y = \sqrt{3} \Rightarrow \operatorname{tg} y = \sqrt{3} \cdot \operatorname{tg} x$$

$$\operatorname{tg} x + \sqrt{3} \operatorname{tg} x = 1 + \sqrt{3} \Rightarrow \operatorname{tg} x (1 + \sqrt{3}) = 1 + \sqrt{3} \Rightarrow$$

$$\Rightarrow \operatorname{tg} x = 1 \Rightarrow x = \arctg(1) \rightarrow \left\{ \begin{array}{l} x = 45^\circ + k_1 \cdot 360^\circ \\ x = 225^\circ + k_1 \cdot 360^\circ \end{array} \right\} \Rightarrow \boxed{x = 45^\circ + k_1 \cdot 180^\circ}$$

$$\operatorname{tg} y = \sqrt{3} \Rightarrow y = \arctg(\sqrt{3}) \rightarrow \left\{ \begin{array}{l} y = 60^\circ + k_2 \cdot 360^\circ \\ y = 240^\circ + k_2 \cdot 360^\circ \end{array} \right\} \Rightarrow \boxed{y = 60^\circ + k_2 \cdot 180^\circ}$$

$$e) \left. \begin{array}{l} x + y = 120 \\ \cos x = \frac{1}{2 \cos y} + \operatorname{sen} x \cdot \operatorname{tg} y \end{array} \right\} y = 120 - x$$

$$\cos x = \frac{1}{2 \cos y} + \operatorname{sen} x \cdot \frac{\operatorname{sen} y}{\cos y}$$

$$\cos x = \frac{1 + 2 \operatorname{sen} x \operatorname{sen} y}{2 \cos y} \Rightarrow 2 \cos x \cos y = 1 + 2 \operatorname{sen} x \operatorname{sen} y$$

$$2 \cos x \cos (120 - x) = 1 + 2 \operatorname{sen} x \cdot \operatorname{sen} (120 - x)$$

$$2 \cos x [\cos 120 \cdot \cos x + \operatorname{sen} 120 \cdot \operatorname{sen} x] = 1 + 2 \operatorname{sen} x [\operatorname{sen} 120 \cos x - \cos 120 \operatorname{sen} x]$$

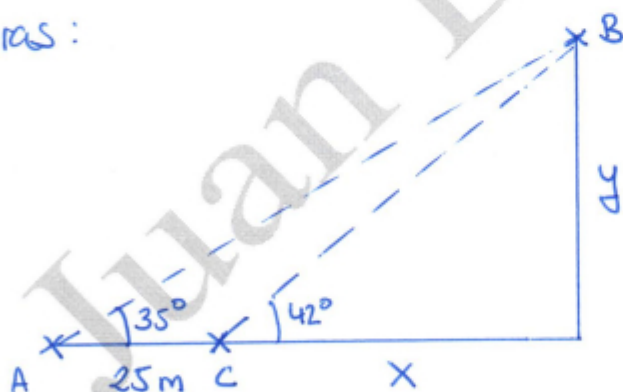
$$2 \cos x \left(-\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \operatorname{sen} x \right) = 1 + 2 \operatorname{sen} x \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \operatorname{sen} x \right) \Rightarrow$$

$$\Rightarrow -\cos^2 x + \sqrt{3} \operatorname{sen} x \cos x = 1 + \sqrt{3} \operatorname{sen} x \cos x + \operatorname{sen}^2 x$$

$$\Rightarrow 0 = 1 + \operatorname{sen}^2 x + \cos^2 x \Rightarrow \boxed{0 \neq 2} \Rightarrow \text{Sin solución!!}$$

17) Averigua los valores de "x" y de "y" en las siguientes figuras:

a)



$$\left. \begin{aligned} \operatorname{tg} 35 &= \frac{y}{25+x} \\ \operatorname{tg} 42 &= \frac{y}{x} \end{aligned} \right\}$$

$$\left. \begin{aligned} y &= (25+x) \cdot \operatorname{tg} 35 \\ y &= x \cdot \operatorname{tg} 42 \end{aligned} \right\} (25+x) \cdot \operatorname{tg} 35 = x \cdot \operatorname{tg} 42 \Rightarrow$$

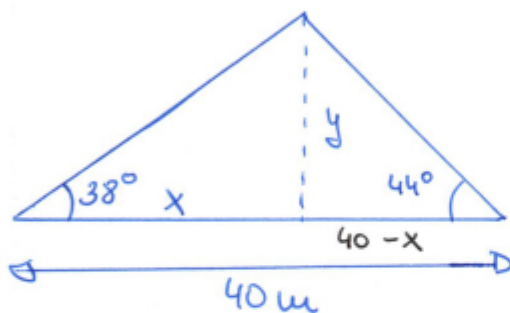
$$\Rightarrow 25 \operatorname{tg} 35 + x \cdot \operatorname{tg} 35 = x \cdot \operatorname{tg} 42 \Rightarrow$$

$$\Rightarrow 25 \operatorname{tg} 35 = x \cdot \operatorname{tg} 42 - x \cdot \operatorname{tg} 35 \Rightarrow$$

$$\Rightarrow x = \frac{25 \cdot \operatorname{tg} 35}{\operatorname{tg} 42 - \operatorname{tg} 35} = 87'44 \text{ m}$$

$$\Rightarrow y = x \cdot \operatorname{tg} 42 = 87'44 \cdot \operatorname{tg} 42 = 78'73 \text{ m}$$

b)



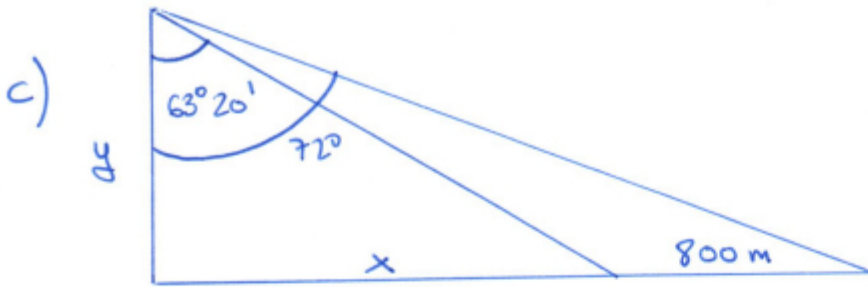
$$\left. \begin{aligned} \operatorname{tg} 38 &= \frac{y}{x} \\ \operatorname{tg} 44 &= \frac{y}{40-x} \end{aligned} \right\}$$

$$\left. \begin{aligned} y &= x \cdot \operatorname{tg} 38 \\ y &= (40-x) \cdot \operatorname{tg} 44 \end{aligned} \right\} \Rightarrow x \operatorname{tg} 38 = (40-x) \operatorname{tg} 44 \Rightarrow$$

$$\Rightarrow x \operatorname{tg} 38 = 38'63 - x \operatorname{tg} 44 \Rightarrow x \operatorname{tg} 38 + x \operatorname{tg} 44 = 38'63$$

$$\Rightarrow x = \frac{38'63}{\operatorname{tg} 38 + \operatorname{tg} 44} = 22'11 \text{ m}$$

$$\Rightarrow y = x \cdot \operatorname{tg} 38 = 22'11 \cdot \operatorname{tg} 38 = 17'27 \text{ m}$$



$$63^{\circ}20' = 63.333^{\circ} = \frac{190^{\circ}}{3}$$

$$\left. \begin{array}{l} \operatorname{tg}(63^{\circ}20') = \frac{x}{y} \\ \operatorname{tg}(72^{\circ}) = \frac{800+x}{y} \end{array} \right\} \begin{array}{l} y = \frac{x}{\operatorname{tg}(63^{\circ}20')} \\ y = \frac{800+x}{\operatorname{tg}(72^{\circ})} \end{array} \Rightarrow$$

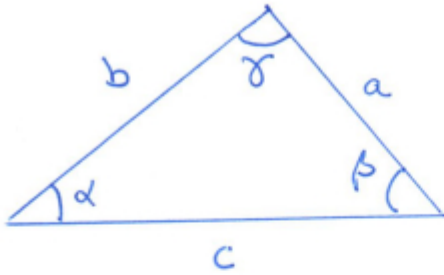
$$\Rightarrow \frac{x}{\operatorname{tg}(63^{\circ}20')} = \frac{800+x}{\operatorname{tg} 72^{\circ}} \Rightarrow x \cdot \operatorname{tg} 72^{\circ} = 800 \operatorname{tg}(63^{\circ}20') + x \cdot \operatorname{tg}(63^{\circ}20')$$

$$\Rightarrow x \operatorname{tg} 72 - x \operatorname{tg}(63^{\circ}20') = 800 \operatorname{tg}(63^{\circ}20') \Rightarrow$$

$$\Rightarrow x = \frac{800 \operatorname{tg}(63^{\circ}20')}{\operatorname{tg} 72^{\circ} - \operatorname{tg}(63^{\circ}20')} = \frac{1592'931}{1'08652} = 1466'08 \text{ m}$$

$$\Rightarrow y = \frac{x}{\operatorname{tg}(63^{\circ}20')} = \frac{1466'08}{\operatorname{tg}(63^{\circ}20')} = 736'29 \text{ m}$$

18) Resuelve los siguientes triángulos:



a) $\alpha = 58^\circ$; $\beta = 46^\circ$; $c = 120\text{ m}$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\boxed{\gamma = 76^\circ}$$

$$\frac{a}{\operatorname{sen} \alpha} = \frac{b}{\operatorname{sen} \beta} = \frac{c}{\operatorname{sen} \gamma} \Rightarrow \frac{a}{\operatorname{sen} 58} = \frac{b}{\operatorname{sen} 46} = \frac{120}{\operatorname{sen} 76} \Rightarrow$$

$$\Rightarrow \boxed{a = \frac{(\operatorname{sen} 58) \cdot 120}{\operatorname{sen} 76} = 104'88\text{ m}}$$

$$\boxed{b = \frac{120 \cdot \operatorname{sen} 46}{\operatorname{sen} 76} = 88'96\text{ m}}$$

b) $a = 183\text{ m}$; $b = 172\text{ m}$; $\alpha = 68^\circ$

$$\frac{a}{\operatorname{sen} \alpha} = \frac{b}{\operatorname{sen} \beta} = \frac{c}{\operatorname{sen} \gamma} \Rightarrow \frac{183}{\operatorname{sen} 68} = \frac{172}{\operatorname{sen} \beta} = \frac{c}{\operatorname{sen} \gamma}$$

$$\operatorname{sen} \beta = \frac{172 \cdot \operatorname{sen} 68}{183} = 0'8714 \Rightarrow \boxed{\beta = 60'628^\circ}$$

$$\alpha + \beta + \gamma = 180^\circ \Rightarrow \boxed{\gamma = 51'372^\circ}$$

$$\boxed{c = \frac{172 \cdot \operatorname{sen}(51'372)}{0'8714} = 154'2\text{ m}}$$

$$c) \ a = 20 \text{ cm}; \ c = 33 \text{ cm}; \ \beta = 53^\circ$$

$$\frac{a}{\operatorname{sen} \alpha} = \frac{b}{\operatorname{sen} \beta} = \frac{c}{\operatorname{sen} \gamma} \Rightarrow \frac{20}{\operatorname{sen} \alpha} = \frac{b}{\operatorname{sen} 53} = \frac{33}{\operatorname{sen} \gamma}$$

↳ No podemos despejar.

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

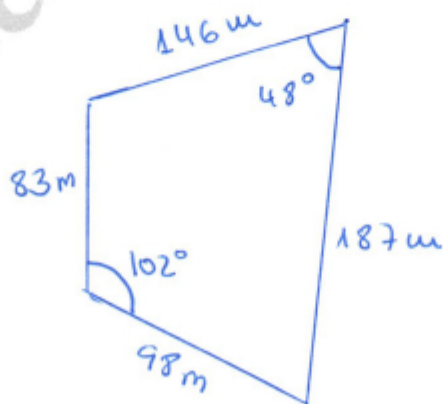
$$b^2 = 20^2 + 33^2 - 2 \cdot 20 \cdot 33 \cdot \cos 53 = 694'60417$$

$$b = \sqrt{694'60417} = 26'355 \text{ cm}$$

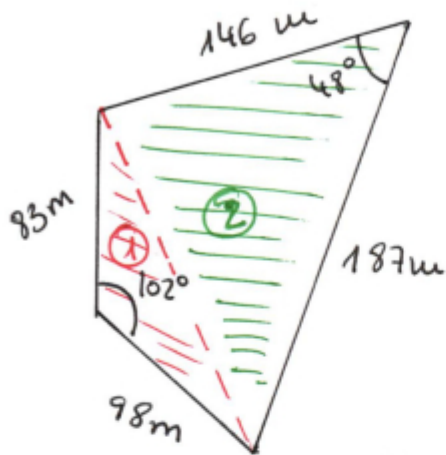
$$\frac{20}{\operatorname{sen} \alpha} = \frac{26'355}{\operatorname{sen} 53} \Rightarrow \operatorname{sen} \alpha = 0'60606 \Rightarrow \alpha = 37'305^\circ$$

$$\alpha + \beta + \gamma = 180^\circ \Rightarrow \gamma = 89'695^\circ$$

19) Halla el área del siguiente cuadrilátero:



Vamos a dividir la figura en dos triángulos



Triángulo 1:

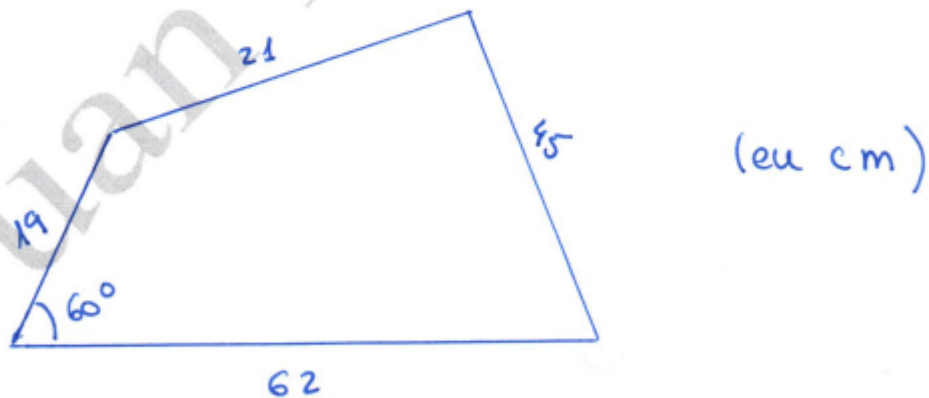
$$A_1 = \frac{1}{2} bc \operatorname{sen} \alpha = \frac{1}{2} 83 \cdot 98 \cdot \operatorname{sen} 102 = 3978'13 \text{ m}^2$$

Triángulo 2:

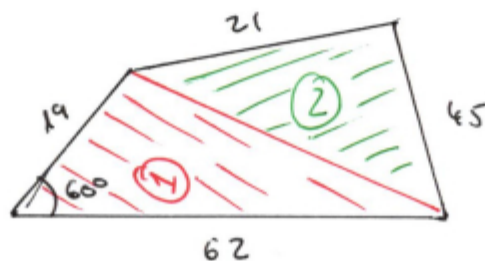
$$A_2 = \frac{1}{2} bc \operatorname{sen} \alpha = \frac{1}{2} 146 \cdot 187 \cdot \operatorname{sen} 48 = 10144'67 \text{ m}^2$$

$$\Rightarrow \boxed{A_{\text{TOTAL}} = A_1 + A_2 = 14122'8 \text{ m}^2}$$

20) Calcular el área de la siguiente figura:



Vamos a dividir la figura en dos triángulos:



Triángulo 1:

$$A_1 = \frac{1}{2} bc \operatorname{sen} \alpha = \frac{1}{2} \cdot 19 \cdot 62 \cdot \operatorname{sen} 60 = 510'089 \text{ cm}^2$$

Triángulo 2:

* Calculamos el lado que falta utilizando el triángulo 1:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha = 19^2 + 62^2 - 2 \cdot 19 \cdot 62 \cdot \cos 60 = 3027$$

$$\Rightarrow a = \sqrt{3027} = 55'018 \text{ cm}$$

* Conocidos los tres lados, se calcula el área utilizando la fórmula de Herón:

$$\text{Semiperímetro} \rightarrow p = \frac{a+b+c}{2} = \frac{55'018 + 21 + 45}{2} = 60'509$$

