

1) TEOREMAS :

1.1. Comprueba que $f(x) = x^2 - 5x + 6$ cumpla las hipótesis del teorema de Rolle en el intervalo $[1, 4]$. Encuentra el punto donde se cumple la tesis del teorema.

1.2. Comprueba si las siguientes funciones cumplen las hipótesis del teorema de Rolle en los intervalos indicados. En caso afirmativo encuentra el punto "c" donde se cumple la tesis del teorema:

a) $f(x) = x - x^3$ en $[-1, 0]$

b) $f(x) = \frac{x^2 - 3x}{x^2 + 1}$ en $[0, 3]$

c) $f(x) = |x^2 - 4|$ en $[-2, 2]$

d) $f(x) = |x+1| + |x-1|$ en $[-1, 1]$

e) $f(x) = x - \frac{|x|}{x}$ en $[-1, 1]$

f) $f(x) = x^3 + 2x^2 - 2x$ en $[-3, -2]$

g) $f(x) = \cos x$ en $[0, 2\pi]$

h) $f(x) = \sqrt{1 + \sin x}$ en $[-\pi, 0]$

i) $f(x) = -x^4 + x^2$ en $[0, 1]$

1.3. Comprueba si las funciones siguientes satisfacen las hipótesis del teorema del valor medio en el intervalo indicado. En caso afirmativo, encuentra el valor intermedio correspondiente.

a) $f(x) = x^3 - x$ en $[-1, 2]$

b) $f(x) = x^3 - 3x + 5$ en $[-2, 0]$

e) $f(x) = \sin\left(\frac{x}{2}\right)$ en $[0, \pi]$

c) $f(x) = (x-2)^3$ en $[0, 3]$

d) $f(x) = \sqrt{1+2x}$ en $\left[\frac{3}{2}, 4\right]$

1.4. Encuentra un punto de la parábola $f(x) = (x-1)^2$ en el que la recta tangente sea paralela a la cuerda que une los puntos $A(2, 1)$ y $B(4, 9)$. Representalo.

1.5. Repite el ejercicio anterior siendo $f(x) = x^2 - 6x$, $A(0, 0)$ y $B(4, -8)$.

1.6. Utiliza el teorema del valor medio para demostrar que la función $f(x) = \sec^2 x - \tan^2 x$ es constante en cualquier intervalo de $\mathbb{R}$.

1.7. Sea la función $f(x) = \begin{cases} x^2 + bx + c & \text{si } x \leq 0 \\ \frac{\ln(1+x)}{x} & \text{si } x > 0 \end{cases}$

a) Determinar c para que $f(x)$ sea continua en 0.

b) Determinar b para que $f(x)$ sea derivable en 0.

c) Utilizando el Teorema del Valor Medio de Lagrange demostrar que existe un punto $c \in [0, e-1]$ tal que

$$f'(c) = \frac{2-e}{(e-1)^2}$$

2) REGLA DE L'HÔPITAL:

2.1. Calcula los siguientes límites:

1) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$

2) $\lim_{x \rightarrow 0} \frac{5x - \operatorname{sen} x}{2x}$

3) $\lim_{x \rightarrow 0} \frac{\operatorname{sen} x}{x}$

4) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$

5) $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 5x + 4}$

6) $\lim_{x \rightarrow 0} \frac{x - \operatorname{sen} x}{\operatorname{tg} x - x}$

7) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

8) $\lim_{x \rightarrow \infty} \frac{\sqrt{1+x}}{x}$

9) $\lim_{x \rightarrow \infty} \frac{\ln x}{x^2}$

10) $\lim_{x \rightarrow \infty} \frac{e^x}{x^3}$

11) $\lim_{x \rightarrow \infty} \frac{x^3}{e^{3x}}$

12) $\lim_{x \rightarrow \infty} \frac{3x^2 - 6x}{x^2 - x - 2}$

13) $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$

14) $\lim_{x \rightarrow 0^+} x \cdot \ln x$

15) $\lim_{x \rightarrow 0} \left(\frac{1}{\operatorname{sen} x} - \frac{1}{x} \right)$

16) $\lim_{x \rightarrow \infty} (e^{-x} \cdot \ln^2 x)$

17) $\lim_{x \rightarrow \infty} (e^{-x} \cdot \sqrt{x})$

18) $\lim_{x \rightarrow \infty} \left(e^x \cdot \frac{1}{x^2} \right)$

- 19) $\lim_{x \rightarrow \infty} x \cdot (e^{1/x} - 1)$ 20) $\lim_{x \rightarrow \frac{\pi}{4}} (1 - \operatorname{tg} x) \sec(2x)$ 21) $\lim_{x \rightarrow 0^+} x^{\operatorname{sen} x}$
- 22) $\lim_{x \rightarrow 0^+} x^x$ 23) $\lim_{x \rightarrow \infty} (1+x)^{1/x}$ 24) $\lim_{x \rightarrow \infty} x^{1/x}$ 25) $\lim_{x \rightarrow 0} (\cos x)^{1/x}$
- 26) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x}\right)^{\operatorname{tg} x}$ 27) $\lim_{x \rightarrow 0} \frac{x \cos x - \operatorname{sen} x}{x^3}$ 28) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 4x}{x - \operatorname{sen} x}$
- 29) $\lim_{x \rightarrow \frac{\pi}{2}} (1 + 2 \cos x)^{1/\cos x}$ 30) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\ln(1+x)} \right)$ 31) $\lim_{x \rightarrow 0} \frac{2x \operatorname{sen} x}{1 - \cos x}$
- 32) $\lim_{x \rightarrow 0} \frac{\ln(\cos x) - 1 + \cos x}{x^2}$ 33) $\lim_{x \rightarrow 0} \frac{\ln(\cos 3x)}{\ln(\cos 2x)}$
- 34) $\lim_{x \rightarrow 0} \frac{e^x - x - \cos x}{\operatorname{sen}^2 x}$ 35) $\lim_{x \rightarrow 0} \frac{(e^x - e^{-x})^2}{x^2}$ 36) $\lim_{x \rightarrow 0} \frac{x \operatorname{sen} x}{1 - \cos x}$
- 37) $\lim_{x \rightarrow 0} \frac{\operatorname{sen} x - x}{\operatorname{tg} x - x}$ 38) $\lim_{x \rightarrow \infty} \left[x \cdot \left(\operatorname{arctg} e^x - \frac{\pi}{2} \right) \right]$
- 39) $\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x})$ 40) $\lim_{x \rightarrow 0} \left(\frac{x^2 + 2}{2 - 2 \operatorname{sen} x} \right)^{\frac{2}{x}}$ 41) $\lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x-1} \right)^x$
- 42) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{x}{x-1} \right)$ 43) $\lim_{x \rightarrow \frac{\pi}{2}^-} (\operatorname{tg} x)^{\cos x}$ 44) $\lim_{x \rightarrow 0} (\cos(2x))^{\frac{1}{\operatorname{sen} x^2}}$
- 45) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\operatorname{sen} x} \right)$ 46) $\lim_{x \rightarrow 0^+} x^{\operatorname{tg} x}$ 47) $\lim_{x \rightarrow \infty} \left(\frac{1}{2x} \right)^{1/x}$
- 48) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\operatorname{arctg} x} \right)$ 49) $\lim_{x \rightarrow 1^-} \left[(x^2 - 1) \cdot \operatorname{tg} \left(\frac{\pi x}{2} \right) \right]$ 50) $\lim_{x \rightarrow \infty} \left(x \cdot \ln \frac{x+1}{x} \right)$

$$51) \lim_{x \rightarrow 0} \frac{\operatorname{sen} x}{4x} \quad 52) \lim_{x \rightarrow 1} \frac{x^2 - 1}{\ln x} \quad 53) \lim_{x \rightarrow 0} \frac{x^3 - x}{\sqrt{x+9} - 3}$$

$$54) \lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} \quad 55) \lim_{x \rightarrow 1} \frac{4x^5 - 4x^2}{-x^2 + 1} \quad 56) \lim_{x \rightarrow 0} \frac{x}{\operatorname{sen} x}$$

$$57) \lim_{x \rightarrow 0} \frac{e^{-x} - e^x}{3 \operatorname{sen} x} \quad 58) \lim_{x \rightarrow 7} \frac{x^2 - 9x + 14}{\ln(x^2 - 6x - 6)} \quad 59) \lim_{x \rightarrow 0} \left(\frac{1}{4x} - \frac{2}{x \cdot e^x} \right)$$

$$60) \lim_{x \rightarrow 0^+} (\operatorname{tg} x)^x \quad 61) \lim_{x \rightarrow \infty} \sqrt[x-2]{\frac{x^2 + 1}{x + 3}} \quad 62) \lim_{x \rightarrow 0} \frac{\operatorname{sen} x}{1 - \cos x}$$

$$63) \lim_{x \rightarrow 0} \frac{2 \operatorname{sen} x}{x^2 - 3x} \quad 64) \lim_{x \rightarrow 0} \frac{x - \operatorname{sen} x}{x \cdot \operatorname{sen} x} \quad 65) \lim_{x \rightarrow 0} \frac{x^4 - \left(\frac{1}{3}\right)x^3}{x - \operatorname{tg} x}$$