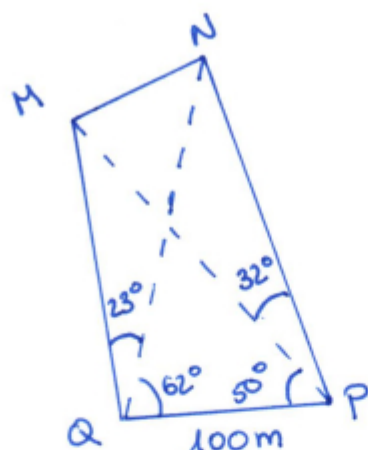
$$A_2 = \sqrt{p(p-a)(p-b)(p-c)} =$$

$$= \sqrt{60'509 \cdot (60'509 - 55'018) \cdot (60'509 - 21) \cdot (60'509 - 45)} =$$

$$= \sqrt{203587'567} = 451'207 \text{ cm}^2$$

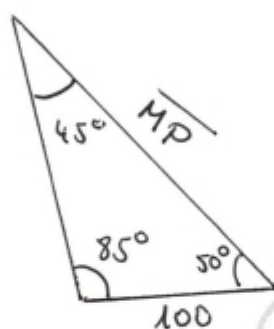
$$\Rightarrow \boxed{A_{\text{TOTAL}} = A_1 + A_2 = 510'089 + 451'207 = 961'296 \text{ cm}^2}$$

21) Calcular la distancia $\overline{MN}$



Triángulo
PQM

$\Rightarrow$

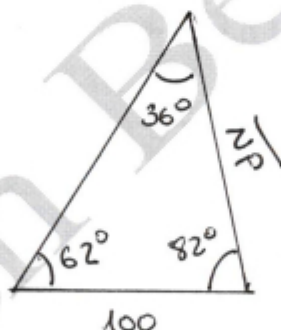


$$\frac{100}{\operatorname{sen} 45} = \frac{\overline{MP}}{\operatorname{sen} 85} \Rightarrow$$

$$\Rightarrow \overline{MP} = \frac{100 \operatorname{sen} 85}{\operatorname{sen} 45} = 140'88 \text{ m}$$

Triángulo
PQN

$\Rightarrow$

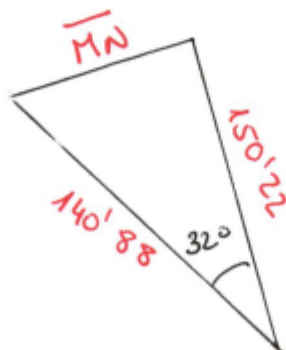


$$\frac{100}{\operatorname{sen} 36} = \frac{\overline{NP}}{\operatorname{sen} 62} \Rightarrow$$

$$\Rightarrow \overline{NP} = \frac{100 \operatorname{sen} 62}{\operatorname{sen} 36} = 150'22 \text{ m}$$

Triángulo
PMN

$\Rightarrow$



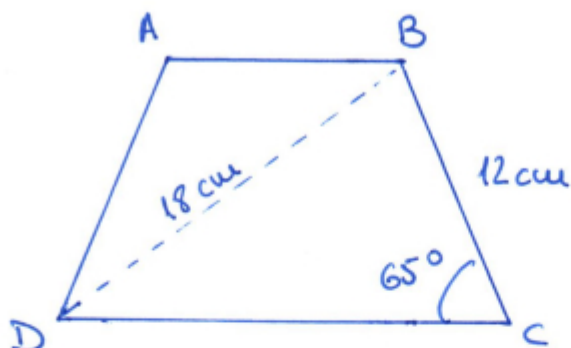
$$\overline{MN}^2 = \overline{MP}^2 + \overline{NP}^2 - 2 \overline{MP} \cdot \overline{NP} \cdot \cos 32$$

$$\overline{MN}^2 = 140'88^2 + 150'22^2 - 2 \cdot 140'88 \cdot 150'22 \cdot \cos$$

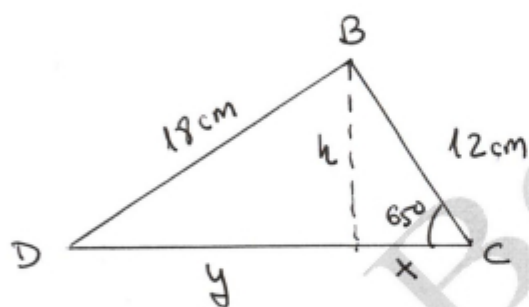
$$\overline{MN}^2 = 6518'75$$

$$\Rightarrow \boxed{\overline{MN} = \sqrt{6518'75} = 80'74 \text{ m}}$$

- 22) Hallar el perímetro y el área del siguiente trapecio isósceles:



Trazamos la altura y tendremos dos triángulos rectángulos



$$\operatorname{sen} 65 = \frac{h}{12} \Rightarrow h = 10'87 \text{ cm}$$

$$\cos 65 = \frac{x}{12} \Rightarrow x = 5'07 \text{ cm}$$

$$y^2 + h^2 = 18^2$$

$$y^2 = 18^2 - h^2 \Rightarrow y = \sqrt{18^2 - 10'87^2} = 14'35 \text{ cm}$$

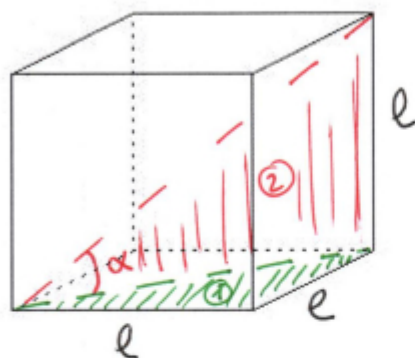
$$\text{Base Mayor} = \overline{DC} = y + x = 19'42 \text{ cm}$$

$$\text{Base menor} = \overline{AB} = \overline{DC} - 2x = 9'28 \text{ cm}$$

$$\boxed{\text{Perímetro} = 19'42 + 9'28 + 2 \cdot 12 = 52'7 \text{ cm}}$$

$$\boxed{\text{Área} = \left(\frac{B_{\text{Mayor}} + B_{\text{menor}}}{2} \right) \cdot h = \left(\frac{19'42 + 9'28}{2} \right) \cdot 10'87 = 155'48 \text{ cm}^2}$$

- ②③ Averigua el ángulo que forma la diagonal de la cara de un cubo con la diagonal del cubo:



Triángulo 1:

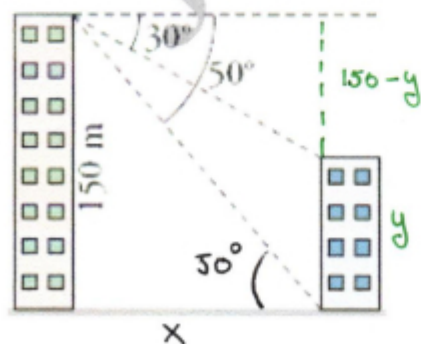
$$\begin{array}{c} x \\ \triangle \\ e \end{array} \quad l \Rightarrow x = \sqrt{l^2 + l^2} = \sqrt{2l^2} = l\sqrt{2}$$

Triángulo 2:

$$\begin{array}{c} \triangle \\ l\sqrt{2} \end{array} \quad l \quad \text{tg } \alpha = \frac{l}{l\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \alpha = \arctg\left(\frac{\sqrt{2}}{2}\right) = 35'26''$$

- ②④ Desde la terraza de un edificio de 150 m de altura medimos los ángulos que se indican en la figura. Calcula la altura del edificio más bajo y la anchura de la calle.



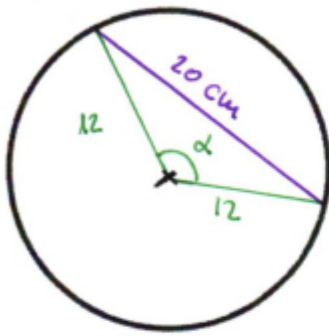
$$\text{tg } 50 = \frac{150}{x} \Rightarrow x = 125'865 \text{ m}$$

$$\text{tg } 30 = \frac{150-y}{x} \Rightarrow 150-y = x \cdot \text{tg } 30$$

$$150-y = 125'865 \cdot \text{tg } 30$$

$$150-y = 72'67 \Rightarrow y = 77'33 \text{ m}$$

- ②5 En una circunferencia de 12 cm de radio trazamos una cuerda de 20 cm de longitud. ¿Cuál es el ángulo correspondiente a esa cuerda?



Con el teorema del coseno:

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$20^2 = 12^2 + 12^2 - 2 \cdot 12 \cdot 12 \cdot \cos \alpha$$

$$400 = 288 - 288 \cos \alpha$$

$$\cos \alpha = -\frac{7}{18} \Rightarrow \alpha = \arccos\left(-\frac{7}{18}\right) = 112' 88''$$

- ②6 Calcular el área de un octógono regular inscrito en una circunferencia de 10 cm de radio.



Como vemos, tenemos 8 triángulos como el representado. Por ello se tendrá

$$\alpha = \frac{360^\circ}{8} = 45^\circ$$

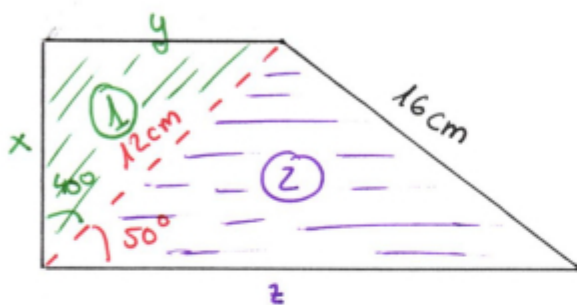
El área del triángulo:

$$A = \frac{1}{2} bc \sin \alpha = \frac{1}{2} \cdot 10 \cdot 10 \cdot \sin 45 = 35'35 \text{ cm}^2$$

Y como en el octógono tenemos 8 triángulitos:

$$A_{\text{oct}} = 8 \cdot A_{\text{tri}} = 8 \cdot 35'35 = 282'8 \text{ cm}^2$$

- (27) De un trapezio rectángulo conocemos el lado oblicuo, que mide 16 cm, la diagonal menor, que mide 12 cm, y el ángulo que ésta forma con la base mayor, de 50° . Calcula el área y el perímetro del trapezio.



Triángulo ①

$$\cos 40 = \frac{x}{12} \Rightarrow x = 9'19 \text{ cm}$$

$$\operatorname{sen} 40 = \frac{y}{12} \Rightarrow y = 7'71 \text{ cm}$$

$$A_1 = \frac{b \cdot h}{2} = \frac{x \cdot y}{2} = 35'43 \text{ cm}^2$$

Triángulo ②

$$16^2 = 12^2 + z^2 - 2 \cdot 12 \cdot z \cdot \cos 50$$

$$256 = 144 + z^2 - 15'427z \Rightarrow z^2 - 15'427z - 112 = 0$$

~~$z = \text{negativo}$~~

$z = 20'81 \text{ cm}$

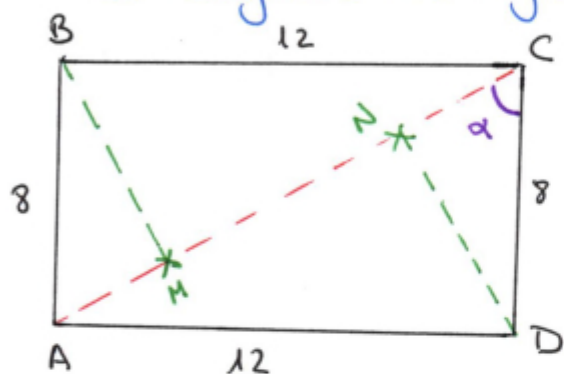
$$A_2 = \frac{1}{2} b c \operatorname{sen} \alpha = \frac{1}{2} \cdot 12 \cdot 20'81 \cdot \operatorname{sen} 50 = 95'65 \text{ cm}^2$$

Por tanto:

$$A = A_1 + A_2 = 35'43 + 95'65 = 131'08 \text{ cm}^2$$

$$P = x + y + z + 16 = 9'19 + 7'71 + 20'81 + 16 = 53'71 \text{ cm}$$

- (28) En un rectángulo ABCD de lados 8 cm y 12 cm, se traza desde B una perpendicular a la diagonal AC, y desde D, otra perpendicular a la misma diagonal. Sean M y N los puntos donde esas perpendiculares cortan a la diagonal. Halla la longitud del segmento $\overline{MN}$.



Triángulo ADC:

$$\overline{AC}^2 = \overline{AD}^2 + \overline{DC}^2 \quad (\text{Pitágoras})$$

$$\overline{AC}^2 = 12^2 + 8^2$$

$$\overline{AC} = \sqrt{208} = 14'42 \text{ cm}$$

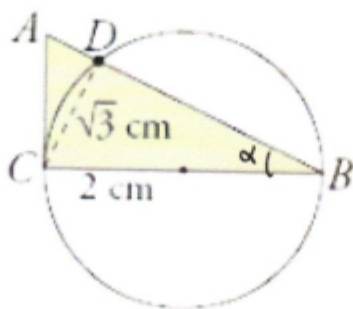
$$\text{tg } \alpha = \frac{12}{8} \Rightarrow \alpha = \arctg\left(\frac{12}{8}\right) = 56'31^\circ$$

Triángulo CND:

$$\cos \alpha = \frac{\overline{CN}}{8} \Rightarrow \overline{CN} = 8 \cdot \cos(56'31^\circ) = 4'44 \text{ cm}$$

$$\Rightarrow \overline{MN} = \overline{AC} - 2 \cdot \overline{CN} = 14'42 - 2 \cdot 4'44 = 5'54 \text{ cm}$$

- (29) El triángulo ABC es rectángulo en C. Sabemos que el radio de la circunferencia mide 2 cm y $\overline{CD} = \sqrt{3}$ cm. Calcula $\overline{AD}$ y $\overline{DB}$.



Triángulo BDC:

$$4^2 = (\sqrt{3})^2 + \overline{BD}^2 \Rightarrow \overline{BD} = \sqrt{13} = 3'6 \text{ cm}$$

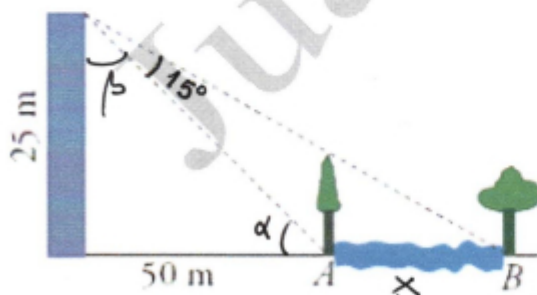
$$\operatorname{sen} \alpha = \frac{\sqrt{3}}{4}; \alpha = \operatorname{arcsen}\left(\frac{\sqrt{3}}{4}\right) = 25'66''$$

Triángulo BCA:

$$\cos \alpha = \frac{4}{\overline{AB}} \Rightarrow \overline{AB} = \frac{4}{\cos(25'66'')} = 4'44 \text{ cm}$$

$$\Rightarrow \overline{AD} = \overline{AB} - \overline{BD} = 4'44 - 3'6 = 0'84 \text{ cm}$$

- ③ Desde una torre de vigilancia de 25 m, observamos dos árboles situados en orillas opuestas de un río bajo un ángulo de 15° . Los dos árboles están alineados con el pie de la torre y la distancia de ésta al río es de 50 m. Calcula la anchura del río.