$$66) \lim_{x \rightarrow 0} \frac{x - 2 \operatorname{sen} x}{\operatorname{tg} x - 2 \operatorname{sen} x} \quad 67) \lim_{x \rightarrow 0} \frac{\ln(e^x + x^3)}{x} \quad 68) \lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{\sqrt[4]{x^3}}$$

$$69) \lim_{x \rightarrow 0} \frac{\ln(\cos(3x))}{x^2} \quad 70) \lim_{x \rightarrow 0} \frac{e^x - e^{\operatorname{sen} x}}{1 - \cos x} \quad 71) \lim_{x \rightarrow 0} \frac{1+x - e^x}{\operatorname{sen}^2 x}$$

$$72) \lim_{x \rightarrow 2} \frac{2^x - x^2}{9 - 3^x} \quad 73) \lim_{x \rightarrow 0} (e^x - x)^{1/x} \quad 74) \lim_{x \rightarrow 0} (\cotg x)^{\operatorname{sen} x}$$

$$75) \lim_{x \rightarrow 0} \left(\frac{2}{\ln(1+x)} - \frac{2}{x} \right) \quad 76) \lim_{x \rightarrow \frac{\pi}{2}} (x - \frac{\pi}{2}) \cdot \operatorname{tg} x \quad 77) \lim_{x \rightarrow \infty} x \cdot e^{4x}$$

$$78) \lim_{x \rightarrow 0} (\operatorname{arcsen} x \cdot \cotg x) \quad 79) \lim_{x \rightarrow \infty} x \cdot \operatorname{arctg} \left(\frac{2}{x} \right) \quad 80) \lim_{x \rightarrow \infty} (\ln x)^{e^{-x}}$$

1) TEOREMAS:

1.1) $f(x) = x^2 - 5x + 6$ es continua y derivable en $\mathbb{R}$, y por tanto, y en concreto, lo es en $[1, 4]$. Además

$$\left. \begin{array}{l} f(1) = 2 \\ f(4) = 2 \end{array} \right\} f(1) = f(4) \Rightarrow \exists c \in]1, 4[/ f'(c) = 0$$

Veámoslo:

$$f'(x) = 2x - 5$$

$$f'(x) = 0 \rightarrow 2x - 5 = 0 \Rightarrow x = \frac{5}{2} \text{ que efectivamente pertenece a }]1, 4[$$

1.2) a) $f(x) = x - x^3 \rightarrow$ Continua y Derivable en $\mathbb{R}$

$$\left. \begin{array}{l} f(-1) = 0 \\ f(0) = 0 \end{array} \right\} f(-1) = f(0) \Rightarrow \exists c \in]-1, 0[/ f'(c) = 0$$

$$f'(x) = 1 - 3x^2$$

$$f'(x) = 0 \rightarrow 1 - 3x^2 = 0 \begin{cases} x = 0.577 \\ x = -0.577 \rightarrow x \in]-1, 0[\end{cases}$$

b) $f(x) = \frac{x^2 - 3x}{x^2 + 1}$; $\text{Dom}(f(x)) = \mathbb{R} \rightarrow$ Continua en $\mathbb{R}$.

$$f'(x) = \frac{(2x - 3)(x^2 + 1) - (x^2 - 3x) \cdot 2x}{(x^2 + 1)^2} = \frac{3x^2 + 2x - 3}{(x^2 + 1)^2}$$

$\hookrightarrow \text{Dom}(f'(x)) = \mathbb{R} \rightarrow$ Derivable en $\mathbb{R}$

$$\left. \begin{array}{l} f(0) = 0 \\ f(3) = 0 \end{array} \right\} f(0) = f(3) \Rightarrow \exists c \in]0, 3[/ f'(c) = 0$$

$$f'(x) = \frac{3x^2 + 2x - 3}{(x^2 + 1)^2}$$

$$f'(x) = 0 \rightarrow 3x^2 + 2x - 3 = 0 \begin{cases} x = 0.721 \rightarrow x \in]0, 3[\\ x = -1.387 \end{cases}$$

$$c) f(x) = |x^2 - 4|$$

$$x^2 - 4 = 0 \begin{cases} x = 2 \\ x = -2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4 & \text{si } x \leq -2 \\ 4 - x^2 & \text{si } -2 < x < 2 \\ x^2 - 4 & \text{si } x \geq 2 \end{cases}$$

Al estar definida $f(x)$ por funciones polinómicas, los únicos puntos en los que puede ser discontinua o no derivable serán en los puntos de encuentro. Así,

En $x = -2$:

$$\left. \begin{array}{l} f(-2) = 0 \\ \lim_{x \rightarrow -2} f(x) \end{array} \right\} \begin{cases} \lim_{x \rightarrow -2^-} x^2 - 4 = 0 \\ \lim_{x \rightarrow -2^+} 4 - x^2 = 0 \end{cases} \Rightarrow \lim_{x \rightarrow -2} f(x) = 0 \Rightarrow \text{Continua en } x = -2$$

En $x = 2$:

$$\left. \begin{array}{l} f(2) = 0 \\ \lim_{x \rightarrow 2} f(x) \end{array} \right\} \begin{cases} \lim_{x \rightarrow 2^-} 4 - x^2 = 0 \\ \lim_{x \rightarrow 2^+} x^2 - 4 = 0 \end{cases} \Rightarrow \lim_{x \rightarrow 2} f(x) = 0 \Rightarrow \text{Continua en } x = 2$$

Veamos la derivabilidad:

$$f'(x) = \begin{cases} 2x & \text{si } x < -2 \\ -2x & \text{si } -2 < x < 2 \\ 2x & \text{si } x > 2 \end{cases}$$

En $x = -2$:

$$\left. \begin{aligned} f'(-2)^- &= 2 \cdot (-2) = -4 \\ f'(-2)^+ &= -2 \cdot (-2) = +4 \end{aligned} \right\} f'(-2)^- \neq f'(-2)^+ \Rightarrow \text{No derivable en } x = -2$$

En $x = 2$:

$$\left. \begin{aligned} f'(2)^- &= -2 \cdot (2) = -4 \\ f'(2)^+ &= 2 \cdot 2 = 4 \end{aligned} \right\} f'(2)^- \neq f'(2)^+ \Rightarrow \text{No derivable en } x = 2$$

En resumen:

$$f(x) = |x^2 - 4| \rightarrow \text{Continua en } \mathbb{R}$$

$$\hookrightarrow \text{Derivable } \forall x \in]-\infty, -2[\cup]-2, 2[\cup]2, +\infty$$

El intervalo pedido! ✓

$$\left. \begin{aligned} f(-2) &= 0 \\ f(2) &= 0 \end{aligned} \right\} \exists c \in]-2, 2[/ f'(c) = 0$$

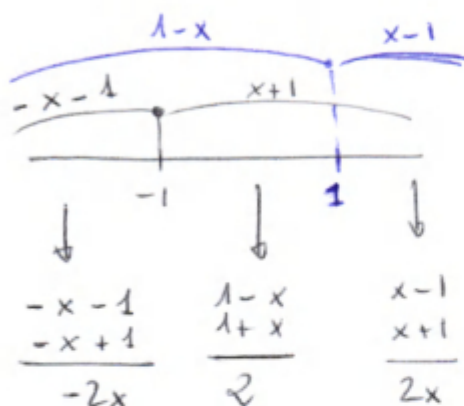
$$f'(x) = -2x \quad \forall x \in]-2, 2[$$

$$f'(x) = 0 \rightarrow -2x = 0 \Rightarrow x = 0 \Rightarrow x \in]-2, 2[$$

$$d) f(x) = |x+1| + |x-1|$$

$$x+1=0 \Rightarrow x=-1$$

$$x-1=0 \Rightarrow x=1$$



$$\Rightarrow f(x) = \begin{cases} -2x & \text{si } x \leq -1 \\ 2 & \text{si } -1 \leq x \leq 1 \\ 2x & \text{si } x > 1 \end{cases}$$

Al estar definida $f(x)$ por funciones polinómicas, los únicos puntos en los que puede ser discontinua o no derivable serán en los puntos de encuentro. Así:

En $x = -1$:

$$f(-1) = 2$$

$$\lim_{x \rightarrow -1} f(x) \rightarrow \begin{cases} \lim_{x \rightarrow -1^-} -2x = 2 \\ \lim_{x \rightarrow -1^+} 2 = 2 \end{cases} \Rightarrow \lim_{x \rightarrow -1} f(x) = 2 \Rightarrow \text{Continua en } x = -1$$

En $x = 1$:

$$f(1) = 2$$

$$\lim_{x \rightarrow 1} f(x) \rightarrow \begin{cases} \lim_{x \rightarrow 1^-} 2 = 2 \\ \lim_{x \rightarrow 1^+} 2x = 2 \end{cases} \Rightarrow \lim_{x \rightarrow 1} f(x) = 2 \Rightarrow \text{Continua en } x = 1$$

Veamos la derivabilidad:

$$f'(x) = \begin{cases} -2 & \text{si } x < -1 \\ 0 & \text{si } -1 < x < 1 \\ 2 & \text{si } x > 1 \end{cases}$$