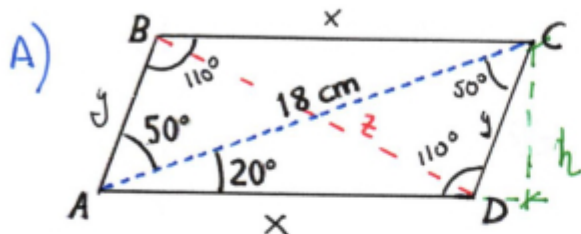
$$\operatorname{tg} \beta = \frac{50}{25} \Rightarrow \beta = \operatorname{arctg}(2) = 63'435''$$

$$\operatorname{tg}(63'435'' + 15'') = \frac{50 + x}{25}$$

$$4'88 = \frac{50 + x}{25} \Rightarrow$$

$$\Rightarrow 122'17 = 50 + x \Rightarrow x = 72'17 \text{ m}$$

- 31) Calcular la longitud de los lados, la longitud de las diagonales, los ángulos, y el área, de los siguientes cuadriláteros:



Triángulo ADC:

$$\frac{18}{\operatorname{sen} 110} = \frac{x}{\operatorname{sen} 50} = \frac{y}{\operatorname{sen} 20} \Rightarrow$$

$$\Rightarrow \begin{cases} x = \frac{18 \cdot \operatorname{sen} 50}{\operatorname{sen} 110} = 14'67 \text{ cm} \\ y = \frac{18 \cdot \operatorname{sen} 20}{\operatorname{sen} 110} = 6'55 \text{ cm} \end{cases}$$

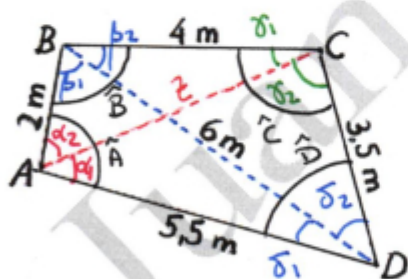
$A = b \cdot h = 14'67 \cdot 18 \operatorname{sen} 20 = 90'314 \text{ cm}^2$

paralelo-gramo

Triángulo ABD:

$$z^2 = 14'67^2 + 6'55^2 - 2 \cdot 14'67 \cdot 6'55 \cdot \cos 70 = 192'383 \Rightarrow z = 13'87 \text{ cm}$$

B)



Triángulo ABD:

$$6^2 = 2^2 + 5.5^2 - 2 \cdot 2 \cdot 5.5 \cdot \cos \hat{A} \Rightarrow$$

$$\Rightarrow \cos \hat{A} = -0'0795 \Rightarrow \hat{A} = 94'56^\circ$$

$$5.5^2 = 2^2 + 6^2 - 2 \cdot 2 \cdot 6 \cdot \cos \beta_1 \Rightarrow$$

$$\cos \beta_1 = 0'40625 \Rightarrow \beta_1 = 66'03^\circ$$

$$\delta_1 = 180 - \hat{A} - \beta_1 = 19'41^\circ$$

Triángulo BCD:

$$6^2 = 4^2 + 3.5^2 - 2 \cdot 4 \cdot 3.5 \cdot \cos \hat{C} \Rightarrow \cos \hat{C} = -0'2768 \Rightarrow \hat{C} = 106'07^\circ$$

$$3.5^2 = 6^2 + 4^2 - 2 \cdot 6 \cdot 4 \cdot \cos \beta_2 \Rightarrow \cos \beta_2 = 0'828 \Rightarrow \beta_2 = 34'09^\circ \Rightarrow \hat{B} = \beta_1 + \beta_2 = 100'12^\circ$$

$$\delta_2 = 180 - \hat{C} - \beta_2 = 39'84^\circ \Rightarrow \hat{D} = \delta_1 + \delta_2 = 59'25^\circ$$

$$\text{Área}_{ABD} = \frac{1}{2} \cdot 2 \cdot 5'5 \cdot \text{sen } \hat{A} = \frac{1}{2} \cdot 2 \cdot 5'5 \cdot \text{sen}(94'56'') = 5'48 \text{ m}^2$$

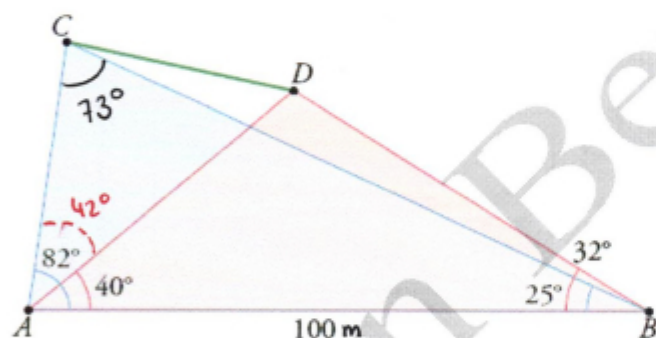
$$\text{Área}_{BCD} = \frac{1}{2} \cdot 4 \cdot 3'5 \cdot \text{sen } \hat{C} = \frac{1}{2} \cdot 4 \cdot 3'5 \cdot \text{sen}(106'07'') = 6'73 \text{ m}^2$$

$$A_{\text{total}} = 5'48 + 6'73 = 12'21 \text{ m}^2$$

Triángulo ABC:

$$z^2 = 2^2 + 4^2 - 2 \cdot 2 \cdot 4 \cdot \cos(100'12'') = 22'81 \Rightarrow z = \overline{AC} = 4'78 \text{ m}$$

(32) Dados los datos reflejados en la siguiente figura, calcular $\overline{CD}$



Triángulo ACB:

$$\hat{C} = 180 - 82 - 25 = 73^\circ$$

$$\frac{100}{\text{sen } 73} = \frac{\overline{AC}}{\text{sen } 25} \Rightarrow \overline{AC} = 44'193 \text{ m}$$

Triángulo ADB:

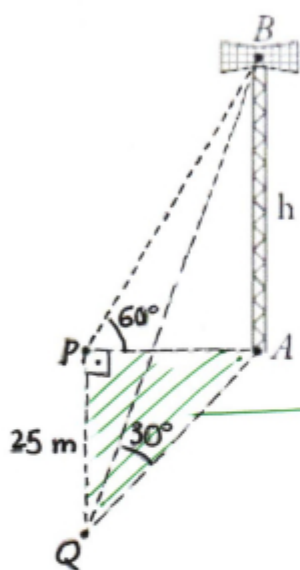
$$\hat{D} = 180 - 40 - 32 = 108^\circ$$

$$\frac{100}{\text{sen } 108} = \frac{\overline{AD}}{\text{sen } 32} \Rightarrow \overline{AD} = 55'72 \text{ m}$$

Triángulo ACD:

$$\overline{CD}^2 = 44'193^2 + 55'72^2 - 2 \cdot 44'193 \cdot 55'72 \cdot \cos 42 = 1397'80 \Rightarrow \overline{CD} = 37'39 \text{ m}$$

- (33) Para medir la altura de una antena, cuyo pie es inaccesible, nos situamos en un punto P al oeste de la antena y la observamos bajo un ángulo de 60° . Caminamos unos 25 m hacia el sur y desde Q el ángulo de observación es de 30° . Halla la altura de la antena.

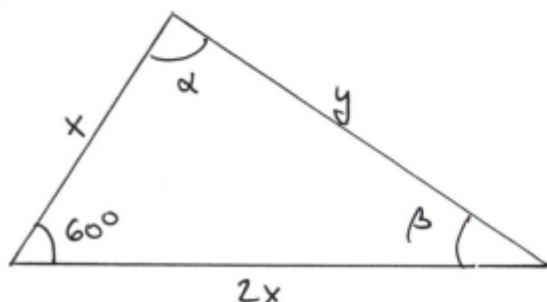


$$\left. \begin{aligned} \operatorname{tg} 60 &= \frac{h}{\overline{PA}} \Rightarrow h = \overline{PA} \cdot \operatorname{tg} 60 = \overline{PA} \cdot \sqrt{3} \\ \operatorname{tg} 30 &= \frac{h}{\overline{QA}} \Rightarrow h = \overline{QA} \cdot \operatorname{tg} 30 = \frac{\overline{QA} \cdot \sqrt{3}}{3} \end{aligned} \right\} \overline{PA} = \frac{\overline{QA}}{3}$$

$$\begin{aligned} \overline{QA}^2 &= 25^2 + \overline{PA}^2 \Rightarrow \overline{QA}^2 = 625 + \frac{\overline{QA}^2}{9} \Rightarrow \\ &\Rightarrow \frac{8 \overline{QA}^2}{9} = 625 \Rightarrow \overline{QA} = 26.52 \text{ m} \end{aligned}$$

$$\Rightarrow \boxed{h = \frac{\overline{QA} \cdot \sqrt{3}}{3} = \frac{26.52 \cdot \sqrt{3}}{3} = 15.31 \text{ m}}$$

- (34) Uno de los lados de un triángulo mide el doble que otro, y el ángulo comprendido entre ellos mide 60° . Halla los otros ángulos.



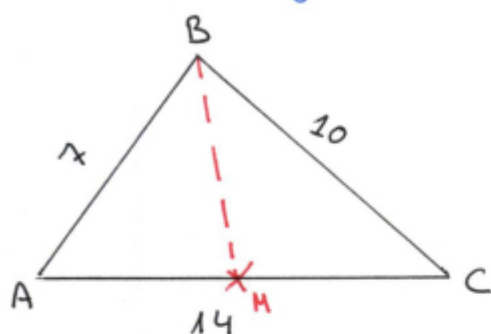
$$y^2 = x^2 + (2x)^2 - 2 \cdot x \cdot 2x \cdot \cos 60$$

$$y^2 = 5x^2 - 4x^2 \cdot \frac{1}{2} = 3x^2$$

$$y = x \cdot \sqrt{3}$$

$$\frac{x \sqrt{3}}{\sin 60} = \frac{x}{\sin \beta} \Rightarrow \sin \beta = \frac{\sin 60}{\sqrt{3}} = \frac{1}{2} \Rightarrow \beta = 30^\circ \Rightarrow \alpha = 90^\circ$$

- 35) En un triángulo ABC de lados $a=10\text{cm}$, $b=14\text{cm}$ y $c=7\text{cm}$, halla la longitud de la mediana que parte de B.



La mediana une un vértice con el punto medio del lado opuesto.

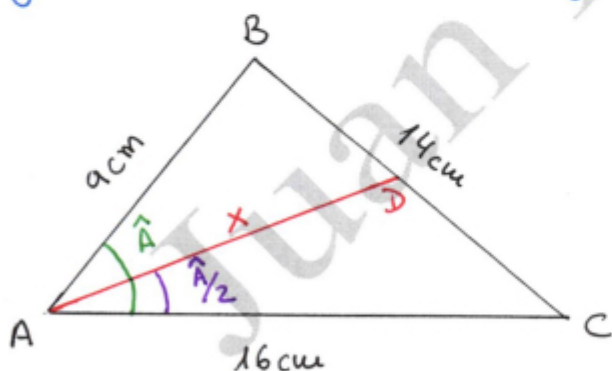
Triángulo ABC:

$$7^2 = 14^2 + 10^2 - 2 \cdot 14 \cdot 10 \cdot \cos \hat{C} \Rightarrow \cos \hat{C} = 0'8821 \Rightarrow \hat{C} = 28'1^\circ$$

Triángulo BMC:

$$\overline{BM}^2 = 10^2 + 7^2 - 2 \cdot 10 \cdot 7 \cdot \cos(28'1^\circ) = 25'50 \Rightarrow \boxed{\overline{BM} = 5'05\text{cm}}$$

- 36) De un triángulo ABC conocemos los tres lados, $a=14\text{cm}$, $b=16\text{cm}$ y $c=9\text{cm}$. Halla la longitud de la bisectriz del ángulo $\hat{A}$.



Triángulo ABC:

$$14^2 = 9^2 + 16^2 - 2 \cdot 9 \cdot 16 \cdot \cos \hat{A} \Rightarrow$$

$$\Rightarrow \cos \hat{A} = 0'4896 \Rightarrow \hat{A} = 60'69^\circ$$

$$9^2 = 16^2 + 14^2 - 2 \cdot 16 \cdot 14 \cdot \cos \hat{C} \Rightarrow \hat{C} = 34'09^\circ$$

Triángulo ADC:

$$\hat{D} = 180 - \frac{\hat{A}}{2} - \hat{C} = 115'565^\circ$$

$$\frac{x}{\sin(34'09^\circ)} = \frac{16}{\sin(115'565^\circ)} \Rightarrow \boxed{x = 9'94\text{cm}}$$

37) Resuelve las siguientes ecuaciones trigonométricas:

a) $2\cos x = \operatorname{tg} x$

$$2\cos x = \frac{\operatorname{sen} x}{\cos x} \Rightarrow 2\cos^2 x = \operatorname{sen} x \Rightarrow$$

$$\Rightarrow 2(1 - \operatorname{sen}^2 x) = \operatorname{sen} x \Rightarrow 2 - 2\operatorname{sen}^2 x = \operatorname{sen} x \Rightarrow$$

$$\Rightarrow 2\operatorname{sen}^2 x + \operatorname{sen} x - 2 = 0 ; \operatorname{sen} x = t$$

$$2t^2 + t - 2 = 0 \begin{cases} \rightarrow t = -1.281 \rightarrow \text{Imposible!!} \\ \rightarrow t = 0.781 \end{cases}$$

$$\operatorname{sen} x = 0.781 \Rightarrow x = \operatorname{arcsen}(0.781) =$$

$$\begin{cases} x = 51.35^\circ + K \cdot 360^\circ \\ x = 128.65^\circ + K \cdot 360^\circ \end{cases}$$

b) $\operatorname{sen}(2x) \cdot \cos x = 6\operatorname{sen}^3 x$

$$2\operatorname{sen} x \cdot \cos x \cdot \cos x = 6\operatorname{sen}^3 x \Rightarrow 2\operatorname{sen} x \cdot \cos^2 x = 6\operatorname{sen}^3 x \Rightarrow$$

$$\Rightarrow 2\operatorname{sen} x (1 - \operatorname{sen}^2 x) = 6\operatorname{sen}^3 x \Rightarrow 2\operatorname{sen} x - 2\operatorname{sen}^3 x = 6\operatorname{sen}^3 x \Rightarrow$$

$$\Rightarrow 8\operatorname{sen}^3 x - 2\operatorname{sen} x = 0 \Rightarrow \underbrace{2\operatorname{sen} x}_0 \left(\underbrace{4\operatorname{sen}^2 x - 1}_0 \right) = 0$$

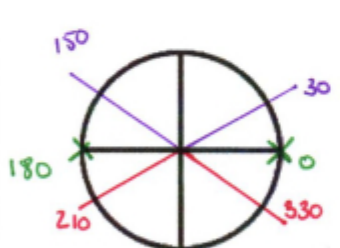
$$\boxed{2\operatorname{sen} x = 0} \Rightarrow x = \operatorname{arcsen}(0) = \begin{cases} x = 0^\circ + K \cdot 360^\circ \\ x = 180^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + K \cdot 180^\circ}$$

$$4 \operatorname{sen}^2 x - 1 = 0 \Rightarrow \operatorname{sen}^2 x = \frac{1}{4} \begin{cases} \Rightarrow \operatorname{sen} x = -\frac{1}{2} \\ \Rightarrow \operatorname{sen} x = +\frac{1}{2} \end{cases}$$

$$\boxed{\operatorname{sen} x = -\frac{1}{2}} \Rightarrow x = \operatorname{arcsen}\left(-\frac{1}{2}\right) \Rightarrow \begin{cases} x = 210^\circ + k \cdot 360^\circ \\ x = 330^\circ + k \cdot 360^\circ \end{cases}$$

$$\boxed{\operatorname{sen} x = +\frac{1}{2}} \Rightarrow x = \operatorname{arcsen}\left(\frac{1}{2}\right) \Rightarrow \begin{cases} x = 30^\circ + k \cdot 360^\circ \\ x = 150^\circ + k \cdot 360^\circ \end{cases}$$

Y si nos fijamos bien, podemos expresar las soluciones de manera más sencilla según:



$$\Rightarrow \begin{cases} x = 0^\circ + k \cdot 180^\circ \\ x = 30^\circ + k \cdot 180^\circ \\ x = 150^\circ + k \cdot 180^\circ \end{cases}$$

c) $4 \operatorname{sen}\left(\frac{x}{2}\right) + 2 \cos x = 3$

$$4 \operatorname{sen}\left(\frac{x}{2}\right) + 2 \cdot \cos\left(2 \cdot \frac{x}{2}\right) = 3$$

$$4 \operatorname{sen}\left(\frac{x}{2}\right) + 2 \cdot \left[\cos^2\left(\frac{x}{2}\right) - \operatorname{sen}^2\left(\frac{x}{2}\right)\right] = 3$$

$$4 \operatorname{sen}\left(\frac{x}{2}\right) + 2 \left[1 - \operatorname{sen}^2\left(\frac{x}{2}\right) - \operatorname{sen}^2\left(\frac{x}{2}\right)\right] = 3$$

$$4 \operatorname{sen}\left(\frac{x}{2}\right) + 2 - 4 \operatorname{sen}^2\left(\frac{x}{2}\right) = 3$$

$$4 \operatorname{sen}^2\left(\frac{x}{2}\right) - 4 \operatorname{sen}\left(\frac{x}{2}\right) + 1 = 0 ; \operatorname{sen}\left(\frac{x}{2}\right) = t$$

$$4t^2 - 4t + 1 = 0 \rightarrow t = 1/2$$

$$\operatorname{sen}\left(\frac{x}{2}\right) = \frac{1}{2} \Rightarrow \frac{x}{2} = \operatorname{arcsen}\left(\frac{1}{2}\right) \Rightarrow \begin{cases} \frac{x}{2} = 30^\circ + K \cdot 360^\circ \\ \frac{x}{2} = 150^\circ + K \cdot 360^\circ \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x = 60^\circ + K \cdot 720^\circ \\ x = 300^\circ + K \cdot 720^\circ \end{cases}$$

$$d) 2 \operatorname{tg}(x) - 3 \operatorname{cotg}(x) - 1 = 0$$

$$2 \operatorname{tg}(x) - \frac{3}{\operatorname{tg}(x)} - 1 = 0 \Rightarrow 2 \operatorname{tg}^2(x) - 3 - \operatorname{tg}(x) = 0$$

$$\Rightarrow 2 \operatorname{tg}^2(x) - \operatorname{tg}(x) - 3 = 0 ; \operatorname{tg}(x) = t$$

$$2t^2 - t - 3 = 0 \begin{cases} t = 3/2 \\ t = -1 \end{cases}$$

$$\boxed{\operatorname{tg}(x) = 3/2} \Rightarrow x = \operatorname{arctg}\left(\frac{3}{2}\right) \Rightarrow \begin{cases} x = 56'31'' + K \cdot 360^\circ \\ x = 236'31'' + K \cdot 360^\circ \end{cases} \Rightarrow$$

$$\Rightarrow \boxed{x = 56'31'' + K \cdot 180^\circ}$$

$$\boxed{\operatorname{tg}(x) = -1} \Rightarrow x = \arctg(-1) \Rightarrow \begin{cases} x = 135^\circ + K \cdot 360^\circ \\ x = 315^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 135^\circ + K \cdot 180^\circ}$$

$$e) \cos^2 x - 3 \operatorname{sen}^2 x = 0$$

$$1 - \operatorname{sen}^2 x - 3 \operatorname{sen}^2 x = 0$$

$$1 - 4 \operatorname{sen}^2 x = 0 \Rightarrow \operatorname{sen}^2 x = \frac{1}{4} \begin{cases} \rightarrow \operatorname{sen} x = -1/2 \\ \rightarrow \operatorname{sen} x = +1/2 \end{cases}$$

$$\boxed{\operatorname{sen}(x) = -1/2} \Rightarrow x = \operatorname{arcsen}(-1/2) \Rightarrow \begin{cases} x = 210^\circ + K \cdot 360^\circ \\ x = 330^\circ + K \cdot 360^\circ \end{cases}$$