Donde es fácil ver que:

$$f'(-1)^- = -2 \neq f'(-1)^+ = 0 \Rightarrow \text{No derivable en } x = -1$$

$$f'(1)^- = 0 \neq f'(1)^+ = 2 \Rightarrow \text{No derivable en } x = 1$$

En resumen:

$$f(x) = |x+1| + |x-1| \rightarrow \text{Continua en } \mathbb{R}$$

$$\hookrightarrow \text{Derivable } \forall x \in]-\infty, -1[\cup]-1, 1[\cup]1, +\infty[$$

El intervalo
pedido! ✓

$$\left. \begin{array}{l} f(-1) = 2 \\ f(1) = 2 \end{array} \right\} \exists c \in]-1, 1[/ f'(c) = 0$$

$$f'(x) = 0 \quad \forall x \in]-1, 1[$$

La derivada se anula en todos los puntos $x \in]-1, 1[$

e) $f(x) = x - \frac{|x|}{x} \Rightarrow$ Es obvio que $\nexists f(x)$ en $x=0$, y por tanto no se pueden satisfacer las hipótesis del teorema de Rolle en $[-1, 1]$

f) $f(x) = x^3 + 2x^2 - 2x \rightarrow$ Continua y derivable en $\mathbb{R}$.

$$\left. \begin{array}{l} f(-3) = -3 \\ f(-2) = 4 \end{array} \right\} f(-3) \neq f(-2) \Rightarrow \text{No se cumplen las condiciones del teorema}$$

g) $f(x) = \cos x \rightarrow \text{Dom}(f(x)) = \mathbb{R} \rightarrow$ Continua en $\mathbb{R}$

$f'(x) = -\sin x \rightarrow \text{Dom}(f'(x)) = \mathbb{R} \rightarrow$ Derivable en $\mathbb{R}$

$$\left. \begin{array}{l} f(0) = \cos 0 = 1 \\ f(2\pi) = \cos(2\pi) = 1 \end{array} \right\} \exists c \in]0, 2\pi[\mid f'(c) = 0$$

$$f'(x) = -\sin x$$

$$f'(x) = 0 \Rightarrow -\sin x = 0 \begin{cases} \rightarrow x = 0 \\ \rightarrow x = \pi \rightarrow x \in]0, 2\pi[\end{cases}$$

h) $f(x) = \sqrt{1 + \sin x}$

$$(\sin x) \in [-1, 1] \Rightarrow (1 + \sin x) \in [0, 2] \Rightarrow$$

$$\Rightarrow \exists \sqrt{1 + \sin x} \quad \forall x \in \mathbb{R} \rightarrow \text{Continua en } \mathbb{R}$$

$$f'(x) = \frac{\cos x}{2\sqrt{1+\sin x}} \quad 1 + \sin x = 0$$

$$\sin x = -1 \rightarrow x = -\pi/2$$

$\Rightarrow \nexists f'(x)$ en $x = -\frac{\pi}{2} \Rightarrow$ No es derivable para $x \in]-\pi, 0[$

$\Rightarrow$ No se cumplen las condiciones del teorema

i) $f(x) = -x^4 + x^2 \rightarrow$ Continua y derivable en $\mathbb{R}$

$$\left. \begin{array}{l} f(0) = 0 \\ f(1) = 0 \end{array} \right\} \exists c \in]0, 1[/ f'(c) = 0$$

$$f'(x) = -4x^3 + 2x$$

$$f'(x) = 0 \rightarrow -4x^3 + 2x = 0 \Rightarrow 2x(-2x^2 + 1) = 0$$

$$\begin{array}{l} \nearrow x = 0 \\ \searrow -2x^2 + 1 = 0 \end{array} \rightarrow \begin{array}{l} x = 0.71 \\ x = -0.71 \end{array}$$

$$\Rightarrow x = 0.71 \in]0, 1[$$

1.3) a) $f(x) = x^3 - x \rightarrow$ Continua y Derivable en $\mathbb{R}$

$$\Rightarrow \exists c \in]-1, 2[/ f'(c) = \frac{f(2) - f(-1)}{2 - (-1)}$$

$$f'(x) = 3x^2 - 1 ; \quad \frac{f(2) - f(-1)}{2 - (-1)} = \frac{6 - 0}{3} = 2$$

$$\Rightarrow 3x^2 - 1 = 2 \Rightarrow 3x^2 = 3$$

$$\begin{array}{l} \nearrow x = -1 \\ \searrow x = +1 \rightarrow x \in]-1, 2[\end{array}$$

b) $f(x) = x^3 - 3x + 5 \rightarrow$ continua y derivable en $\mathbb{R}$.

$$\Rightarrow \exists c \in]-2, 0[/ f'(c) = \frac{f(0) - f(-2)}{0 - (-2)}$$

$$f'(x) = 3x^2 - 3 ; \frac{f(0) - f(-2)}{0 - (-2)} = \frac{5 - 3}{2} = 1$$

$$\Rightarrow 3x^2 - 3 = 1 \Rightarrow 3x^2 = 4 \begin{cases} x = 1.1547 \\ x = -1.1547 \rightarrow x \in]-2, 0[\end{cases}$$

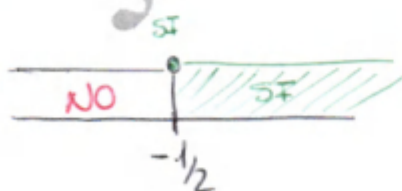
c) $f(x) = (x-2)^3 \rightarrow$ Continua y derivable en $\mathbb{R}$.

$$\Rightarrow \exists c \in]0, 3[/ f'(c) = \frac{f(3) - f(0)}{3 - 0}$$

$$f'(x) = 3(x-2)^2 ; \frac{f(3) - f(0)}{3 - 0} = \frac{1 - (-8)}{3} = \frac{9}{3} = 3$$

$$\Rightarrow 3(x-2)^2 = 3 \Rightarrow (x-2)^2 = 1 \begin{cases} x-2 = 1 \Rightarrow x = 3 \\ x-2 = -1 \Rightarrow x = 1 \rightarrow x \in]0, 3[\end{cases}$$

d) $f(x) = \sqrt{1+2x}$; $1+2x \geq 0 \Rightarrow 1+2x = 0 \Rightarrow x = -1/2$



$$\Rightarrow \text{Dom}(f(x)) = [-1/2, +\infty[$$

$$\Rightarrow \text{Continua en } [\frac{3}{2}, 4]$$

$$f'(x) = \frac{1}{\sqrt{1+2x}}$$

$$\Rightarrow \text{Dom}(f'(x)) =]-1/2, +\infty[$$

$$\Rightarrow \text{Derivable en }]\frac{3}{2}, 4[$$

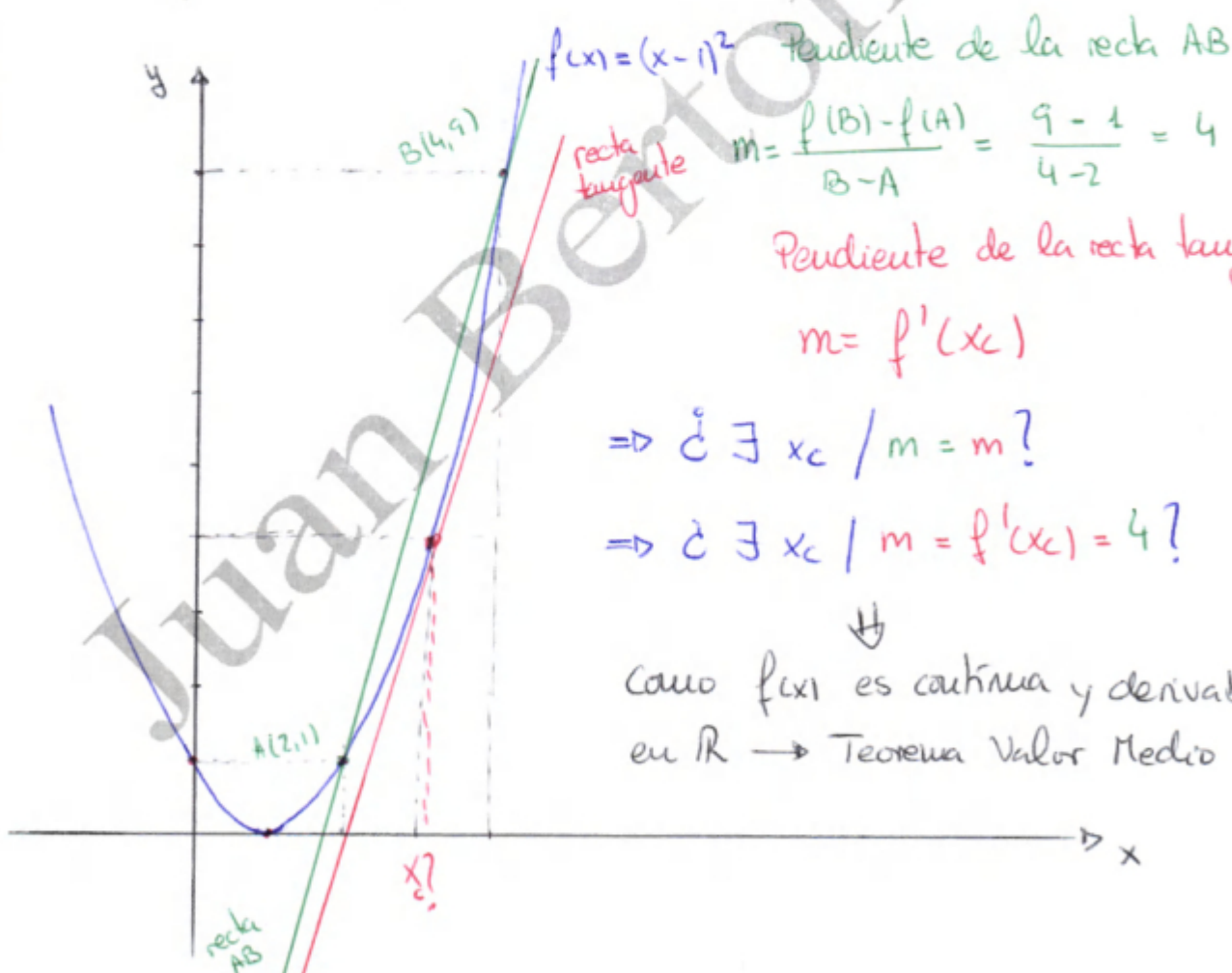
$$\Rightarrow \exists c \in]3\frac{1}{2}, 4[\mid f'(c) = \frac{f(4) - f(3\frac{1}{2})}{4 - 3\frac{1}{2}}$$

$$f'(x) = \frac{1}{\sqrt{1+2x}} \quad ; \quad \frac{f(4) - f(3\frac{1}{2})}{4 - 3\frac{1}{2}} = \frac{3 - 2}{5/2} = \frac{1}{5/2} = \frac{2}{5}$$

$$\Rightarrow \frac{1}{\sqrt{1+2x}} = \frac{2}{5} \Rightarrow \sqrt{1+2x} = \frac{5}{2} \Rightarrow 1+2x = \frac{25}{4} \Rightarrow$$

$$\Rightarrow x = \frac{21}{8} \in]3\frac{1}{2}, 4[$$

1.4)

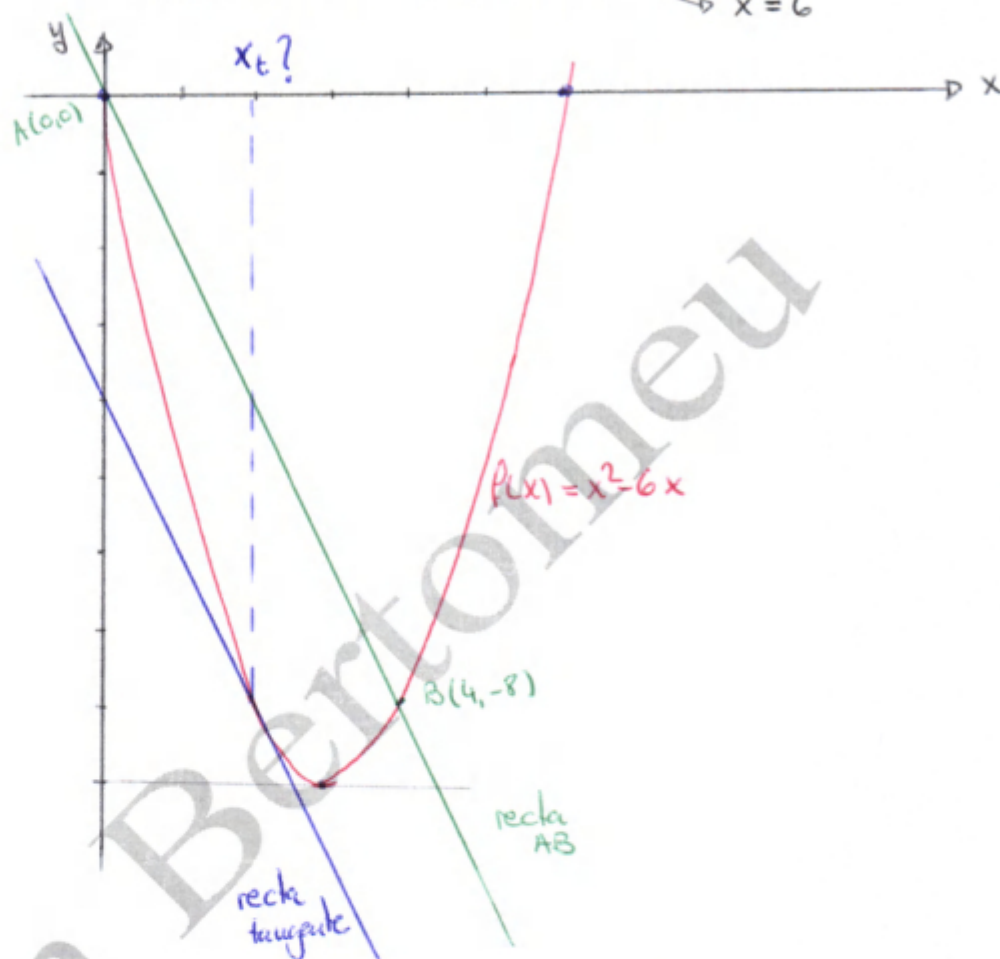


$$\left. \begin{array}{l} f'(x) = 2(x-1) \\ m = 4 \end{array} \right\} 2(x-1) = 4 \Rightarrow x = 3 \in]2, 4[$$

1.5) $f(x) = x^2 - 6x$

- Parábola Cóncava
- X vértice = 3
- Puntos de corte $\Rightarrow x^2 - 6x = 0 \Rightarrow x = 0$ and $x = 6$

x	y = x ² - 6x
0	0
3	-9
4	-8
6	0



Pendiente recta AB $\rightarrow m = \frac{f(B) - f(A)}{B - A} = \frac{-8 - 0}{4 - 0} = -2$

Pendiente recta tangente $\rightarrow m = f'(x_t)$

$f'(x_t) = -2$

$$\left. \begin{array}{l} f'(x) = 2x - 6 \\ f'(x_t) = -2 \end{array} \right\} 2x - 6 = -2 \Rightarrow 2x = 4 \Rightarrow x_t = 2 \in]0, 4[$$

1.6) Tenemos que probar que $f'(x) = 0$ para cualquier intervalo real. Tenemos que saber:

$$y = \sec x \longrightarrow y' = \sec x \cdot \operatorname{tg} x$$

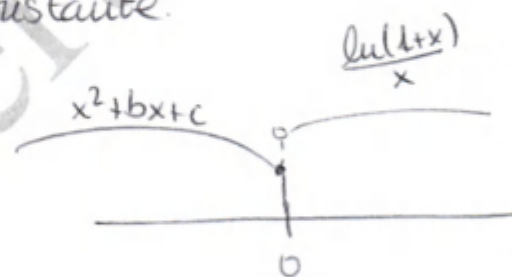
$$y = \operatorname{tg} x \longrightarrow y' = \frac{1}{\cos^2 x}$$

Así, como $f(x) = \sec^2 x - \operatorname{tg}^2 x$, se tendrá:

$$\begin{aligned} f'(x) &= 2\sec x \cdot \sec x \cdot \operatorname{tg} x - 2\operatorname{tg} x \cdot \frac{1}{\cos^2 x} = \\ &= 2\sec^2 x \cdot \operatorname{tg} x - 2\sec^2 x \cdot \operatorname{tg} x = 0 \quad \left(\sec x = \frac{1}{\cos x} \right) \end{aligned}$$

$\Rightarrow$ Como $f'(x) = 0 \Rightarrow f(x)$ es constante.

$$1.7) f(x) = \begin{cases} x^2 + bx + c & \text{si } x \leq 0 \\ \frac{\ln(1+x)}{x} & \text{si } x > 0 \end{cases}$$



Si $x < 0 \rightarrow f(x) = x^2 + bx + c \rightarrow$ Continua y Derivable en $\mathbb{R}$

Si $x > 0 \rightarrow f(x) = \frac{\ln(1+x)}{x} \rightarrow \begin{cases} 1+x > 0 \rightarrow x > -1 \\ x \neq 0 \end{cases} \Rightarrow \text{Continua }]0, +\infty[$

$$\rightarrow f'(x) = \frac{\frac{1}{1+x} \cdot x - \ln(1+x)}{x^2} = \frac{x - (1+x)\ln(1+x)}{x^2(1+x)}$$

$$\begin{cases} 1+x > 0 \rightarrow x > -1 \\ x^2(1+x) \neq 0 \rightarrow \begin{cases} x \neq 0 \\ x \neq -1 \end{cases} \end{cases} \Rightarrow \text{Derivable }]0, +\infty[$$

Si $x = 0$: Continuidad

$$f(0) = c$$

$$\lim_{x \rightarrow 0} f(x) \rightarrow \begin{cases} \lim_{x \rightarrow 0^-} x^2 + bx + c = c \\ \lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{1}{1+x} = 1 \end{cases}$$

$$\Rightarrow \boxed{\text{Si } c = 1} \Rightarrow \lim_{x \rightarrow 0} f(x) = 1 = f(0) \Rightarrow \text{Continua en } x = 0$$

Derivabilidad:

$$f'(x) = \begin{cases} 2x + b & \text{si } x \leq 0 \\ \frac{x - (1+x)\ln(1+x)}{x^2(1+x)} & \text{si } x > 0 \end{cases}$$

$$\left. \begin{aligned} f'(0)^- &= 2 \cdot 0 + b = b \\ f'(0)^+ &= \frac{0}{0} \end{aligned} \right\} \text{ Como al calcular } f'(0)^+ \text{ se presenta una} \\ \text{indeterminación (obvio, pues ya habíamos} \\ \text{razonado que } f(x) = \frac{\ln(1+x)}{x} \text{ no era derivable} \\ \text{en } x=0), \text{ tendremos que aplicar la definición} \\ \text{de derivada lateral.}$$

$$f'(0)^+ = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^+} \frac{\frac{\ln(1+x)}{x} - 1}{x} = \lim_{x \rightarrow 0^+} \frac{\ln(1+x) - x}{x^2} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x} - 1}{2x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{-\frac{1}{1+x^2}}{2} = -\frac{1}{2}$$

Por lo tanto, si $b = -1/2$ la función será derivable en $x=0$

c) Hemos visto que para $c=1$, la función es continua en $x=0$, y por tanto continua en $\mathbb{R}$.