Agrupando
 $\Rightarrow$

$$\boxed{\operatorname{sen}(x) = 1/2} \Rightarrow x = \operatorname{arcsen}(1/2) \Rightarrow \begin{cases} x = 30^\circ + K \cdot 360^\circ \\ x = 150^\circ + K \cdot 360^\circ \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} x = 150^\circ + K \cdot 180^\circ \\ x = 30^\circ + K \cdot 180^\circ \end{cases}}$$

$$f) \operatorname{sen}(2x+60^\circ) + \operatorname{sen}(x+30^\circ) = 0$$

$$\operatorname{sen}(2(x+30)) + \operatorname{sen}(x+30) = 0$$

$$2 \operatorname{sen}(x+30) \cdot \cos(x+30) + \operatorname{sen}(x+30) = 0$$

$$\underbrace{\operatorname{sen}(x+30)}_{!!} \left[\underbrace{2 \cos(x+30) + 1}_{!!} \right] = 0$$

$$\boxed{\operatorname{sen}(x+30) = 0} \Rightarrow x+30 = \operatorname{arcsen}(0) \Rightarrow \begin{cases} x+30 = 0 + k \cdot 360^\circ \\ x+30 = 180 + k \cdot 360^\circ \end{cases} \Rightarrow$$

$$\Rightarrow x+30 = 0 + k \cdot 180^\circ$$

$$\boxed{x = -30^\circ + k \cdot 180^\circ}$$

$$\boxed{2\cos(x+30) + 1 = 0} \Rightarrow \cos(x+30) = -1/2 \Rightarrow x+30 = \arccos(-1/2)$$

$$\Rightarrow \begin{cases} x+30^\circ = 120^\circ + k \cdot 360^\circ \Rightarrow x = 90^\circ + k \cdot 360^\circ \\ x+30^\circ = 240^\circ + k \cdot 360^\circ \Rightarrow x = 210^\circ + k \cdot 360^\circ \end{cases}$$

$$g) \operatorname{sen}^2 x - \cos^2 x = \frac{1}{2}$$

$$\cos^2 x - \operatorname{sen}^2 x = -1/2 \Rightarrow \cos(2x) = -1/2 \Rightarrow 2x = \arccos(-1/2)$$

$$\Rightarrow \begin{cases} 2x = 120^\circ + k \cdot 360^\circ \\ 2x = 240^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \begin{cases} x = 60^\circ + k \cdot 180^\circ \\ x = 120^\circ + k \cdot 180^\circ \end{cases}$$

$$h) \cos x \cdot \operatorname{tg} x = 1/2$$

$$\cancel{\cos x} \cdot \frac{\operatorname{sen} x}{\cancel{\cos x}} = 1/2 \Rightarrow \operatorname{sen} x = 1/2 \Rightarrow x = \operatorname{arcsen}(1/2) \Rightarrow$$

$$\Rightarrow \begin{cases} x = 30^\circ + k \cdot 360^\circ \\ x = 150^\circ + k \cdot 360^\circ \end{cases}$$

$$i) \cos(2x) + \operatorname{sen}(2x) = 1$$

$$\cos^2(x) - \operatorname{sen}^2(x) + 2\operatorname{sen}(x) \cdot \cos(x) = 1$$

$$\cancel{1} - \operatorname{sen}^2 x - \operatorname{sen}^2(x) + 2\operatorname{sen}(x) \cdot \cos(x) = \cancel{1}$$

$$- 2\operatorname{sen}^2(x) + 2\operatorname{sen}(x) \cdot \cos(x) = 0$$

$$\underbrace{2\operatorname{sen}(x)}_{!!} \left[\underbrace{-\operatorname{sen}(x) + \cos(x)}_{!!} \right] = 0$$

$$\boxed{2\operatorname{sen}(x) = 0} \Rightarrow x = \operatorname{arcsen}(0) \Rightarrow \begin{cases} x = 0^\circ + k \cdot 360^\circ \\ x = 180^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + k \cdot 180^\circ}$$

$$\boxed{-\operatorname{sen}(x) + \cos(x) = 0} \Rightarrow \operatorname{sen}(x) = \cos(x) \Rightarrow \frac{\operatorname{sen}(x)}{\cos(x)} = 1 \Rightarrow$$

$$\Rightarrow \operatorname{tg}(x) = 1 \Rightarrow x = \operatorname{arctg}(1) \Rightarrow \begin{cases} x = 45^\circ + k \cdot 360^\circ \\ x = 225^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 45^\circ + k \cdot 180^\circ}$$

$$j) \cos(2x) - \operatorname{sen}(2x) = 0$$

$$\cos(2x) = \operatorname{sen}(2x) \Rightarrow 1 = \frac{\operatorname{sen}(2x)}{\cos(2x)} \Rightarrow \operatorname{tg}(2x) = 1$$

$$\Rightarrow 2x = \operatorname{arctg}(1) \Rightarrow \begin{cases} 2x = 45^\circ + k \cdot 360^\circ \\ 2x = 225^\circ + k \cdot 360^\circ \end{cases} \Rightarrow 2x = 45^\circ + k \cdot 180^\circ$$

$$\Rightarrow \boxed{x = 22.5^\circ + k \cdot 90^\circ}$$

$$k) \operatorname{sen}(2x) + \cos(x) = 1$$

$$2\operatorname{sen}x \cos x + \cos(x) = 1$$

$$(2\operatorname{sen}x \cos x)^2 = (1 - \cos x)^2 \Rightarrow 4\operatorname{sen}^2 x \cos^2 x = 1 - 2\cos x + \cos^2 x$$

$$\Rightarrow 4(1 - \cos^2 x) \cos^2 x = 1 - 2\cos x + \cos^2 x \Rightarrow$$

$$\Rightarrow 4\cos^2 x - 4\cos^4 x = 1 - 2\cos x + \cos^2 x \Rightarrow$$

$$\Rightarrow -4\cos^4 x + 3\cos^2 x + 2\cos x - 1 = 0; \cos x = t$$

$$-4t^4 + 3t^2 + 2t - 1 = 0$$

1	-4	0	3	2	-1
		-4	-4	-1	1
	-4	-4	-1	1	0

$$-4t^3 - 4t^2 - t + 1 = 0 \begin{cases} \rightarrow t = 0.34781 \\ \rightarrow t = \text{Imaginarias} \end{cases}$$

↓
sacadas con ordenador!!

$$\cos x = 1 \Rightarrow x = \arccos(1) \Rightarrow x = 0^\circ + k \cdot 360^\circ$$

$$\cos x = 0'34781 \Rightarrow x = \arccos(0'34781) \Rightarrow \begin{cases} x = 69'6466^\circ + k \cdot 360^\circ \\ x = 290'3534^\circ + k \cdot 360^\circ \end{cases}$$

Ojo!! $\rightarrow$ Como hemos elevado al cuadrado, tenemos que comprobar las soluciones!!

$$x = 0 \Rightarrow \operatorname{sen}(2 \cdot 0) + \cos(0) = 1 \quad \checkmark$$

$$x = 69'6466^\circ \Rightarrow \operatorname{sen}(2 \cdot 69'6466) + \cos(69'6466) = 1 \quad \checkmark$$

$$x = 290'3534^\circ \Rightarrow \operatorname{sen}(2 \cdot 290'3534^\circ) + \cos(290'3534^\circ) = -0'3 \quad \times$$

$$\begin{cases} x = 0^\circ + k \cdot 360^\circ \\ x = 69'6466 + k \cdot 360^\circ \end{cases}$$

$$e) \operatorname{sen} x \cdot \cotg x = -1/4$$

$$\cancel{\operatorname{sen} x} \cdot \frac{\cos x}{\cancel{\operatorname{sen} x}} = -1/4 \Rightarrow \cos x = -1/4 \Rightarrow x = \arccos(-1/4) \Rightarrow$$

$$\Rightarrow \begin{cases} x = 104'4775^\circ + k \cdot 360^\circ \\ x = 255'5225^\circ + k \cdot 360^\circ \end{cases}$$

$$m) \operatorname{sen}(2x) + \operatorname{sen}(2x) = 0$$

$$2 \operatorname{sen}(2x) = 0 \Rightarrow \operatorname{sen}(2x) = 0 \Rightarrow 2x = \arcsen(0) \Rightarrow$$

$$\Rightarrow \begin{cases} 2x = 0^\circ + k \cdot 360^\circ \\ 2x = 180^\circ + k \cdot 360^\circ \end{cases} \Rightarrow 2x = 0^\circ + k \cdot 180^\circ \Rightarrow \boxed{x = 0^\circ + k \cdot 90^\circ}$$

$$n) \operatorname{tg}(x) + \operatorname{sen}(x) = 0$$

$$\frac{\operatorname{sen}(x)}{\cos(x)} + \operatorname{sen}(x) = 0 \Rightarrow \frac{\operatorname{sen}(x) + \operatorname{sen}(x) \cdot \cos(x)}{\cos(x)} = 0 \Rightarrow$$

$$\Rightarrow \underbrace{\operatorname{sen}(x)}_{\neq 0} \left(\underbrace{1 + \cos(x)}_{\neq 0} \right) = 0$$

$$\boxed{\operatorname{sen}(x) = 0} \Rightarrow x = \arcsen(0) \Rightarrow \begin{cases} x = 0^\circ + k \cdot 360^\circ \\ x = 180^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + k \cdot 180^\circ}$$

$$\boxed{1 + \cos x = 0} \Rightarrow \cos(x) = -1 \Rightarrow \boxed{x = \arccos(-1) = 180^\circ + k \cdot 360^\circ}$$

y agrupando en una única expresión:

$$\boxed{x = 0 + k \cdot 180^\circ}$$

$$o) \operatorname{tg}(x) - \operatorname{sen}(2x) = 0$$

$$\frac{\operatorname{sen}(x)}{\cos(x)} - 2\operatorname{sen}(x)\cos(x) = 0 \Rightarrow \frac{\operatorname{sen}(x) - 2\operatorname{sen}(x)\cos^2(x)}{\cos(x)} = 0 \Rightarrow$$

$$\Rightarrow \underbrace{\operatorname{sen}(x)}_{0} \left[\underbrace{1 - 2\cos^2(x)}_{0} \right] = 0$$

$$\boxed{\operatorname{sen}(x) = 0} \Rightarrow x = \operatorname{arcsen}(0) \Rightarrow \begin{cases} x = 0^\circ + k \cdot 360^\circ \\ x = 180^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + k \cdot 180^\circ}$$

$$\boxed{1 - 2\cos^2(x) = 0} \Rightarrow 2\cos^2(x) = 1 \Rightarrow \cos^2(x) = \frac{1}{2} \begin{cases} \rightarrow \cos x = -\frac{\sqrt{2}}{2} \\ \rightarrow \cos x = +\frac{\sqrt{2}}{2} \end{cases}$$

$$\cos x = -\frac{\sqrt{2}}{2} \Rightarrow x = \arccos\left(-\frac{\sqrt{2}}{2}\right) = \begin{cases} x = 135^\circ + k \cdot 360^\circ \\ x = 225^\circ + k \cdot 360^\circ \end{cases}$$

$$\cos x = +\frac{\sqrt{2}}{2} \Rightarrow x = \arccos\left(\frac{\sqrt{2}}{2}\right) = \begin{cases} x = 45^\circ + k \cdot 360^\circ \\ x = 315^\circ + k \cdot 360^\circ \end{cases}$$

Agrupamos
 $\Rightarrow$

$$\Rightarrow \boxed{\begin{cases} x = 45^\circ + k \cdot 180^\circ \\ x = 135^\circ + k \cdot 180^\circ \end{cases}}$$

$$p) \frac{\operatorname{sen}(60^\circ - x)}{\cos x} = 1$$

$$\operatorname{sen}(60 - x) = \cos x \Rightarrow$$

$$\Rightarrow \operatorname{sen} 60 \cdot \cos x - \cos 60 \operatorname{sen} x = \cos x$$

$$\frac{\sqrt{3}}{2} \cdot \cos x - \frac{1}{2} \operatorname{sen} x = \cos x$$

$$\frac{\sqrt{3}}{2} \cdot \cos x - \cos x = \frac{1}{2} \operatorname{sen} x$$

$$\cos x \left(\frac{\sqrt{3}}{2} - 1 \right) = \frac{1}{2} \operatorname{sen} x$$

$$\frac{\operatorname{sen} x}{\cos x} = 2 \left(\frac{\sqrt{3}}{2} - 1 \right) \Rightarrow \operatorname{tg} x = (\sqrt{3} - 2)$$

$$\Rightarrow x = \operatorname{arctg}(\sqrt{3} - 2) \Rightarrow \begin{cases} x = 165^\circ + k \cdot 360^\circ \\ x = 345^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 165^\circ + k \cdot 180^\circ}$$

$$q) \operatorname{tg}\left(\frac{\pi}{4} - x\right) + \operatorname{tg} x - 1 = 0$$

$$\frac{\operatorname{tg}\left(\frac{\pi}{4}\right) - \operatorname{tg} x}{1 + \operatorname{tg}\left(\frac{\pi}{4}\right) \cdot \operatorname{tg} x} + \operatorname{tg} x - 1 = 0 \Rightarrow \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} + \operatorname{tg} x - 1 = 0$$

$$\Rightarrow \frac{1 - \operatorname{tg} x + (\operatorname{tg} x - 1)(\operatorname{tg} x + 1)}{1 + \operatorname{tg} x} = 0 \Rightarrow \cancel{1} - \operatorname{tg} x + \operatorname{tg}^2 x - \cancel{1} = 0$$

$$\Rightarrow \operatorname{tg}^2 x - \operatorname{tg} x = 0 \Rightarrow \underbrace{\operatorname{tg} x}_0 (\underbrace{\operatorname{tg} x - 1}_0) = 0$$

$$\boxed{\operatorname{tg} x = 0} \Rightarrow x = \operatorname{arctg}(0) \Rightarrow \begin{cases} x = 0^\circ + k \cdot 360^\circ \\ x = 180^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + k \cdot 180^\circ}$$

$$\boxed{\operatorname{tg} x - 1 = 0} \Rightarrow x = \operatorname{arctg}(1) \Rightarrow \begin{cases} x = 45^\circ + k \cdot 360^\circ \\ x = 225^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 45^\circ + k \cdot 180^\circ}$$

$$r) \operatorname{sen}(x+30^\circ) + \cos(x+60^\circ) = 1 + \cos(2x)$$

$$\operatorname{sen} x \cdot \cos 30 + \cos x \operatorname{sen} 30 + \cos x \cdot \cos 60 - \operatorname{sen} x \cdot \operatorname{sen} 60 = 1 + \cos(2x)$$

$$\frac{\sqrt{3}}{2} \operatorname{sen} x + \frac{1}{2} \cos x + \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \operatorname{sen} x = 1 + \cos^2 x - \operatorname{sen}^2 x$$

$$\cos x = \cos^2 x + \operatorname{sen}^2 x + \cos^2 x - \operatorname{sen}^2 x$$

$$\cos x = 2\cos^2 x \Rightarrow 2\cos^2 x - \cos x = 0 \Rightarrow \underbrace{\cos x}_{!!} (\underbrace{2\cos x - 1}_{!!}) = 0$$

$$\boxed{\cos x = 0} \Rightarrow x = \arccos(0) \Rightarrow \begin{cases} x = 90^\circ + k \cdot 360^\circ \\ x = 270^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 90^\circ + k \cdot 180^\circ}$$