También hemos visto que la función será derivable en $]0, +\infty[$ independientemente del valor de b .

Por tanto, se satisfacen las hipótesis del Teorema del Valor medio en $[0, e-1]$, y el teorema nos asegura que

existe $c \in]0, e-1[$ / $f'(c) = \frac{f(e-1) - f(0)}{e-1-0}$. Veámoslo:

$$f(e-1) = \frac{\ln(1+e-1)}{e-1} = \frac{1}{e-1}$$

$$f(0) = c = 1$$

$$\Rightarrow f'(c) = \frac{\frac{1}{e-1} - 1}{e-1} = \frac{\frac{1-e+1}{e-1}}{e-1} = \frac{2-e}{(e-1)^2}, \text{ como queríamos demostrar}$$

2) REGLA DE L'HOPITAL

$$2.1) 1) \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{e^x}{1} = 1$$

$$2) \lim_{x \rightarrow 0} \frac{5x - \sin x}{2x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{5 - \cos x}{2} = \frac{4}{2} = 2$$

$$3) \lim_{x \rightarrow 0} \frac{\sin x}{x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\cos x}{1} = 1$$

$$4) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\sin x}{1} = 0$$

$$5) \lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 5x + 4} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 1} \frac{1/x}{2x - 5} = -1/3$$

$$6) \lim_{x \rightarrow 0} \frac{x - \sin x}{\tan x - x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{1 - \cos x}{1 + \tan^2 x - 1} = \left[\frac{0}{0} \right] \stackrel{L'H}{=}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{2 \tan x (1 + \tan^2 x)} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 0} \frac{\cancel{\sin x}}{2 \cdot \frac{\cancel{\sin x}}{\cos x} (1 + \tan^2 x)} =$$

$$= \lim_{x \rightarrow 0} \frac{\cos x}{2(1 + \tan^2 x)} = \frac{1}{2(1+0)} = \frac{1}{2}$$

$$7) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\sin x}{2x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\cos x}{2} = \frac{1}{2}$$

8) $\lim_{x \rightarrow \infty} \frac{\sqrt{1+x}}{x} = 0$, porque el " ∞ " del denominador es de mayor orden. No obstante, también se puede aplicar L'Hôpital.

$$\hookrightarrow \lim_{x \rightarrow \infty} \frac{\sqrt{1+x}}{x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{\frac{1}{2\sqrt{1+x}}}{1} = \frac{0}{1} = 0$$

9) $\lim_{x \rightarrow \infty} \frac{\ln x}{x^2} = 0$, porque el " ∞ " del denominador es de mayor orden. No obstante se puede ver con L'Hôpital.

$$\hookrightarrow \lim_{x \rightarrow \infty} \frac{\ln x}{x^2} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{1/x}{2x} = \lim_{x \rightarrow \infty} \frac{1}{2x^2} = 0$$

10) $\lim_{x \rightarrow \infty} \frac{e^x}{x^3} = \infty$, porque el " ∞ " del numerador es de mayor orden. No obstante, apliquemos L'Hôpital.

$$\hookrightarrow \lim_{x \rightarrow \infty} \frac{e^x}{x^3} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{3x^2} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{6x} = \left[\frac{\infty}{\infty} \right] =$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{6} = \infty$$

11) $\lim_{x \rightarrow \infty} \frac{x^3}{e^{3x}} = 0$, porque el " ∞ " del denominador es de mayor orden. No obstante, apliquemos L'Hôpital.

$$\hookrightarrow \lim_{x \rightarrow \infty} \frac{x^3}{e^{3x}} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{3x^2}{3e^{3x}} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{6x}{9e^{3x}} = \left[\frac{\infty}{\infty} \right] =$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{6}{27e^{3x}} = 0$$

12) $\lim_{x \rightarrow \infty} \frac{3x^2 - 6x}{x^2 - x - 2} = \lim_{x \rightarrow \infty} \frac{3x^2}{x^2} = 3$

$$\hookrightarrow \lim_{x \rightarrow \infty} \frac{3x^2 - 6x}{x^2 - x - 2} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{6x - 6}{2x - 1} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{6}{2} = 3$$

13) $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$, por orden de magnitud de los infinitos.

$$\hookrightarrow \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{1/x}{1} = \frac{0}{1} = 0$$

14) $\lim_{x \rightarrow 0^+} x \cdot \ln x = [0 \cdot (-\infty)] = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2} =$

$$= \lim_{x \rightarrow 0^+} -x = 0$$

$$15) \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = [\infty - \infty] = \lim_{x \rightarrow 0} \frac{x - \sin x}{x \sin x} = \left[\frac{0}{0} \right] =$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{1 - \cos x}{1 \sin x + x \cos x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\sin x}{\cos x + \cos x + 1 \cdot (-\sin x)} = \frac{0}{2} = 0$$

$$16) \lim_{x \rightarrow \infty} (e^{-x} \cdot \ln^2 x) = \lim_{x \rightarrow \infty} \frac{\ln^2 x}{e^x} = 0, \text{ por orden de magnitud de los infinitos.}$$

$$\begin{aligned} \hookrightarrow \lim_{x \rightarrow \infty} (e^{-x} \cdot \ln^2 x) &= [0 \cdot \infty] = \lim_{x \rightarrow \infty} \frac{\ln^2 x}{e^x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \\ &= \lim_{x \rightarrow \infty} \frac{2 \ln x \cdot \frac{1}{x}}{e^x} = \lim_{x \rightarrow \infty} \frac{2 \ln x}{x \cdot e^x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{2 \cdot \frac{1}{x}}{e^x + x e^x} = \\ &= \lim_{x \rightarrow \infty} \frac{2}{x e^x + x^2 e^x} = \left[\frac{2}{\infty} \right] = 0 \end{aligned}$$

$$17) \lim_{x \rightarrow \infty} (e^{-x} \cdot \sqrt{x}) = \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{e^x} = 0, \text{ por orden de magnitud de los infinitos.}$$

$$\begin{aligned} \hookrightarrow \lim_{x \rightarrow \infty} (e^{-x} \cdot \sqrt{x}) &= [0 \cdot \infty] = \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{e^x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{2} \sqrt{x}}{e^x} = \\ &= \lim_{x \rightarrow \infty} \frac{1}{2 \sqrt{x} \cdot e^x} = \left[\frac{1}{\infty} \right] = 0 \end{aligned}$$

$$18) \lim_{x \rightarrow \infty} \left(e^x \cdot \frac{1}{x^2} \right) = \lim_{x \rightarrow \infty} \frac{e^x}{x^2} = \infty, \text{ por orden de magnitud de los infinitos.}$$

$$\hookrightarrow \lim_{x \rightarrow \infty} \frac{e^x}{x^2} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{2x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{2} = \infty$$

$$19) \lim_{x \rightarrow \infty} x \cdot (e^{1/x} - 1) = [\infty \cdot 0] = \lim_{x \rightarrow \infty} \frac{e^{1/x} - 1}{1/x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \\ = \lim_{x \rightarrow \infty} \frac{e^{1/x} \cdot (-1/x^2)}{(-1/x^2)} = \lim_{x \rightarrow \infty} e^{1/x} = e^0 = 1$$

$$20) \lim_{x \rightarrow \frac{\pi}{4}} (1 - \tan x) \sec(2x) = [0 \cdot \infty] = \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{\cos(2x)} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \\ \text{sec } x = \frac{1}{\cos x} \\ = \lim_{x \rightarrow \frac{\pi}{4}} \frac{+ (1 + \tan^2 x)}{+ \sec(2x) \cdot 2} = \frac{2}{2} = 1$$

$$21) \lim_{x \rightarrow 0^+} x^{\sin x} = [0^0] \Rightarrow A = \lim_{x \rightarrow 0^+} x^{\sin x}$$

$$\ln A = \ln \lim_{x \rightarrow 0^+} x^{\sin x} = \lim_{x \rightarrow 0^+} \ln x^{\sin x} = \lim_{x \rightarrow 0^+} \sin x \cdot \ln x$$

$$\lim_{x \rightarrow 0^+} \sin x \cdot \ln x = [0 \cdot -\infty] = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/\sin x} = \left[\frac{-\infty}{\infty} \right] \stackrel{L'H}{=} \\ = \lim_{x \rightarrow 0^+} \frac{1/x}{-1/\cos x} = \lim_{x \rightarrow 0^+} \frac{\sin^2 x}{-x \cdot \cos x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{2 \sin x \cos x}{-1 \cos x + x \sin x} = \frac{0}{-1} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow 0^+} x^{\sin x} = e^0 = 1$$