$$\boxed{2\cos x - 1 = 0} \Rightarrow x = \arccos\left(\frac{1}{2}\right) \Rightarrow \begin{cases} x = 60^\circ + k \cdot 360^\circ \\ x = 300^\circ + k \cdot 360^\circ \end{cases}$$

$$s) \quad \operatorname{tg}(x+45^\circ) + \operatorname{tg}(x-45^\circ) = 2 \operatorname{ctg} x$$

$$\frac{\operatorname{tg} x + \operatorname{tg} 45}{1 - \operatorname{tg} x \cdot \operatorname{tg} 45} + \frac{\operatorname{tg} x - \operatorname{tg} 45}{1 + \operatorname{tg} x \cdot \operatorname{tg} 45} = \frac{2}{\operatorname{tg} x}$$

$$\frac{\operatorname{tg} x + 1}{1 - \operatorname{tg} x} + \frac{\operatorname{tg} x - 1}{1 + \operatorname{tg} x} = \frac{2}{\operatorname{tg} x}$$

$$\frac{(1 + \operatorname{tg} x)^2 + (\operatorname{tg} x - 1)(1 - \operatorname{tg} x)}{(1 - \operatorname{tg} x)(1 + \operatorname{tg} x)} = \frac{2}{\operatorname{tg} x}$$

$$\frac{(1 + \operatorname{tg} x)^2 - (1 - \operatorname{tg} x)^2}{(1 - \operatorname{tg} x)(1 + \operatorname{tg} x)} = \frac{2}{\operatorname{tg} x}$$

$$\frac{1 + 2\operatorname{tg} x + \operatorname{tg}^2 x - (1 - 2\operatorname{tg} x + \operatorname{tg}^2 x)}{1 - \operatorname{tg}^2 x} = \frac{2}{\operatorname{tg} x} \Rightarrow$$

$$\frac{4\operatorname{tg} x}{1 - \operatorname{tg}^2 x} = \frac{2}{\operatorname{tg} x} \Rightarrow 4\operatorname{tg}^2 x = 2 - 2\operatorname{tg}^2 x \Rightarrow$$

$$\Rightarrow 6\operatorname{tg}^2 x = 2 \Rightarrow \operatorname{tg}^2 x = \frac{1}{3} \begin{cases} \rightarrow \operatorname{tg} x = -\frac{\sqrt{3}}{3} \\ \rightarrow \operatorname{tg} x = +\frac{\sqrt{3}}{3} \end{cases}$$

$$\boxed{\operatorname{tg} x = -\frac{\sqrt{3}}{3}} \Rightarrow x = \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) \Rightarrow \begin{cases} x = 150^\circ + k \cdot 360^\circ \\ x = 330^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 150^\circ + k \cdot 180^\circ}$$

$$\boxed{\operatorname{tg} x = +\frac{\sqrt{3}}{3}} \Rightarrow x = \operatorname{arctg}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow \begin{cases} x = 30^\circ + k \cdot 360^\circ \\ x = 210^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 30^\circ + k \cdot 180^\circ}$$

$$t) \quad 4 \operatorname{sen}[2(x+30)] = 1$$

$$\operatorname{sen}[2(x+30)] = \frac{1}{4} \Rightarrow 2(x+30) = \operatorname{arcsen}\left(\frac{1}{4}\right) \Rightarrow$$

$$\Rightarrow \begin{cases} 2(x+30) = 14^\circ 47' 75'' + k \cdot 360^\circ \\ 2(x+30) = 165^\circ 52' 25'' + k \cdot 360^\circ \end{cases} \Rightarrow \begin{cases} x+30 = 7^\circ 23' 87.5'' + k \cdot 180^\circ \\ x+30 = 82^\circ 76' 12.5'' + k \cdot 180^\circ \end{cases} \Rightarrow$$

$$\Rightarrow \boxed{\begin{cases} x = -22^\circ 76' 12.5'' + k \cdot 180^\circ \\ x = 52^\circ 76' 12.5'' + k \cdot 180^\circ \end{cases}}$$

$$u) \quad \operatorname{tg}\left(\frac{\pi}{4} - x\right) - 1 = 0$$

$$\operatorname{tg}\left(\frac{\pi}{4} - x\right) = 1 \Rightarrow \frac{\pi}{4} - x = \operatorname{arctg}(1) \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{\pi}{4} - x = \frac{\pi}{4} + k \cdot 2\pi \\ \frac{\pi}{4} - x = \frac{5\pi}{4} + k \cdot 2\pi \end{cases} \Rightarrow \frac{\pi}{4} - x = \frac{\pi}{4} + k \cdot \pi \Rightarrow$$

$$\Rightarrow x = \frac{\pi}{4} - \left(\frac{\pi}{4} + k\pi\right) = -k\pi \Rightarrow \boxed{x = k\pi}$$

$$v) \operatorname{sen}(x+30) - \cos x = 0$$

$$\operatorname{sen} x \cdot \cos 30 + \cos x \cdot \operatorname{sen} 30 - \cos x = 0$$

$$\frac{\sqrt{3}}{2} \cdot \operatorname{sen} x + \frac{1}{2} \cos x - \cos x = 0$$

$$\frac{\sqrt{3}}{2} \operatorname{sen} x - \frac{1}{2} \cos x = 0$$

$$\frac{\sqrt{3}}{2} \operatorname{sen} x = \frac{1}{2} \cos x \Rightarrow \frac{\operatorname{sen} x}{\cos x} = \frac{1}{\sqrt{3}} \Rightarrow \operatorname{tg} x = \frac{\sqrt{3}}{3} \Rightarrow$$

$$\Rightarrow x = \operatorname{arctg}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow \begin{cases} x = 30^\circ + k \cdot 360^\circ \\ x = 210^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 30^\circ + k \cdot 180^\circ}$$

$$w) 4 \operatorname{sen} x - \sec x = 0$$

$$4 \operatorname{sen} x - \frac{1}{\cos x} = 0 \Rightarrow 4 \operatorname{sen} x = \frac{1}{\cos x} \Rightarrow 4 \operatorname{sen} x \cos x = 1$$

$$\Rightarrow 2 \cdot 2 \cdot \operatorname{sen} x \cos x = 1 \Rightarrow 2 \cdot \operatorname{sen}(2x) = 1 \Rightarrow \operatorname{sen}(2x) = \frac{1}{2}$$

$$\Rightarrow 2x = \operatorname{arcsen}\left(\frac{1}{2}\right) \Rightarrow \begin{cases} 2x = 30^\circ + k \cdot 360^\circ \\ 2x = 150^\circ + k \cdot 360^\circ \end{cases} \Rightarrow$$

$$\Rightarrow \boxed{\begin{cases} x = 15^\circ + k \cdot 180^\circ \\ x = 75^\circ + k \cdot 180^\circ \end{cases}}$$

$$x) \frac{\cos^2 x}{2\cos x + \operatorname{sen} x} = \operatorname{sen} x$$

$$\cos^2 x = 2\operatorname{sen} x \cos x + \operatorname{sen}^2 x \Rightarrow$$

$$\Rightarrow \cos^2 x - \operatorname{sen}^2 x = 2\operatorname{sen} x \cos x$$

$$\cos(2x) = \operatorname{sen}(2x) \Rightarrow \frac{\operatorname{sen}(2x)}{\cos(2x)} = 1 \Rightarrow \operatorname{tg}(2x) = 1$$

$$\Rightarrow 2x = \arctg(1) \Rightarrow \begin{cases} 2x = 45^\circ + k \cdot 360^\circ \\ 2x = 225^\circ + k \cdot 360^\circ \end{cases} \Rightarrow 2x = 45^\circ + k \cdot 180^\circ \Rightarrow$$

$$\Rightarrow \boxed{x = 22.5^\circ + k \cdot 90^\circ}$$

$$y) \frac{1}{\cos x + \operatorname{sen} x} + 2\operatorname{sen} x = 2\cos x$$

$$\frac{1 + 2\operatorname{sen} x \cos x + 2\operatorname{sen}^2 x}{\cos x + \operatorname{sen} x} = 2\cos x \Rightarrow$$

$$\Rightarrow 1 + \cancel{2\operatorname{sen} x \cos x} + 2\operatorname{sen}^2 x = 2\cos^2 x + \cancel{2\operatorname{sen} x \cos x} \Rightarrow$$

$$\Rightarrow 1 = 2\cos^2 x - 2\operatorname{sen}^2 x \Rightarrow 1 = 2(\cos^2 x - \operatorname{sen}^2 x) \Rightarrow 1 = 2\cos(2x)$$

$$\Rightarrow \cos(2x) = \frac{1}{2} \Rightarrow 2x = \arccos\left(\frac{1}{2}\right) \Rightarrow \begin{cases} 2x = 60^\circ + k \cdot 360^\circ \\ 2x = 300^\circ + k \cdot 360^\circ \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x = 30^\circ + k \cdot 180^\circ \\ x = 150^\circ + k \cdot 180^\circ \end{cases}$$

$$z) \operatorname{sen}(x) \cdot [\operatorname{sen}(x) - 1] = 5 \cos^2 x - 4$$

$$\operatorname{sen}^2(x) - \operatorname{sen}(x) = 5(1 - \operatorname{sen}^2(x)) - 4$$

$$\operatorname{sen}^2(x) - \operatorname{sen}(x) = 5 - 5\operatorname{sen}^2(x) - 4$$

$$6\operatorname{sen}^2(x) - \operatorname{sen}(x) - 1 = 0; \operatorname{sen}(x) = t$$

$$6t^2 - t - 1 = 0 \begin{cases} \rightarrow t = 1/2 \\ \rightarrow t = -1/3 \end{cases}$$

$$\boxed{\operatorname{sen} x = 1/2} \Rightarrow x = \operatorname{arcsen}\left(\frac{1}{2}\right) = \begin{cases} x = 30^\circ + K \cdot 360^\circ \\ x = 150^\circ + K \cdot 360^\circ \end{cases}$$

$$\boxed{\operatorname{sen} x = -1/3} \Rightarrow x = \operatorname{arcsen}\left(-\frac{1}{3}\right) = \begin{cases} x = 199'47^\circ + K \cdot 360^\circ \\ x = 340'53^\circ + K \cdot 360^\circ \end{cases}$$

$$\alpha) 2 \cos(x) - 1 = \sec(x)$$

$$2 \cos(x) - 1 = \frac{1}{\cos(x)} \Rightarrow 2 \cos^2(x) - \cos(x) = 1; \cos x = t$$

$$2t^2 - t - 1 = 0 \begin{cases} \rightarrow t = 1 \\ \rightarrow t = -1/2 \end{cases}$$

$$\boxed{\cos x = 1} \Rightarrow x = \arccos(1) = 0^\circ + K \cdot 360^\circ$$

$$\boxed{\cos x = -1/2} \Rightarrow x = \arccos(-1/2) \Rightarrow \begin{cases} x = 120^\circ + K \cdot 360^\circ \\ x = 240^\circ + K \cdot 360^\circ \end{cases}$$

$$\beta) 2\cos x + \operatorname{sen} x = 1$$

$$2\cos x + \sqrt{1 - \cos^2 x} = 1 \Rightarrow \left(\sqrt{1 - \cos^2 x}\right)^2 = (1 - 2\cos x)^2 \Rightarrow$$

$$\Rightarrow 1 - \cos^2 x = 1 - 4\cos x + 4\cos^2 x \Rightarrow$$

$$\Rightarrow 5\cos^2 x - 4\cos x = 0 \Rightarrow \underbrace{\cos x}_0 (5\underbrace{\cos x - 4}_0) = 0$$

$$\boxed{\cos x = 0} \Rightarrow x = \arccos(0) \Rightarrow \begin{cases} x = 90^\circ + k \cdot 360^\circ \\ x = 270^\circ + k \cdot 360^\circ \end{cases}$$

$$\boxed{5\cos x - 4 = 0} \Rightarrow x = \arccos\left(\frac{4}{5}\right) \Rightarrow \begin{cases} x = 36^\circ 52' + k \cdot 360^\circ \\ x = 323^\circ 8' + k \cdot 360^\circ \end{cases}$$

Como hemos elevado al cuadrado, hay que comprobar las soluciones:

$$x = 90^\circ \rightarrow 2 \cdot \cos 90 + \operatorname{sen} 90 = 1 \quad \checkmark$$

$$x = 270^\circ \rightarrow 2 \cdot \cos 270 + \operatorname{sen} 270 = -1 \quad \times$$

$$x = 36^\circ 52' \rightarrow 2 \cdot \cos 36^\circ 52' + \operatorname{sen} 36^\circ 52' = 2.2 \quad \times$$

$$x = 323^\circ 8' \rightarrow 2 \cdot \cos 323^\circ 8' + \operatorname{sen} 323^\circ 8' = 1 \quad \checkmark$$

$$\boxed{\begin{cases} x = 90^\circ + k \cdot 360^\circ \\ x = 323^\circ 8' + k \cdot 360^\circ \end{cases}}$$

$$8) \operatorname{sen} x + \cos x = 0$$

$$\operatorname{sen} x = -\cos x \Rightarrow \frac{\operatorname{sen} x}{\cos x} = -1 \Rightarrow \operatorname{tg} x = -1 \Rightarrow$$

$$\Rightarrow x = \operatorname{arctg}(-1) \Rightarrow \begin{cases} x = 135^\circ + K \cdot 360^\circ \\ x = 315^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 135^\circ + K \cdot 180^\circ}$$

(38) Resuelve los siguientes sistemas de ecuaciones trigonométricas:

$$a) \begin{cases} x + y = 60 \\ \cos x = \operatorname{sen} y \end{cases} \quad y = 60 - x$$

$$\cos x = \operatorname{sen}(60 - x) \Rightarrow$$

$$\cos x = \operatorname{sen} 60 \cdot \cos x - \cos 60 \cdot \operatorname{sen} x \Rightarrow$$

$$\cos x = \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \operatorname{sen} x \Rightarrow$$

$$\cos x - \frac{\sqrt{3}}{2} \cos x = -\frac{1}{2} \operatorname{sen} x \Rightarrow$$

$$\Rightarrow \cos x \left(1 - \frac{\sqrt{3}}{2} \right) = -\frac{1}{2} \operatorname{sen} x \Rightarrow \left(\frac{2 - \sqrt{3}}{2} \right) \cos x = -\frac{1}{2} \operatorname{sen} x$$

$$\Rightarrow \frac{\operatorname{sen} x}{\cos x} = \sqrt{3} - 2 \Rightarrow \operatorname{tg}(x) = \sqrt{3} - 2 \Rightarrow$$

$$\Rightarrow x = \operatorname{arctg}(\sqrt{3} - 2) \Rightarrow \begin{cases} x = 165^\circ + K \cdot 360^\circ \\ x = 345^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 165^\circ + K \cdot 180^\circ}$$

$$\Rightarrow \boxed{y = 60 - x = 60 - (165^\circ + k \cdot 180^\circ) = -105^\circ - k \cdot 180^\circ}$$

$$\Rightarrow \boxed{(x, y) = (165^\circ + k \cdot 180^\circ, -105^\circ - k \cdot 180^\circ)}$$

$$\left. \begin{array}{l} b) \ y - x = 30^\circ \\ \operatorname{tg} y = \operatorname{tg}(y - 3x) \end{array} \right\} y = 30 + x$$

$$\operatorname{tg}(30 + x) = \operatorname{tg}(30 + x - 3x)$$

$$\operatorname{tg}(30 + x) = \operatorname{tg}(30 - 2x)$$

$$\frac{\operatorname{tg} 30 + \operatorname{tg} x}{1 - \operatorname{tg} 30 \cdot \operatorname{tg} x} = \frac{\operatorname{tg} 30 - \operatorname{tg}(2x)}{1 + \operatorname{tg} 30 \cdot \operatorname{tg} 2x} \Rightarrow$$

$$\Rightarrow \frac{\frac{\sqrt{3}}{3} + \operatorname{tg} x}{1 - \frac{\sqrt{3}}{3} \cdot \operatorname{tg} x} = \frac{\frac{\sqrt{3}}{3} - \operatorname{tg}(2x)}{1 + \frac{\sqrt{3}}{3} \cdot \operatorname{tg}(2x)} \Rightarrow \frac{\frac{\sqrt{3} + 3 \operatorname{tg} x}{3}}{\frac{3 - \sqrt{3} \operatorname{tg} x}{3}} = \frac{\frac{\sqrt{3} - 3 \operatorname{tg}(2x)}{3}}{\frac{3 + \sqrt{3} \operatorname{tg}(2x)}{3}}$$

$$\Rightarrow \frac{\sqrt{3} + 3 \operatorname{tg} x}{3 - \sqrt{3} \operatorname{tg} x} = \frac{\sqrt{3} - 3 \operatorname{tg}(2x)}{3 + \sqrt{3} \operatorname{tg}(2x)} \Rightarrow$$

$$\Rightarrow \frac{\sqrt{3} + 3 \operatorname{tg} x}{3 - \sqrt{3} \operatorname{tg} x} = \frac{\sqrt{3} - 3 \cdot \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}}{3 + \sqrt{3} \cdot \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}} \Rightarrow$$

$$\Rightarrow \frac{\sqrt{3} + 3 \operatorname{tg} x}{3 - \sqrt{3} \operatorname{tg} x} = \frac{\frac{\sqrt{3} - \sqrt{3} \cdot \operatorname{tg}^2 x - 6 \operatorname{tg} x}{\cancel{1 - \operatorname{tg}^2 x}}}{\frac{3 - 3 \operatorname{tg}^2 x + 2\sqrt{3} \operatorname{tg} x}{\cancel{1 - \operatorname{tg}^2 x}}} \Rightarrow$$

$$\Rightarrow \frac{\sqrt{3} + 3 \operatorname{tg} x}{3 - \sqrt{3} \operatorname{tg} x} = \frac{\sqrt{3} - \sqrt{3} \cdot \operatorname{tg}^2 x - 6 \operatorname{tg} x}{3 - 3 \operatorname{tg}^2 x + 2\sqrt{3} \operatorname{tg} x} \Rightarrow$$

$$\Rightarrow (\sqrt{3} + 3 \operatorname{tg} x) \cdot (3 - 3 \operatorname{tg}^2 x + 2\sqrt{3} \operatorname{tg} x) = (3 - \sqrt{3} \operatorname{tg} x) (\sqrt{3} - \sqrt{3} \cdot \operatorname{tg}^2 x - 6 \operatorname{tg} x)$$

$$\Rightarrow \cancel{3\sqrt{3}} - \cancel{3\sqrt{3}\operatorname{tg}^2 x} + 6 \operatorname{tg} x + 9 \operatorname{tg} x - 9 \operatorname{tg}^3 x + \cancel{6\sqrt{3}\operatorname{tg}^2 x} =$$

$$= \cancel{3\sqrt{3}} - \cancel{3\sqrt{3}\operatorname{tg}^2 x} - 18 \operatorname{tg} x - 3 \operatorname{tg} x + 3 \operatorname{tg}^3 x + \cancel{6\sqrt{3}\operatorname{tg}^2 x} \Rightarrow$$

$$\Rightarrow 15 \operatorname{tg} x - 9 \operatorname{tg}^3 x = -21 \operatorname{tg} x + 3 \operatorname{tg}^3 x \Rightarrow$$

$$\Rightarrow 12 \operatorname{tg}^3 x - 36 \operatorname{tg} x = 0 \Rightarrow \underbrace{12 \operatorname{tg} x}_{!!} (\underbrace{\operatorname{tg}^2 x - 3}_{!!}) = 0$$

$$\boxed{12 \operatorname{tg} x = 0} \Rightarrow x = \operatorname{arctg}(0) \Rightarrow \begin{cases} x = 0^\circ + K \cdot 360^\circ \\ x = 180^\circ + K \cdot 360^\circ \end{cases} \Rightarrow x = 0^\circ + K \cdot 180^\circ$$

$$\Rightarrow y = 30^\circ + x = 30^\circ + (0^\circ + K \cdot 180^\circ) = 30^\circ + K \cdot 180^\circ$$

$$\boxed{\operatorname{tg}^2 x - 3 = 0} \Rightarrow \operatorname{tg}^2 x = 3 \begin{cases} \rightarrow \operatorname{tg} x = +\sqrt{3} \\ \rightarrow \operatorname{tg} x = -\sqrt{3} \end{cases}$$

$$\operatorname{tg} x = +\sqrt{3} \Rightarrow x = \arctg(\sqrt{3}) \Rightarrow \begin{cases} x = 60^\circ + k \cdot 360^\circ \\ x = 240^\circ + k \cdot 360^\circ \end{cases} \Rightarrow x = 60^\circ + k \cdot 180^\circ$$

$$\Rightarrow y = 30^\circ + x = 30^\circ + (60^\circ + k \cdot 180^\circ) = 90^\circ + k \cdot 180^\circ$$

$$\operatorname{tg} x = -\sqrt{3} \Rightarrow x = \arctg(-\sqrt{3}) \Rightarrow \begin{cases} x = 120^\circ + k \cdot 360^\circ \\ x = 300^\circ + k \cdot 360^\circ \end{cases} \Rightarrow x = 120^\circ + k \cdot 180^\circ$$

$$\Rightarrow y = 30^\circ + x = 30^\circ + (120^\circ + k \cdot 180^\circ) = 150^\circ + k \cdot 180^\circ$$

Las soluciones válidas son por tanto:

$$\begin{aligned} (x, y) &= (0^\circ + k \cdot 180^\circ, 30^\circ + k \cdot 180^\circ) \\ (x, y) &= (120^\circ + k \cdot 180^\circ, 150^\circ + k \cdot 180^\circ) \end{aligned}$$

Nota! $\rightarrow$ la solución $(x, y) = (60^\circ + k \cdot 180^\circ, 90^\circ + k \cdot 180^\circ)$ no es una solución válida pues $\nexists \operatorname{tg} 90^\circ$

$$\downarrow$$

2ª ecuación $\rightarrow \operatorname{tg} y = \operatorname{tg}(y - 3x)$

$$c) \begin{cases} \operatorname{sen} x + 2 \cos y = 2 \\ 3 \operatorname{sen} x - 4 \cos y = 1 \end{cases} \xrightarrow{2E_1} \begin{cases} 2 \operatorname{sen} x + 4 \cos y = 4 \\ 3 \operatorname{sen} x - 4 \cos y = 1 \end{cases}$$

$$5 \operatorname{sen} x = 5$$

$$\operatorname{sen} x = 1$$

$$\operatorname{sen} x = 1 \Rightarrow x = \arcsen(1) = 90^\circ + k \cdot 360^\circ$$

$$\operatorname{sen} x + 2 \cos y = 2$$

$$1 + 2 \cos y = 2 \rightarrow 2 \cos y = 1 \rightarrow \cos y = 1/2$$

$$\Rightarrow y = \arccos(1/2) \Rightarrow \begin{cases} y = 60^\circ + k \cdot 360^\circ \\ y = 300^\circ + k \cdot 360^\circ \end{cases} \Rightarrow$$

Las soluciones:

$$(x, y) = (90^\circ + k_1 \cdot 360^\circ, 60^\circ + k_2 \cdot 360^\circ)$$

$$(x, y) = (90^\circ + k_1 \cdot 360^\circ, 300^\circ + k_2 \cdot 360^\circ)$$

$$d) \begin{cases} 4 \operatorname{sen} x + \cos^2 y = 2 \\ 2 \operatorname{sen} x + \cos y = 1 \end{cases} \xrightarrow{-2E_2} \begin{cases} 4 \operatorname{sen} x + \cos^2 y = 2 \\ -4 \operatorname{sen} x - 2 \cos y = -2 \end{cases}$$

$$\hline \cos^2 y - 2 \cos y = 0$$

$$\Rightarrow \cos y (\cos y - 2) = 0 \begin{cases} \rightarrow \cos y = 0 \\ \rightarrow \cos y - 2 = 0 \Rightarrow \cos y = 2 \rightarrow \text{Imposible} \end{cases}$$

$$\cos y = 0 \rightarrow y = \arccos(0) \Rightarrow \begin{cases} y = 90^\circ + k_1 \cdot 360^\circ \\ y = 270^\circ + k_1 \cdot 360^\circ \end{cases} \Rightarrow y = 90^\circ + k_1 \cdot 180^\circ$$

$$2\operatorname{sen} x + \cos y = 1$$

$$2\operatorname{sen} x = 1$$

$$\operatorname{sen} x = 1/2 \Rightarrow x = \operatorname{arc} \operatorname{sen}(1/2) \Rightarrow \begin{cases} x = 30^\circ + k_2 \cdot 360^\circ \\ x = 150^\circ + k_2 \cdot 360^\circ \end{cases}$$

Las soluciones:

$$(x, y) = (30^\circ + k_2 \cdot 360^\circ, 90^\circ + k_1 \cdot 180^\circ)$$

$$(x, y) = (150^\circ + k_2 \cdot 360^\circ, 90^\circ + k_1 \cdot 180^\circ)$$

③9) Resuelve las siguientes ecuaciones trigonométricas:

a) $2 \cdot \sin^2(x) = 1$

$$\sin^2(x) = \frac{1}{2} \Rightarrow \sin(x) = \pm \frac{1}{\sqrt{2}} \begin{cases} \rightarrow \sin(x) = \frac{1}{\sqrt{2}} \\ \rightarrow \sin(x) = -\frac{1}{\sqrt{2}} \end{cases}$$

$$\boxed{\sin(x) = \frac{1}{\sqrt{2}}}$$

$$x = \arcsin\left(\frac{1}{\sqrt{2}}\right) \Rightarrow \begin{cases} x = 45^\circ + K \cdot 360^\circ \\ x = 135^\circ + K \cdot 360^\circ \end{cases}$$

$$\boxed{\sin(x) = -\frac{1}{\sqrt{2}}}$$

$$x = \arcsin\left(-\frac{1}{\sqrt{2}}\right) \Rightarrow \begin{cases} x = -45^\circ + K \cdot 360^\circ = 315^\circ + K \cdot 360^\circ \\ x = 225^\circ + K \cdot 360^\circ \end{cases}$$

Si te fijas bien, puedes agrupar estas cuatro soluciones en una única expresión:

$$\boxed{x = 45^\circ + K \cdot 90^\circ \text{ con } K \in \mathbb{N}}$$

b) $3 \cdot \tan^2(x) - 1 = 0$

$$\tan^2(x) = \frac{1}{3} \Rightarrow \tan(x) = \pm \frac{1}{\sqrt{3}} \begin{cases} \rightarrow \tan(x) = \frac{1}{\sqrt{3}} \\ \rightarrow \tan(x) = -\frac{1}{\sqrt{3}} \end{cases}$$

$$\boxed{\tan(x) = \frac{1}{\sqrt{3}}}$$

$$x = \arctan\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \begin{cases} x = 30^\circ + K \cdot 360^\circ \\ x = 210^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 30^\circ + K \cdot 180^\circ}$$

$$\boxed{\operatorname{tg}(x) = -\frac{1}{\sqrt{3}}}$$

$$x = \operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right) \Rightarrow \begin{cases} x = 150^\circ + k \cdot 360^\circ \\ x = 330^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 150^\circ + k \cdot 180^\circ}$$

$$c) 1 - 4 \cdot \cos^2(x) = 0$$

$$\cos^2(x) = \frac{1}{4} \Rightarrow \cos(x) = \pm \frac{1}{2} \begin{cases} \cos(x) = \frac{1}{2} \\ \cos(x) = -\frac{1}{2} \end{cases}$$

$$\boxed{\cos(x) = \frac{1}{2}}$$

$$x = \arccos\left(\frac{1}{2}\right) \Rightarrow \begin{cases} x = 60^\circ + k \cdot 360^\circ \\ x = 300^\circ + k \cdot 360^\circ \end{cases}$$

$$\boxed{\cos(x) = -\frac{1}{2}}$$

$$x = \arccos\left(-\frac{1}{2}\right) \Rightarrow \begin{cases} x = 120^\circ + k \cdot 360^\circ \\ x = 240^\circ + k \cdot 360^\circ \end{cases}$$

$$\text{Y agrupando soluciones} \Rightarrow \boxed{\begin{cases} x = 60^\circ + k \cdot 180^\circ \\ x = 120^\circ + k \cdot 180^\circ \end{cases}}$$

$$d) 3 \operatorname{tg}(x) + 4 = 0$$

$$\operatorname{tg}(x) = -\frac{4}{3} \Rightarrow$$

$$\Rightarrow x = \operatorname{arctg}\left(-\frac{4}{3}\right) \Rightarrow \begin{cases} x = 126^\circ 87' + k \cdot 360^\circ \\ x = 306^\circ 87' + k \cdot 360^\circ \end{cases} \Rightarrow$$

$$\Rightarrow \boxed{x = 126^\circ 87' + k \cdot 180^\circ}$$

$$e) 2\cos^2(x) - \sin^2(x) = 1$$

$$2 \cdot (1 - \sin^2(x)) - \sin^2(x) = 1$$

$$2 - 2\sin^2(x) - \sin^2(x) = 1 \Rightarrow -3\sin^2(x) = -1 \Rightarrow$$

$$\Rightarrow \sin^2(x) = \frac{1}{3} \Rightarrow \sin(x) = \pm \frac{1}{\sqrt{3}} \begin{cases} \sin(x) = \frac{1}{\sqrt{3}} \\ \sin(x) = -\frac{1}{\sqrt{3}} \end{cases}$$

$$\boxed{\sin(x) = \frac{1}{\sqrt{3}}}$$

$$x = \arcsin\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \begin{cases} x = 35'26^\circ + K \cdot 360^\circ \\ x = 144'74^\circ + K \cdot 360^\circ \end{cases}$$

$$\boxed{\sin(x) = -\frac{1}{\sqrt{3}}}$$

$$x = \arcsin\left(-\frac{1}{\sqrt{3}}\right) \Rightarrow \begin{cases} x = 215'26^\circ + K \cdot 360^\circ \\ x = 324'74^\circ + K \cdot 360^\circ \end{cases}$$

$$\text{Y agrupando soluciones} \Rightarrow \boxed{\begin{cases} x = 35'26^\circ + K \cdot 180^\circ \\ x = 144'74^\circ + K \cdot 180^\circ \end{cases}}$$

$$f) \sin^2(x) - \sin(x) = 0$$

$$\underbrace{\sin(x)}_{=0} \cdot \underbrace{[\sin(x) - 1]}_{=0} = 0 \begin{cases} \sin(x) = 0 \\ \sin(x) - 1 = 0 \Rightarrow \sin(x) = 1 \end{cases}$$

$$\boxed{\sin(x) = 0}$$

$$x = \arcsin(0) \rightarrow \begin{cases} x = 0^\circ + K \cdot 360^\circ \\ x = 180^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + K \cdot 180^\circ}$$

$$\boxed{\text{sen}(x) = 1}$$

$$\boxed{x = \arcsen(1) = 90^\circ + k \cdot 360^\circ}$$

$$g) 2 \cdot \cos^2(x) - \sqrt{3} \cdot \cos(x) = 0$$

$$\underbrace{\cos(x)}_{\neq 0} \cdot \underbrace{[2 \cos(x) - \sqrt{3}]}_{\neq 0} = 0 \begin{cases} \cos(x) = 0 \\ 2 \cos(x) - \sqrt{3} = 0 \rightarrow \cos(x) = \frac{\sqrt{3}}{2} \end{cases}$$

$$\boxed{\cos(x) = 0}$$

$$x = \arccos(0) \Rightarrow \begin{cases} x = 90^\circ + k \cdot 360^\circ \\ x = 270^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 90^\circ + k \cdot 180^\circ}$$

$$\boxed{\cos(x) = \frac{\sqrt{3}}{2}}$$

$$x = \arccos\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \begin{cases} x = 30^\circ + k \cdot 360^\circ \\ x = 330^\circ + k \cdot 360^\circ \end{cases}$$

$$h) \cos^2(x) - \sin^2(x) = 0$$

$$\sin^2(x) = \cos^2(x) \Rightarrow \frac{\sin^2(x)}{\cos^2(x)} = 1 \Rightarrow \tan^2(x) = 1 \begin{cases} \tan(x) = 1 \\ \tan(x) = -1 \end{cases}$$

$$\boxed{\tan(x) = 1}$$

$$x = \arctan(1) \Rightarrow \begin{cases} x = 45^\circ + k \cdot 360^\circ \\ x = 225^\circ + k \cdot 360^\circ \end{cases} \Rightarrow x = 45^\circ + k \cdot 180^\circ$$

$$\boxed{\tan(x) = -1}$$

$$x = \arctan(-1) \Rightarrow \begin{cases} x = 135^\circ + k \cdot 360^\circ \\ x = 315^\circ + k \cdot 360^\circ \end{cases} \Rightarrow x = 135^\circ + k \cdot 180^\circ$$

y podemos agrupar las soluciones en la expresión:

$$x = 45^\circ + K \cdot 90^\circ$$

i) $\sin^2(x) - \cos^2(x) = 1$

$$\sin^2(x) - (1 - \sin^2(x)) = 1 \Rightarrow 2\sin^2(x) = 2 \Rightarrow \sin^2(x) = 1$$

$$\Rightarrow \sin(x) = \pm 1 \begin{cases} \sin(x) = 1 \\ \sin(x) = -1 \end{cases}$$

$$\boxed{\sin(x) = 1}$$

$$x = \arcsen(1) \Rightarrow x = 90^\circ + K \cdot 360^\circ$$

$$\boxed{\sin(x) = -1}$$

$$x = \arcsen(-1) \Rightarrow x = 270^\circ + K \cdot 360^\circ$$

$$\Rightarrow \boxed{x = 90^\circ + K \cdot 180^\circ}$$

j) $2 \cdot \cos^2(x) + \sin(x) = 1$

$$2 \cdot (1 - \sin^2(x)) + \sin(x) = 1 \Rightarrow 2 - 2\sin^2(x) + \sin(x) = 1 \Rightarrow$$

$$\Rightarrow 2\sin^2(x) - \sin(x) - 1 = 0 ; \sin(x) = t$$

$$\Rightarrow 2t^2 - t - 1 = 0 ; t = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \begin{cases} t = 1 \\ t = -1/2 \end{cases}$$

$$\boxed{\sin(x) = 1}$$

$$x = \arcsen(1) \Rightarrow \boxed{x = 90^\circ + K \cdot 360^\circ}$$

$$\boxed{\text{sen}(x) = -1/2}$$

$$x = \arcsen(-1/2) \Rightarrow \begin{cases} x = -30^\circ + K \cdot 360^\circ = 330^\circ + K \cdot 360^\circ \\ x = 210^\circ + K \cdot 360^\circ \end{cases}$$