$$22) \lim_{x \rightarrow 0^+} x^x = [0^0] \Rightarrow A = \lim_{x \rightarrow 0^+} x^x$$

$$\ln A = \ln \lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} \ln x^x = \lim_{x \rightarrow 0^+} x \cdot \ln x$$

$$\lim_{x \rightarrow 0^+} x \cdot \ln x = [0 \cdot (-\infty)] = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} = \left[\frac{-\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2} =$$

$$= \lim_{x \rightarrow 0^+} -x = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow 0^+} x^x = e^0 = 1$$

$$23) \lim_{x \rightarrow \infty} (1+x)^{1/x} = [\infty^0] \Rightarrow A = \lim_{x \rightarrow \infty} (1+x)^{1/x}$$

$$\ln A = \ln \lim_{x \rightarrow \infty} (1+x)^{1/x} = \lim_{x \rightarrow \infty} \ln (1+x)^{1/x} = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln(1+x)$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln(1+x) = [0 \cdot \infty] = \lim_{x \rightarrow \infty} \frac{\ln(1+x)}{x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{1}{1+x} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow \infty} (1+x)^{1/x} = e^0 = 1$$

$$24) \lim_{x \rightarrow \infty} x^{1/x} = [\infty^0] \Rightarrow A = \lim_{x \rightarrow \infty} x^{1/x}$$

$$\ln A = \ln \lim_{x \rightarrow \infty} x^{1/x} = \lim_{x \rightarrow \infty} \ln x^{1/x} = \lim_{x \rightarrow \infty} \frac{1}{x} \ln x$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln x = \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow \infty} x^{1/x} = e^0 = 1$$

$$25) \lim_{x \rightarrow 0} (\cos x)^{1/x} = [1^\infty] \Rightarrow A = \lim_{x \rightarrow 0} (\cos x)^{1/x}$$

$$\ln A = \lim_{x \rightarrow 0} \ln (\cos x)^{1/x} = \lim_{x \rightarrow 0} \ln(\cos x)^{1/x} = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \ln(\cos x)$$

$$\lim_{x \rightarrow 0} \frac{1}{x} \cdot \ln(\cos x) = [\infty \cdot 0] = \lim_{x \rightarrow 0} \frac{\ln(\cos x)}{x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} =$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{\cos x}(-\sin x)}{1} = \lim_{x \rightarrow 0} -\frac{\sin x}{\cos x} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow 0} (\cos x)^{1/x} = e^0 = 1$$

$$26) \lim_{x \rightarrow 0^+} \left(\frac{1}{x} \right)^{\tan x} = [\infty^0] \Rightarrow A = \lim_{x \rightarrow 0^+} \left(\frac{1}{x} \right)^{\tan x}$$

$$\ln A = \lim_{x \rightarrow 0^+} \ln \left(\frac{1}{x} \right)^{\tan x} = \lim_{x \rightarrow 0^+} \ln \left(\frac{1}{x} \right)^{\tan x} = \lim_{x \rightarrow 0^+} \tan x \cdot \ln \left(\frac{1}{x} \right)$$

$$\lim_{x \rightarrow 0^+} \tan x \cdot \ln \left(\frac{1}{x} \right) = [0 \cdot \infty] = \lim_{x \rightarrow 0^+} \frac{\ln(1/x)}{\frac{1}{\tan x}} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} =$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{1/x} \cdot \frac{-1}{x^2}}{\frac{1}{\tan^2 x}} = \lim_{x \rightarrow 0^+} \frac{\tan^2 x}{x(1+\tan^2 x)} = \lim_{x \rightarrow 0^+} \frac{\frac{\sin^2 x}{\cos^2 x}}{x \cdot \left(1 + \frac{\sin^2 x}{\cos^2 x} \right)} =$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{\sin^2 x}{\cos^2 x}}{x \cdot \left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x} \right)} = \lim_{x \rightarrow 0^+} \frac{\sin^2 x}{x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} 2 \sin x \cos x = 0$$

$\cos^2 x + \sin^2 x = 1$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow 0^+} \left(\frac{1}{x} \right)^{\tan x} = e^0 = 1$$

$$27) \lim_{x \rightarrow 0} \frac{x \cos x - \operatorname{sen} x}{x^3} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\cancel{\cos x} - x \cancel{\operatorname{sen} x} - \cancel{\cos x}}{3x^2} =$$

$$= \lim_{x \rightarrow 0} \frac{-\operatorname{sen} x}{3x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{-\cos x}{3} = -\frac{1}{3}$$

$$28) \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 4x}{x - \operatorname{sen} x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 4}{1 - \cos x} = \left[\frac{-2}{0} \right] = -\infty$$

$$\Rightarrow \begin{cases} \lim_{x \rightarrow 0^-} f(x) = \frac{-}{+} = -\infty \\ \lim_{x \rightarrow 0^+} f(x) = \frac{-}{+} = -\infty \end{cases} \Rightarrow \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 4x}{x - \operatorname{sen} x} = -\infty$$

$$29) \lim_{x \rightarrow \frac{\pi}{2}} (1 + 2\cos x)^{\frac{1}{\cos x}} = \left[1^\infty \right] \Rightarrow A = \lim_{x \rightarrow \frac{\pi}{2}} (1 + 2\cos x)^{\frac{1}{\cos x}}$$

$$\ln A = \lim_{x \rightarrow \frac{\pi}{2}} \ln (1 + 2\cos x)^{\frac{1}{\cos x}} = \lim_{x \rightarrow \frac{\pi}{2}} \ln (1 + 2\cos x)^{\frac{1}{\cos x}} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{\cos x} \cdot \ln (1 + 2\cos x)$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln (1 + 2\cos x)}{\cos x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{1}{1+2\cos x} \cdot (-2\operatorname{sen} x)}{-\operatorname{sen} x} =$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{2}{1 + 2\cos x} = 2$$

$$\Rightarrow \ln A = 2 \Rightarrow A = \lim_{x \rightarrow \frac{\pi}{2}} (1 + 2\cos x)^{\frac{1}{\cos x}} = e^2$$

$$30) \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\ln(1+x)} \right) = [\infty - \infty] = \lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x \cdot \ln(1+x)} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \frac{1}{0}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{1+x} - 1}{\ln(1+x) + \frac{x}{1+x}} = \lim_{x \rightarrow 0} \frac{\frac{1-1-x}{1+x}}{\frac{(1+x)\ln(1+x) + x}{1+x}} = \lim_{x \rightarrow 0} \frac{-x}{(1+x)(\ln(1+x) + x)} = \frac{0}{0}$$

$$= \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{-1}{\ln(1+x) + \underbrace{\frac{(1+x)}{(1+x)}}_1 + 1} = -\frac{1}{2}$$

$$31) \lim_{x \rightarrow 0} \frac{2x \operatorname{sen} x}{1 - \cos x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{2 \operatorname{sen} x + 2x \cos x}{\operatorname{sen} x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \frac{2 \cos x + 2 \cos x - 2x \operatorname{sen} x}{\cos x} = \frac{4}{1} = 4$$

$$32) \lim_{x \rightarrow 0} \frac{\ln(\cos x) - 1 + \cos x}{x^2} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\cos x} \cdot (-\operatorname{sen} x) - \operatorname{sen} x}{2x} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \frac{\operatorname{sen} x + \operatorname{sen} x \cos x}{-2x \cos x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\cos x + \cos^2 x - \operatorname{sen}^2 x}{-2 \cos x + 2x \operatorname{sen} x} = \frac{2}{-2} = -1$$

$$33) \lim_{x \rightarrow 0} \frac{\ln(\cos 3x)}{\ln(\cos 2x)} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\cos 3x} \cdot -\operatorname{sen} 3x \cdot 3}{\frac{1}{\cos 2x} \cdot -\operatorname{sen} 2x \cdot 2} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 0} \frac{3}{2} \cdot \frac{\cos 2x}{\cos 3x} \cdot \frac{\operatorname{sen} 3x}{\operatorname{sen} 2x} = \frac{3}{2} \cdot 1 \cdot \lim_{x \rightarrow 0} \frac{\operatorname{sen} 3x}{\operatorname{sen} 2x} \stackrel{L'H}{=} \frac{0}{0}$$

$$= \frac{3}{2} \cdot 1 \cdot \lim_{x \rightarrow 0} \frac{\cos 3x \cdot 3}{\cos 2x \cdot 2} = \frac{9}{4}$$

$$34) \lim_{x \rightarrow 0} \frac{e^x - x - \cos x}{\sin^2 x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{e^x - 1 + \sin x}{2 \sin x \cos x} = \left[\frac{0}{0} \right] =$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{e^x + \cos x}{2 [\cos^2 x - \sin^2 x]} = \frac{2}{2} = 1$$

$$35) \lim_{x \rightarrow 0} \frac{(e^x - e^{-x})^2}{x^2} = \left[\frac{0}{0} \right] \stackrel{\text{solo operaciones}}{=} \lim_{x \rightarrow 0} \frac{e^{2x} - 2e^x \cdot e^{-x} + e^{-2x}}{x^2} =$$