Y si te fijas bien, podemos agrupar las tres soluciones en única expresión según:

$$\boxed{x = 90^\circ + K \cdot 120^\circ}$$

$$K) \quad 3 \cdot \text{tg}^2(x) - \sqrt{3} \cdot \text{tg}(x) = 0$$

$$\underbrace{\text{tg}(x)}_{\text{0}} \cdot \underbrace{[3 \cdot \text{tg}(x) - \sqrt{3}]}_{\text{0}} = 0 \begin{cases} \text{tg}(x) = 0 \\ 3 \text{tg}(x) - \sqrt{3} = 0 \Rightarrow \text{tg}(x) = \frac{\sqrt{3}}{3} \end{cases}$$

$$\boxed{\text{tg}(x) = 0}$$

$$x = \arctg(0) \Rightarrow \begin{cases} x = 0^\circ + K \cdot 360^\circ \\ x = 180^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + K \cdot 180^\circ}$$

$$\boxed{\text{tg}(x) = \frac{\sqrt{3}}{3}}$$

$$x = \arctg\left(\frac{\sqrt{3}}{3}\right) \Rightarrow \begin{cases} x = 30^\circ + K \cdot 360^\circ \\ x = 210^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 30^\circ + K \cdot 180^\circ}$$

$$e) \sin\left(\frac{\pi}{6} - x\right) + \cos\left(\frac{\pi}{3} - x\right) = \frac{1}{2}$$

$$\sin\left(\frac{\pi}{6}\right) \cdot \cos(x) - \cos\left(\frac{\pi}{6}\right) \cdot \sin(x) + \cos\left(\frac{\pi}{3}\right) \cdot \cos(x) + \sin\left(\frac{\pi}{3}\right) \cdot \sin(x) = \frac{1}{2}$$

$$\frac{1}{2} \cos(x) - \frac{\sqrt{3}}{2} \sin(x) + \frac{1}{2} \cos(x) + \frac{\sqrt{3}}{2} \sin(x) = \frac{1}{2}$$

$$\cos(x) = \frac{1}{2} \Rightarrow$$

$$x = \arccos\left(\frac{1}{2}\right) \Rightarrow$$

$$\begin{cases} x = 60^\circ + K \cdot 360^\circ \\ x = 300^\circ + K \cdot 360^\circ \end{cases}$$

$$m) \sin(2x) - 2 \cdot \cos^2(x) = 0$$

$$2 \sin(x) \cdot \cos(x) - 2 \cos^2(x) = 0$$

$$\cos(x) \cdot [2 \sin(x) - 2 \cos(x)] = 0$$

$$\cos(x) = 0$$

$$2 \sin(x) - 2 \cos(x) = 0$$

$$\cos(x) = 0$$

$$x = \arccos(0) \Rightarrow \begin{cases} x = 90^\circ + K \cdot 360^\circ \\ x = 270^\circ + K \cdot 360^\circ \end{cases}$$

$$\Rightarrow x = 90^\circ + K \cdot 180^\circ$$

$$2 \sin(x) - 2 \cos(x) = 0$$

$$2 \sin(x) = 2 \cos(x) \Rightarrow \frac{\sin(x)}{\cos(x)} = 1 \Rightarrow \operatorname{tg}(x) = 1 \Rightarrow$$

$$\Rightarrow x = \operatorname{arctg}(1) \Rightarrow \begin{cases} x = 45^\circ + K \cdot 360^\circ \\ x = 225^\circ + K \cdot 360^\circ \end{cases}$$

$$\Rightarrow x = 45^\circ + K \cdot 180^\circ$$

$$n) \cos(2x) - 3 \cdot \sin(x) + 1 = 0$$

$$\cos^2(x) - \sin^2(x) - 3 \cdot \sin(x) + 1 = 0$$

$$1 - \sin^2(x) - \sin^2(x) - 3 \sin(x) + 1 = 0$$

$$-2\sin^2(x) - 3\sin(x) + 2 = 0 ; \sin(x) = t$$

$$-2t^2 - 3t + 2 = 0 ; t = \frac{3 \pm \sqrt{9+16}}{-4} = \frac{3 \pm 5}{-4}$$

Impossible!

$$t = -2$$

$$t = 1/2$$

$$\boxed{\sin(x) = \frac{1}{2}}$$

$$x = \arcsin\left(\frac{1}{2}\right) \Rightarrow$$

$$\begin{cases} x = 30^\circ + K \cdot 360^\circ \\ x = 150^\circ + K \cdot 360^\circ \end{cases}$$

$$o) \sin\left(\frac{\pi}{4} + x\right) - \sqrt{2} \cdot \sin(x) = 0$$

$$\sin\left(\frac{\pi}{4}\right) \cdot \cos(x) + \cos\left(\frac{\pi}{4}\right) \sin(x) - \sqrt{2} \cdot \sin(x) = 0$$

$$\frac{\sqrt{2}}{2} \cdot \cos(x) + \frac{\sqrt{2}}{2} \sin(x) - \sqrt{2} \cdot \sin(x) = 0$$

$$\frac{\sqrt{2}}{2} \cdot \cos(x) - \frac{\sqrt{2}}{2} \sin(x) = 0 \Rightarrow \cancel{\frac{\sqrt{2}}{2}} \cos(x) = \cancel{\frac{\sqrt{2}}{2}} \sin(x) \Rightarrow$$

$$\Rightarrow \frac{\sin(x)}{\cos(x)} = 1 \Rightarrow \operatorname{tg}(x) = 1 \Rightarrow$$

$$\Rightarrow x = \operatorname{arctg}(1) \Rightarrow \begin{cases} x = 45^\circ + K \cdot 360^\circ \\ x = 225^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 45^\circ + K \cdot 180^\circ}$$

$$p) \cos^2\left(\frac{x}{2}\right) + \cos(x) - \frac{1}{2} = 0$$

$$\frac{1 + \cos(x)}{2} + \cos(x) - \frac{1}{2} = 0 \Rightarrow 1 + \cos(x) + 2\cos(x) - 1 = 0$$

$$3\cos(x) = 0 \Rightarrow \cos(x) = 0 \Rightarrow$$

$$\Rightarrow x = \arccos(0) \Rightarrow \begin{cases} x = 90^\circ + K \cdot 360^\circ \\ x = 270^\circ + K \cdot 360^\circ \end{cases} \Rightarrow x = 90^\circ + K \cdot 180^\circ$$

$$q) \operatorname{tg}^2\left(\frac{x}{2}\right) + 1 = \cos(x)$$

$$\frac{1 - \cos(x)}{1 + \cos(x)} + 1 = \cos(x) \Rightarrow 1 - \cancel{\cos(x)} + 1 + \cancel{\cos(x)} = \cos(x) + \cos^2(x)$$

$$\Rightarrow \cos^2(x) + \cos(x) - 2 = 0; \cos(x) = t$$

$$\Rightarrow t^2 + t - 2 = 0; t = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} \begin{cases} t = -2 \\ t = 1 \end{cases}$$

Impossible

$$\boxed{\cos(x) = 1}$$

$$x = \arccos(1) \Rightarrow x = 0^\circ + K \cdot 360^\circ$$

$$r) 2 \cdot \sin^2\left(\frac{x}{2}\right) + \cos(2x) = 0$$

$$\cancel{2} \cdot \frac{1 - \cos(x)}{\cancel{2}} + \cos^2(x) - \sin^2(x) = 0$$

$$1 - \cos(x) + \cos^2(x) - (1 - \cos^2(x)) = 0$$

$$2\cos^2(x) - \cos(x) = 0$$

$$\underbrace{\cos(x)}_0 \cdot \underbrace{[2\cos(x) - 1]}_0 = 0 \begin{cases} \rightarrow \cos(x) = 0 \\ \rightarrow 2\cos(x) - 1 = 0 \Rightarrow \cos(x) = \frac{1}{2} \end{cases}$$

$$\boxed{\cos(x) = 0}$$

$$x = \arccos(0) \Rightarrow \begin{cases} x = 90^\circ + K \cdot 360^\circ \\ x = 270^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 90^\circ + K \cdot 180^\circ}$$

$$\boxed{\cos(x) = \frac{1}{2}}$$

$$x = \arccos\left(\frac{1}{2}\right) \Rightarrow \begin{cases} x = 60^\circ + K \cdot 360^\circ \\ x = 300^\circ + K \cdot 360^\circ \end{cases}$$

$$s) \quad 4 \cdot \sin^2(x) \cdot \cos^2(x) + 2 \cdot \cos^2(x) - 2 = 0$$

$$4 \cdot (1 - \cos^2(x)) \cdot \cos^2(x) + 2 \cos^2(x) - 2 = 0$$

$$4 \cos^2(x) - 4 \cos^4(x) + 2 \cos^2(x) - 2 = 0$$

$$-4 \cos^4(x) + 6 \cos^2(x) - 2 = 0$$

$$2 \cos^4(x) - 3 \cos^2(x) + 1 = 0 ; \cos^2(x) = t$$

$$2 \cdot t^2 - 3t + 1 = 0 ; t = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} \begin{cases} t = 1 \\ t = 1/2 \end{cases}$$

$$\boxed{\cos^2(x) = 1}$$

$$\cos(x) = \pm 1 \begin{cases} \rightarrow \cos(x) = 1 \Rightarrow x = \arccos(1) = 0^\circ + K \cdot 360^\circ \\ \rightarrow \cos(x) = -1 \Rightarrow x = \arccos(-1) = 180^\circ + K \cdot 360^\circ \end{cases} \Rightarrow$$

$$\Rightarrow \boxed{x = 0^\circ + K \cdot 180^\circ}$$

$$\cos^2(x) = \frac{1}{2}$$

$$\cos(x) = \pm \frac{1}{\sqrt{2}} \begin{cases} \cos(x) = \frac{1}{\sqrt{2}} \\ \cos(x) = -\frac{1}{\sqrt{2}} \end{cases}$$

$$x = \arccos\left(\frac{1}{\sqrt{2}}\right) \Rightarrow \begin{cases} x = 45^\circ + K \cdot 360^\circ \\ x = 315^\circ + K \cdot 360^\circ \end{cases}$$

$$x = \arccos\left(-\frac{1}{\sqrt{2}}\right) \Rightarrow \begin{cases} x = 135^\circ + K \cdot 360^\circ \\ x = 225^\circ + K \cdot 360^\circ \end{cases}$$

$$\Rightarrow x = 45^\circ + K \cdot 90^\circ$$

$$t) \sin(x) + \cos(x) = 0$$

$$\sin(x) = -\cos(x) \Rightarrow \frac{\sin(x)}{\cos(x)} = -1 \Rightarrow \operatorname{tg}(x) = -1 \Rightarrow$$

$$\Rightarrow x = \operatorname{arctg}(-1) \Rightarrow \begin{cases} x = 135^\circ + K \cdot 360^\circ \\ x = 315^\circ + K \cdot 360^\circ \end{cases} \Rightarrow x = 135^\circ + K \cdot 180^\circ$$

$$u) \sin^2(x) - 2\sqrt{3} \cdot \sin(x) \cdot \cos(x) + 3\cos^2(x) = 0$$

$$\sin^2(x) - 2\sqrt{3} \cdot \sin(x) \cdot \cos(x) = -3 \cdot \cos^2(x)$$

$$\frac{\sin^2(x)}{\cos^2(x)} - \frac{2\sqrt{3} \sin(x) \cos(x)}{\cos^2(x)} = -3 \Rightarrow$$

$$\Rightarrow \operatorname{tg}^2(x) - 2\sqrt{3} \operatorname{tg}(x) + 3 = 0 \Rightarrow$$

$$\operatorname{tg}(x) = t \rightarrow t^2 - 2\sqrt{3} \cdot t + 3 = 0$$

$$t = \frac{2\sqrt{3} \pm \sqrt{12 - 12}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\Rightarrow \operatorname{tg}(x) = \sqrt{3}$$

$$x = \operatorname{arctg}(\sqrt{3}) \Rightarrow \begin{cases} x = 60^\circ + k \cdot 360^\circ \\ x = 240^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 60^\circ + k \cdot 180^\circ}$$

$$v) \operatorname{sen}^2(x) + \operatorname{sen}(x) \cdot \cos(x) = 0$$

$$\underbrace{\operatorname{sen}(x)}_{0} \cdot \left[\underbrace{\operatorname{sen}(x) + \cos(x)}_{0} \right] = 0 \begin{cases} \nearrow \operatorname{sen}(x) = 0 \\ \searrow \operatorname{sen}(x) + \cos(x) = 0 \end{cases}$$

$$\boxed{\operatorname{sen}(x) = 0}$$

$$x = \operatorname{arcsen}(0) \Rightarrow \begin{cases} x = 0^\circ + k \cdot 360^\circ \\ x = 180^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + k \cdot 180^\circ}$$

$$\boxed{\operatorname{sen}(x) + \cos(x) = 0}$$

$$\operatorname{sen}(x) = -\cos(x) \Rightarrow \frac{\operatorname{sen}(x)}{\cos(x)} = -1 \Rightarrow \operatorname{tg}(x) = -1$$

$$x = \operatorname{arctg}(-1) \Rightarrow \begin{cases} x = 135^\circ + k \cdot 360^\circ \\ x = 315^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 135^\circ + k \cdot 180^\circ}$$

$$w) \sqrt{3} \cdot \cos\left(\frac{3\pi}{2} + x\right) + \cos(x - \pi) = 2$$

$$\sqrt{3} \cdot \left[\cancel{\cos\left(\frac{3\pi}{2}\right)} \cdot \cos(x) - \cancel{\sin\left(\frac{3\pi}{2}\right)} \cdot \sin(x) \right] + \cos(x) \cdot \cancel{\cos(\pi)} + \sin(x) \cdot \cancel{\sin(\pi)} = 2$$

$$\sqrt{3} \cdot \sin(x) - \cos(x) = 2$$

$$\sqrt{3} \cdot \sin(x) - \sqrt{1 - \sin^2(x)} = 2$$

$$\sqrt{3} \cdot \sin(x) - 2 = \sqrt{1 - \sin^2(x)} \Rightarrow \left(\sqrt{3} \cdot \sin(x) - 2\right)^2 = \left(\sqrt{1 - \sin^2(x)}\right)^2$$

↑ elevamos

$$\Rightarrow 3 \cdot \sin^2(x) - 4\sqrt{3} \cdot \sin(x) + 4 = 1 - \sin^2(x)$$

$$\Rightarrow 4 \cdot \sin^2(x) - 4\sqrt{3} \cdot \sin(x) + 3 = 0; \sin(x) = t$$

$$4 \cdot t^2 - 4\sqrt{3} \cdot t + 3 = 0$$

$$t = \frac{4\sqrt{3} \pm \sqrt{48 - 48}}{8} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$$

$$\sin(x) = \frac{\sqrt{3}}{2}$$

$$x = \arcsin\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \begin{cases} x = 60^\circ + k \cdot 360^\circ \xrightarrow{\text{radianes}} x = \frac{\pi}{3} + 2k\pi \\ x = 120^\circ + k \cdot 360^\circ \xrightarrow{\text{radianes}} x = \frac{2\pi}{3} + 2k\pi \end{cases}$$