$$= \lim_{x \rightarrow 0} \frac{e^{2x} + e^{-2x} - 2}{x^2} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{2e^{2x} - 2e^{-2x}}{2x} = \lim_{x \rightarrow 0} \frac{e^{2x} - e^{-2x}}{x} =$$

$$= \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} 2e^{2x} + 2e^{-2x} = 2 + 2 = 4$$

$$36) \lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\sin x + x \cos x}{1 + \sin x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=}$$

$$= \lim_{x \rightarrow 0} \frac{\cos x + \cos x - x \sin x}{\cos x} = \frac{2}{1} = 2$$

$$37) \lim_{x \rightarrow 0} \frac{\sin x - x}{\tan x - x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\cos x - 1}{1 + \tan^2 x - 1} = \left[\frac{0}{0} \right] \stackrel{L'H}{=}$$

$$= \lim_{x \rightarrow 0} \frac{-\sin x}{2 \tan x (1 + \tan^2 x)} = \lim_{x \rightarrow 0} \frac{-\sin x}{2 \cdot \frac{\sin x}{\cos x} (1 + \tan^2 x)} =$$

$$= \lim_{x \rightarrow 0} \frac{-\cos x}{2 (1 + \tan^2 x)} = \frac{-1}{2}$$

$$38) \lim_{x \rightarrow \infty} \left[x \cdot \left(\arctan e^x - \frac{\pi}{2} \right) \right] = [\infty \cdot 0] = \lim_{x \rightarrow \infty} \frac{\arctan e^x - \frac{\pi}{2}}{1/x} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{1+(e^x)^2} \cdot e^x}{-1/x^2} = \lim_{x \rightarrow \infty} \frac{-x^2 \cdot e^x}{1+e^{2x}} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=}$$

↳ Este límite es 0 por orden de magnitud!

$$= \lim_{x \rightarrow \infty} - \frac{(2x \cdot e^x + x^2 \cdot e^x)}{2e^{2x}} = \lim_{x \rightarrow \infty} \frac{e^x (-2x - x^2)}{2(e^x)^2} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=}$$

$$= \lim_{x \rightarrow \infty} - \frac{2 - 2x}{2e^x} = \lim_{x \rightarrow \infty} - \frac{1-x}{e^x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} - \frac{1}{e^x} = 0$$

$$39) \lim_{x \rightarrow \infty} (\sqrt{x+\sqrt{x}} - \sqrt{x}) = [\infty - \infty] =$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x+\sqrt{x}} - \sqrt{x})(\sqrt{x+\sqrt{x}} + \sqrt{x})}{\sqrt{x+\sqrt{x}} + \sqrt{x}} = \lim_{x \rightarrow \infty} \frac{(\sqrt{x+\sqrt{x}})^2 - (\sqrt{x})^2}{\sqrt{x+\sqrt{x}} + \sqrt{x}} =$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x}} + \sqrt{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{2\sqrt{x}} = \frac{1}{2}$$

Cogiendo los inputs más altos.

$\left[\frac{\infty}{\infty} \right] \rightarrow$ Por L'Hôpital saldría demasiado complejo al ser las derivadas muy largas. No lo hago!!

$$40) \lim_{x \rightarrow 0} \left(\frac{x^2 + 2}{2 - 2\sin x} \right)^{2/x} = [1^\infty] \Rightarrow A = \lim_{x \rightarrow 0} \left(\frac{x^2 + 2}{2 - 2\sin x} \right)^{2/x}$$

$$\ln A = \ln \lim_{x \rightarrow 0} \left(\frac{x^2 + 2}{2 - 2\sin x} \right)^{2/x} = \lim_{x \rightarrow 0} \ln \left(\frac{x^2 + 2}{2 - 2\sin x} \right)^{2/x} = \lim_{x \rightarrow 0} \frac{2}{x} \ln \left(\frac{x^2 + 2}{2 - 2\sin x} \right)$$

$$\lim_{x \rightarrow 0} \frac{2}{x} \ln \left(\frac{x^2 + 2}{2 - 2\sin x} \right) = [\infty \cdot 0] = \lim_{x \rightarrow 0} \frac{2 \cdot \ln \left(\frac{x^2 + 2}{2 - 2\sin x} \right)}{x} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \underbrace{2 \cdot \frac{2 - 2\sin x}{x^2 + 2}}_{\frac{2}{2} = 1} \cdot \underbrace{\frac{2x(2 - 2\sin x) - (x^2 + 2) \cdot (-2\cos x)}{(2 - 2\sin x)^2}}_{\frac{(-2) \cdot (-2)}{2} = 2} = 1 \cdot 2 = 2$$

$$\Rightarrow \ln A = 2 \Rightarrow A = \lim_{x \rightarrow 0} \left(\frac{x^2 + 2}{2 - 2\sin x} \right)^{2/x} = e^2$$

$$41) \lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x-1} \right)^x = [1^\infty] \Rightarrow A = \lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x-1} \right)^x$$

$$\ln A = \ln \lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x-1} \right)^x = \lim_{x \rightarrow \infty} \ln \left(\frac{3x+1}{3x-1} \right)^x = \lim_{x \rightarrow \infty} x \cdot \ln \left(\frac{3x+1}{3x-1} \right)$$

$$\lim_{x \rightarrow \infty} x \cdot \ln \left(\frac{3x+1}{3x-1} \right) = [\infty \cdot 0] = \lim_{x \rightarrow \infty} \frac{\ln \left(\frac{3x+1}{3x-1} \right)}{1/x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{3x-1}{3x+1} \cdot \frac{3(3x-1) - 3(3x+1)}{(3x-1)^2}}{-1/x^2} = \lim_{x \rightarrow \infty} \frac{6x^2}{9x^2 - 1} = \frac{6}{9} = \frac{2}{3}$$

$$\Rightarrow \ln A = \frac{2}{3} \Rightarrow A = \lim_{x \rightarrow \infty} \left(\frac{3x+1}{3x-1} \right)^x = e^{2/3}$$

$$42) \lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{x}{x-1} \right) = [\infty - \infty] = \lim_{x \rightarrow 1} \frac{x - 1 - x \ln x}{(x-1) \ln x} =$$

$$= \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 1} \frac{1 - (\ln x + x \cdot \frac{1}{x})}{\ln x + \frac{x-1}{x}} = \lim_{x \rightarrow 1} \frac{-\ln x}{\ln x + \frac{x-1}{x}} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 1} \frac{-1/x}{1/x + \frac{x+1-x}{x^2}} = \frac{-1}{1+1} = -\frac{1}{2}$$

$$43) \lim_{x \rightarrow \frac{\pi}{2}^-} (\operatorname{tg} x)^{\cos x} = [\infty^0] \Rightarrow A = \lim_{x \rightarrow \frac{\pi}{2}^-} (\operatorname{tg} x)^{\cos x}$$

$$\ln A = \ln \lim_{x \rightarrow \frac{\pi}{2}^-} (\operatorname{tg} x)^{\cos x} = \lim_{x \rightarrow \frac{\pi}{2}^-} \ln (\operatorname{tg} x)^{\cos x} = \lim_{x \rightarrow \frac{\pi}{2}^-} \cos x \cdot \ln (\operatorname{tg} x)$$

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \cos x \cdot \ln (\operatorname{tg} x) = [0 \cdot \infty] = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\ln (\operatorname{tg} x)}{1/\cos x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\frac{\cos x}{\operatorname{sen} x} \cdot (1 + \operatorname{tg}^2 x)}{\frac{\operatorname{sen} x}{\cos^2 x}} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\frac{\cos x}{\operatorname{sen} x} \cdot (\cos^2 x + \operatorname{sen}^2 x)}{\frac{\operatorname{sen} x}{\cos^2 x}} =$$

$$= \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\cos x}{\operatorname{sen}^2 x} = \frac{0}{1} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow \frac{\pi}{2}^-} (\operatorname{tg} x)^{\cos x} = e^0 = 1$$

$$44) \lim_{x \rightarrow 0} (\cos(2x))^{\frac{1}{\operatorname{sen} x^2}} = [1^\infty] \Rightarrow A = \lim_{x \rightarrow 0} (\cos(2x))^{\frac{1}{\operatorname{sen} x^2}}$$

$$\ln A = \ln \lim_{x \rightarrow 0} (\cos(2x))^{\frac{1}{\operatorname{sen} x^2}} = \lim_{x \rightarrow 0} \ln (\cos(2x))^{\frac{1}{\operatorname{sen} x^2}} = \lim_{x \rightarrow 0} \frac{1}{\operatorname{sen} x^2} \cdot \ln(\cos 2x)$$

$$\lim_{x \rightarrow 0} \frac{1}{\operatorname{sen} x^2} \cdot \ln(\cos(2x)) = [\infty \cdot 0] = \lim_{x \rightarrow 0} \frac{\ln(\cos 2x)}{\operatorname{sen} x^2} = \left[\frac{0}{0} \right] \stackrel{L'H}{=}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{\cos(2x)} \cdot (-\sin(2x)) \cdot 2}{\cos x^2 \cdot 2x} = \lim_{x \rightarrow 0} \frac{-\operatorname{tg}(2x)}{x \cdot \cos x^2} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} =$$