Como hemos elevado al cuadrado, hay que comprobar las soluciones

$$x = \frac{\pi}{3} \rightarrow \sqrt{3} \cdot \cos\left(\frac{3\pi}{2} + \frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3} - \pi\right) = 1 \neq 2 \rightarrow \text{No válida } \times$$

$$x = \frac{2\pi}{3} \rightarrow \sqrt{3} \cdot \cos\left(\frac{3\pi}{2} + \frac{2\pi}{3}\right) + \cos\left(\frac{2\pi}{3} - \pi\right) = 2 \rightarrow \text{Válida } \checkmark$$

$$x) \cos\left(\frac{5\pi}{6} - x\right) + \sin(x) - \sqrt{3} \cdot \cos(x) = 0$$

$$\cos\left(\frac{5\pi}{6}\right) \cdot \cos(x) + \sin\left(\frac{5\pi}{6}\right) \cdot \sin(x) + \sin(x) - \sqrt{3} \cdot \cos(x) = 0$$

$$-\frac{\sqrt{3}}{2} \cdot \cos(x) + \frac{1}{2} \sin(x) + \sin(x) - \sqrt{3} \cdot \cos(x) = 0$$

$$\frac{1}{2} \sin(x) + \sin(x) = \frac{\sqrt{3}}{2} \cos(x) + \sqrt{3} \cos(x)$$

$$\frac{3}{2} \sin(x) = \frac{3\sqrt{3}}{2} \cos(x) \Rightarrow$$

$$\Rightarrow \frac{\sin(x)}{\cos(x)} = \sqrt{3} \Rightarrow \operatorname{tg}(x) = \sqrt{3} \Rightarrow$$

$$\Rightarrow x = \operatorname{arctg}(\sqrt{3}) \Rightarrow \begin{cases} x = 60^\circ + K \cdot 360^\circ \\ x = 240^\circ + K \cdot 360^\circ \end{cases}$$

$$\Rightarrow x = 60^\circ + K \cdot 180^\circ$$

↓ radianes

$$x = \frac{\pi}{3} + K\pi$$

$$y) \sin\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) = 1$$

$$\sin\left(\frac{\pi}{4}\right) \cdot \cos(x) + \cos\left(\frac{\pi}{4}\right) \cdot \sin(x) + \cos\left(\frac{\pi}{4}\right) \cdot \cos(x) - \sin\left(\frac{\pi}{4}\right) \cdot \sin(x) = 1$$

$$\frac{\sqrt{2}}{2} \cdot \cos(x) + \frac{\sqrt{2}}{2} \cdot \sin(x) + \frac{\sqrt{2}}{2} \cos(x) - \frac{\sqrt{2}}{2} \sin(x) = 1$$

$$\sqrt{2} \cos(x) = 1 \Rightarrow \cos(x) = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow x = \arccos\left(\frac{1}{\sqrt{2}}\right) \Rightarrow \begin{cases} x = 45^\circ + K \cdot 360^\circ \xrightarrow{\text{rad}} x = \frac{\pi}{4} + K \cdot 2\pi \\ x = 315^\circ + K \cdot 360^\circ \xrightarrow{\text{rad}} x = \frac{7\pi}{4} + K \cdot 2\pi \end{cases}$$

$$z) \cos\left(\frac{\pi}{3} - x\right) - \sqrt{3} \cdot \sin\left(\frac{\pi}{3} - x\right) = 1$$

$$\cos\left(\frac{\pi}{3}\right) \cdot \cos(x) + \sin\left(\frac{\pi}{3}\right) \cdot \sin(x) - \sqrt{3} \cdot \left[\sin\left(\frac{\pi}{3}\right) \cdot \cos(x) - \cos\left(\frac{\pi}{3}\right) \cdot \sin(x)\right] = 1$$

$$\frac{1}{2} \cos(x) + \frac{\sqrt{3}}{2} \sin(x) - \sqrt{3} \cdot \left[\frac{\sqrt{3}}{2} \cos(x) - \frac{1}{2} \sin(x)\right] = 1$$

$$\frac{1}{2} \cos(x) + \frac{\sqrt{3}}{2} \sin(x) - \frac{3}{2} \cos(x) + \frac{\sqrt{3}}{2} \sin(x) = 1$$

$$\sqrt{3} \sin(x) - \cos(x) = 1$$

$$\sqrt{3} \sin(x) - (\sqrt{1 - \sin^2(x)}) = 1$$

$$\sqrt{3} \sin(x) - 1 = \sqrt{1 - \sin^2(x)} \xRightarrow{\text{Elevamos}} (\sqrt{3} \sin(x) - 1)^2 = (\sqrt{1 - \sin^2(x)})^2$$

$$\Rightarrow 3 \sin^2(x) - 2\sqrt{3} \sin(x) + 1 = 1 - \sin^2(x) \Rightarrow$$

$$\Rightarrow 4 \sin^2(x) - 2\sqrt{3} \sin(x) = 0 \Rightarrow \underbrace{\sin(x)}_0 \cdot \underbrace{[4\sin(x) - 2\sqrt{3}]}_0 = 0$$

$$\boxed{\sin(x) = 0}$$

$$x = \arcsin(0) \Rightarrow \begin{cases} x = 0^\circ + k \cdot 360^\circ \rightarrow x = 0 + k \cdot 2\pi \\ x = 180^\circ + k \cdot 360^\circ \rightarrow x = \pi + k \cdot 2\pi \end{cases}$$

$$\boxed{4 \sin(x) - 2\sqrt{3} = 0}$$

$$4 \sin(x) = 2\sqrt{3} \Rightarrow \sin(x) = \frac{\sqrt{3}}{2} \Rightarrow$$

$$\Rightarrow x = \arcsin\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \begin{cases} x = 60^\circ + k \cdot 360^\circ \rightarrow x = \frac{\pi}{3} + k \cdot 2\pi \\ x = 120^\circ + k \cdot 360^\circ \rightarrow x = \frac{2\pi}{3} + k \cdot 2\pi \end{cases}$$

Y como hemos elevado al cuadrado, hay que comprobar las soluciones:

$$x = 0 \rightarrow \cos\left(\frac{\pi}{3}\right) - \sqrt{3} \cdot \sin\left(\frac{\pi}{3}\right) = -1 \neq 1 \quad \times \text{ No válida}$$

$$x = \pi \rightarrow \cos\left(\frac{\pi}{3} - \pi\right) - \sqrt{3} \cdot \sin\left(\frac{\pi}{3} - \pi\right) = 1 \quad \checkmark \text{ Válida}$$

$$x = \frac{\pi}{3} \rightarrow \cos(0) - \sqrt{3} \cdot \sin(0) = 1 \quad \checkmark \text{ Válida}$$

$$x = \frac{2\pi}{3} \rightarrow \cos\left(\frac{\pi}{3} - \frac{2\pi}{3}\right) - \sqrt{3} \cdot \sin\left(\frac{\pi}{3} - \frac{2\pi}{3}\right) = 2 \quad \times \text{ No válida}$$

Las soluciones por tanto son:

$$x = \pi + k \cdot 2\pi \quad \wedge \quad x = \frac{\pi}{3} + k \cdot 2\pi$$

④ Resuelve las siguientes ecuaciones trigonométricas:

a) $\cos(2x) + 3 \cdot \sin(x) = 2$

$$\cos^2(x) - \sin^2(x) + 3 \cdot \sin(x) = 2$$

$$1 - \sin^2(x) - \sin^2(x) + 3 \sin(x) - 2 = 0$$

$$-2\sin^2(x) + 3\sin(x) - 1 = 0 ; \sin(x) = t$$

$$-2t^2 + 3t - 1 = 0 ; t = \frac{-3 \pm \sqrt{9-8}}{-4} = \frac{-3 \pm 1}{-4} \begin{cases} t = 1/2 \\ t = 1 \end{cases}$$

$$\boxed{\sin(x) = 1}$$

$$\boxed{x = \arcsen(1) = 90^\circ + k \cdot 360^\circ}$$

$$\boxed{\text{sen}(x) = \frac{1}{2}}$$

$$x = \arcsen\left(\frac{1}{2}\right) \rightarrow \begin{cases} x = 30^\circ + k \cdot 360^\circ \\ x = 150^\circ + k \cdot 360^\circ \end{cases}$$

$$b) \text{tg}(2x) \cdot \text{tg}(x) = 1$$

$$\frac{2 \cdot \text{tg}(x)}{1 - \text{tg}^2(x)} \cdot \text{tg}(x) = 1 \Rightarrow 2 \text{tg}^2(x) = 1 - \text{tg}^2(x) \Rightarrow$$

$$\Rightarrow 3 \text{tg}^2(x) = 1 \Rightarrow \text{tg}^2(x) = \frac{1}{3} \begin{cases} \text{tg}(x) = -\frac{1}{\sqrt{3}} \\ \text{tg}(x) = \frac{1}{\sqrt{3}} \end{cases}$$

$$\boxed{\text{tg}(x) = -\frac{1}{\sqrt{3}}}$$

$$x = \arctg\left(-\frac{1}{\sqrt{3}}\right) \Rightarrow \begin{cases} x = 150^\circ + k \cdot 360^\circ \\ x = 330^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 150^\circ + k \cdot 180^\circ}$$

$$\boxed{\text{tg}(x) = \frac{1}{\sqrt{3}}}$$

$$x = \arctg\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \begin{cases} x = 30^\circ + k \cdot 360^\circ \\ x = 210^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 30^\circ + k \cdot 180^\circ}$$

$$c) \cos(x) \cdot \cos(2x) + 2 \cdot \cos^2(x) = 0$$

$$\cos(x) \cdot [\cos(2x) + 2\cos(x)] = 0 \Rightarrow$$

$$\cos(x) \cdot [\cos^2(x) - \sin^2(x) + 2\cos(x)] = 0 \Rightarrow$$

$$\cos(x) \cdot [\cos^2(x) - (1 - \cos^2(x)) + 2\cos(x)] = 0 \Rightarrow$$

$$\underbrace{\cos(x)}_0 \cdot [\underbrace{2\cos^2(x) + 2\cos(x) - 1}_0] = 0$$

$$\boxed{\cos(x) = 0}$$

$$x = \arccos(0) \Rightarrow \begin{cases} x = 90^\circ + K \cdot 360^\circ \\ x = 270^\circ + K \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 90^\circ + K \cdot 180^\circ}$$

$$\boxed{2\cos^2(x) + 2\cos(x) - 1 = 0}$$

$$\cos(x) = t$$

$$2t^2 + 2t - 1 = 0 ; t = \frac{-2 \pm \sqrt{4 + 8}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

$$\rightarrow t = \frac{-1 - \sqrt{3}}{2} \approx -1'37 \rightarrow \text{Imposible!!}$$

$$\rightarrow t = \frac{-1 + \sqrt{3}}{2} \approx 0'36$$

$$\cos(x) = \frac{-1 + \sqrt{3}}{2}$$

$$x = \arccos\left(\frac{-1 + \sqrt{3}}{2}\right) \Rightarrow \begin{cases} x = 68'53^\circ + K \cdot 360^\circ \\ x = 291'47^\circ + K \cdot 360^\circ \end{cases}$$

$$d) 2 \cdot \operatorname{sen}(x) = \operatorname{tg}(2x)$$

$$2 \cdot \operatorname{sen}(x) = \frac{\operatorname{sen}(2x)}{\cos(2x)} \rightarrow 2 \operatorname{sen}(x) = \frac{2 \operatorname{sen}(x) \cos(x)}{\cos^2(x) - \operatorname{sen}^2(x)} \Rightarrow$$

$$\Rightarrow 2 \cdot \operatorname{sen}(x) \cdot (\cos^2(x) - \operatorname{sen}^2(x)) = 2 \operatorname{sen}(x) \cdot \cos(x) \Rightarrow$$

$$\Rightarrow 2 \operatorname{sen}(x) \cdot (\cos^2(x) - \operatorname{sen}^2(x)) - 2 \operatorname{sen}(x) \cdot \cos(x) = 0 \Rightarrow$$

$$\Rightarrow 2 \operatorname{sen}(x) \cdot [\cos^2(x) - \operatorname{sen}^2(x) - \cos(x)] = 0 \Rightarrow$$

$$\Rightarrow 2 \operatorname{sen}(x) \cdot [\cos^2(x) - (1 - \cos^2(x)) - \cos(x)] = 0 \Rightarrow$$

$$\Rightarrow \underbrace{2 \operatorname{sen}(x)}_{=0} \cdot \left[\underbrace{2 \cos^2(x) - \cos(x) - 1}_{=0} \right] = 0$$

$$\boxed{2 \operatorname{sen}(x) = 0}$$

$$\operatorname{sen}(x) = 0 \rightarrow x = \arcsen(0) \rightarrow \begin{cases} x = 0^\circ + k \cdot 360^\circ \\ x = 180^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + k \cdot 180^\circ}$$

$$\boxed{2 \cos^2(x) - \cos(x) - 1 = 0}$$

$$\cos(x) = t \rightarrow 2t^2 - t - 1 = 0; t = \frac{1 \pm \sqrt{1+8}}{4} \begin{cases} t = 1 \\ t = -1/2 \end{cases}$$

$$\boxed{\cos(x) = 1}$$

$$x = \arccos(1) \Rightarrow x = 0^\circ + k \cdot 360^\circ$$

Esta solución ya
está incluida aquí

$$\cos(x) = -\frac{1}{2}$$

$$x = \arccos\left(-\frac{1}{2}\right) \Rightarrow$$

$$\begin{cases} x = 120^\circ + k \cdot 360^\circ \\ x = 240^\circ + k \cdot 360^\circ \end{cases}$$

$$e) \sqrt{3} \cdot \sin\left(\frac{x}{2}\right) + \cos(x) - 1 = 0$$

$$\sqrt{3} \cdot \sin\left(\frac{x}{2}\right) + \cos\left(2 \cdot \frac{x}{2}\right) - 1 = 0$$

$$\sqrt{3} \cdot \sin\left(\frac{x}{2}\right) + \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) - 1 = 0$$

$$\sqrt{3} \cdot \sin\left(\frac{x}{2}\right) + \cancel{1} - \sin^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) - \cancel{1} = 0$$

$$\sqrt{3} \cdot \sin\left(\frac{x}{2}\right) - 2\sin^2\left(\frac{x}{2}\right) = 0$$

$$\underbrace{\sin\left(\frac{x}{2}\right)}_{=0} \cdot \left[\underbrace{\sqrt{3} - 2\sin\left(\frac{x}{2}\right)}_{=0} \right] = 0 \begin{cases} \rightarrow \sin\left(\frac{x}{2}\right) = 0 \\ \rightarrow \sin\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2} \end{cases}$$

$$\sin\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = \arcsin(0) \Rightarrow \begin{cases} \frac{x}{2} = 0^\circ + k \cdot 360^\circ \\ \frac{x}{2} = 180^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \frac{x}{2} = 0^\circ + k \cdot 180^\circ$$

$$\Rightarrow x = 0^\circ + k \cdot 360^\circ$$

$$\sin\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{x}{2} = \arcsin\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \begin{cases} \frac{x}{2} = 60^\circ + k \cdot 360^\circ \rightarrow x = 120^\circ + k \cdot 720^\circ \\ \frac{x}{2} = 120^\circ + k \cdot 360^\circ \rightarrow x = 240^\circ + k \cdot 720^\circ \end{cases}$$

$$f) \sin(2x) \cdot \cos(x) = 6 \cdot \sin^3(x)$$

$$2 \sin(x) \cdot \cos(x) \cdot \cos(x) = 6 \cdot \sin^3(x) \Rightarrow$$

$$\Rightarrow 6 \sin^3(x) - 2 \sin(x) \cdot \cos^2(x) = 0 \Rightarrow$$

$$\Rightarrow \underbrace{2 \sin(x)}_0 \cdot \left[\underbrace{3 \sin^2(x) - \cos^2(x)}_0 \right] = 0$$

$$\boxed{2 \sin(x) = 0}$$

$$\sin(x) = 0 \rightarrow x = \arcsin(0) \Rightarrow \begin{cases} x = 0^\circ + k \cdot 360^\circ \\ x = 180^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 0^\circ + k \cdot 180^\circ}$$

$$\boxed{3 \cdot \sin^2(x) - \cos^2(x) = 0}$$

$$3 \sin^2(x) = \cos^2(x) \Rightarrow 3 \tan^2(x) = 1 \Rightarrow \tan^2(x) = \frac{1}{3} \begin{cases} \tan(x) = -\frac{1}{\sqrt{3}} \\ \tan(x) = \frac{1}{\sqrt{3}} \end{cases}$$

$$\tan(x) = -\frac{1}{\sqrt{3}}$$

$$x = \arctan\left(-\frac{1}{\sqrt{3}}\right) \Rightarrow \begin{cases} x = 150^\circ + k \cdot 360^\circ \\ x = 330^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 150^\circ + k \cdot 180^\circ}$$

$$\tan(x) = \frac{1}{\sqrt{3}}$$

$$x = \arctan\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \begin{cases} x = 30^\circ + k \cdot 360^\circ \\ x = 210^\circ + k \cdot 360^\circ \end{cases} \Rightarrow \boxed{x = 30^\circ + k \cdot 180^\circ}$$