$$= \lim_{x \rightarrow 0} \frac{-(1 + \operatorname{tg}^2(2x)) \cdot 2}{\cos x^2 - 2x^2 \cdot \sin x^2} = \frac{-2}{1} = -2$$

$$\Rightarrow \ln A = -2 \Rightarrow A = \lim_{x \rightarrow 0} [\cos(2x)]^{\frac{1}{\sin x^2}} = e^{-2} = \frac{1}{e^2}$$

$$45) \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right) = [\infty - \infty] = \lim_{x \rightarrow 0} \frac{\sin x - x}{x \sin x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} =$$

$$= \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x + x \cos x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} = \lim_{x \rightarrow 0} \frac{-\sin x}{\cos x + \cos x - x \sin x} = \frac{0}{2} = 0$$

$$46) \lim_{x \rightarrow 0^+} x^{\operatorname{tg} x} = [0^0] \Rightarrow A = \lim_{x \rightarrow 0^+} x^{\operatorname{tg} x}$$

$$\ln A = \ln \lim_{x \rightarrow 0^+} x^{\operatorname{tg} x} = \lim_{x \rightarrow 0^+} \ln x^{\operatorname{tg} x} = \lim_{x \rightarrow 0^+} \operatorname{tg} x \cdot \ln x$$

$$\lim_{x \rightarrow 0^+} \operatorname{tg} x \cdot \ln x = [0 \cdot (-\infty)] = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/\operatorname{tg} x} = \left[\frac{-\infty}{\infty} \right] \stackrel{L'H}{=} =$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{1}{\operatorname{tg}^2 x}}} = \lim_{x \rightarrow 0^+} \frac{-\operatorname{tg}^2 x}{x(1 + \operatorname{tg}^2 x)} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} =$$

$$= \lim_{x \rightarrow 0^+} \frac{-2 \operatorname{tg} x (1 + \operatorname{tg}^2 x)}{(1 + \operatorname{tg}^2 x) + x \cdot 2 \operatorname{tg} x (1 + \operatorname{tg}^2 x)} = \frac{0}{1 + 0} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow 0^+} x^{\operatorname{tg} x} = e^0 = 1$$

$$47) \lim_{x \rightarrow \infty} \left(\frac{1}{2x} \right)^{1/x} = [0^0] \Rightarrow A = \lim_{x \rightarrow \infty} \left(\frac{1}{2x} \right)^{1/x}$$

$$\ln A = \ln \lim_{x \rightarrow \infty} \left(\frac{1}{2x} \right)^{1/x} = \lim_{x \rightarrow \infty} \ln \left(\frac{1}{2x} \right)^{1/x} = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln \left(\frac{1}{2x} \right)$$

$$\lim_{x \rightarrow \infty} \frac{1}{x} \cdot \ln \left(\frac{1}{2x} \right) = [0 \cdot (-\infty)] = \lim_{x \rightarrow \infty} \frac{\ln(1/2x)}{x} = \left[\frac{-\infty}{\infty} \right] \stackrel{L'H}{=}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow \infty} \left(\frac{1}{2x} \right)^{1/x} = e^0 = 1$$

$$48) \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\arctg x} \right) = [\infty - \infty] = \lim_{x \rightarrow 0^+} \frac{\arctg x - x}{x \arctg x} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x^2} - 1}{\arctg x + \frac{x}{1+x^2}} = \lim_{x \rightarrow 0^+} \frac{\frac{-x^2}{1+x^2}}{\frac{(1+x^2) \cdot \arctg x + x}{1+x^2}} =$$

$$= \lim_{x \rightarrow 0^+} \frac{-x^2}{(1+x^2) \arctg x + x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{-2x}{2x \arctg x + 1 + 1} = \frac{0}{2} = 0$$

$$49) \lim_{x \rightarrow 1^-} \left[(x^2-1) \cdot \lg \left(\frac{\pi x}{2} \right) \right] = [0 \cdot \infty] = \lim_{x \rightarrow 1^-} \frac{(x^2-1) \cdot \operatorname{sen} \left(\frac{\pi x}{2} \right)}{\cos \left(\frac{\pi x}{2} \right)} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 1^-} \frac{2x \operatorname{sen} \left(\frac{\pi}{2} x \right) + (x^2-1) \cdot \cos \left(\frac{\pi}{2} x \right) \cdot \frac{\pi}{2}}{-\operatorname{sen} \left(\frac{\pi}{2} x \right) \cdot \frac{\pi}{2}} = \frac{2}{-\pi/2} = -\frac{4}{\pi}$$

$$50) \lim_{x \rightarrow \infty} \left(x \cdot \ln \left(\frac{x+1}{x} \right) \right) = [\infty \cdot 0] = \lim_{x \rightarrow \infty} \frac{\ln \left(\frac{x+1}{x} \right)}{1/x} = \left[\frac{0}{0} \right] =$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{\frac{x}{x+1} \cdot \frac{x-x+1}{x^2}}{+1/x^2} = \lim_{x \rightarrow \infty} \frac{x^2}{x(x+1)} = 1$$

$$51) \lim_{x \rightarrow 0} \frac{\sec x}{4x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\cos x}{4} = \frac{1}{4}$$

$$52) \lim_{x \rightarrow 1} \frac{x^2-1}{\ln x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 1} \frac{2x}{1/x} = 2$$

$$53) \lim_{x \rightarrow 0} \frac{x^3 - x}{\sqrt{x+9} - 3} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{3x^2 - 1}{\frac{1}{2\sqrt{x+9}}} = \frac{-1}{1/6} = -6$$

$$54) \lim_{x \rightarrow 0} \frac{\lg x}{x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{1 + \lg^2 x}{1} = 1$$

$$55) \lim_{x \rightarrow 1} \frac{4x^5 - 4x^2}{-x^2 + 1} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 1} \frac{20x^4 - 8x}{-2x} = \frac{12}{-2} = -6$$

$$56) \lim_{x \rightarrow 0} \frac{x}{\sec x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{1}{\cos x} = 1$$

$$57) \lim_{x \rightarrow 0} \frac{e^{-x} - e^x}{3 \sec x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{-e^{-x} - e^x}{3 \cos x} = \frac{-2}{3}$$

$$58) \lim_{x \rightarrow 7} \frac{x^2 - 9x + 14}{\ln(x^2 - 6x - 6)} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 7} \frac{2x - 9}{\frac{2x - 6}{x^2 - 6x - 6}} =$$

$$= \lim_{x \rightarrow 7} \frac{(2x - 9)(x^2 - 6x - 6)}{2x - 6} = \frac{5}{8}$$

$$59) \lim_{x \rightarrow 0} \left(\frac{1}{4x} - \frac{2}{x \cdot e^x} \right) = [\infty - \infty] = \lim_{x \rightarrow 0} \frac{e^x - 8}{4x e^x} =$$

$$= \left[\frac{-7}{0} \right] \rightarrow \begin{cases} \lim_{x \rightarrow 0^-} \frac{e^x - 8}{4x e^x} = +\infty \\ \lim_{x \rightarrow 0^+} \frac{e^x - 8}{4x e^x} = -\infty \end{cases} \Rightarrow \nexists \lim_{x \rightarrow 0} \left(\frac{1}{4x} - \frac{2}{x \cdot e^x} \right)$$

$$60) \lim_{x \rightarrow 0^+} (\operatorname{tg} x)^x = [0^0] \Rightarrow A = \lim_{x \rightarrow 0^+} (\operatorname{tg} x)^x$$

$$\ln A = \lim_{x \rightarrow 0^+} \ln (\operatorname{tg} x)^x = \lim_{x \rightarrow 0^+} \ln (\operatorname{tg} x)^x = \lim_{x \rightarrow 0^+} x \cdot \ln (\operatorname{tg} x)$$

$$\lim_{x \rightarrow 0^+} x \cdot \ln (\operatorname{tg} x) = [0 \cdot (-\infty)] = \lim_{x \rightarrow 0^+} \frac{\ln (\operatorname{tg} x)}{1/x} = \left[\frac{-\infty}{\infty} \right] \stackrel{L'H}{=} \frac{-\infty}{\infty}$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{\operatorname{tg} x} \cdot (1 + \operatorname{tg}^2 x)}{-1/x^2} = \lim_{x \rightarrow 0^+} \frac{-x^2 \cdot (1 + \operatorname{tg}^2 x)}{\operatorname{tg} x} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0^+} \frac{-2x(1 + \operatorname{tg}^2 x) - x^2 \cdot 2 \operatorname{tg} x \cdot (1 + \operatorname{tg}^2 x)}{1 + \operatorname{tg}^2 x} = \frac{0}{1} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow 0^+} (\operatorname{tg} x)^x = e^0 = 1$$

$$61) \lim_{x \rightarrow \infty} \sqrt[x-2]{\frac{x^2+1}{x+3}} = \lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x+3} \right)^{\frac{1}{x-2}} = [\infty^0]$$

$$A = \lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x+3} \right)^{\frac{1}{x-2}}$$

$$\ln A = \ln \lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x+3} \right)^{\frac{1}{x-2}} = \lim_{x \rightarrow \infty} \frac{1}{x-2} \cdot \ln \left(\frac{x^2+1}{x+3} \right) = [0 \cdot \infty]$$

$$\lim_{x \rightarrow \infty} \frac{\ln \left(\frac{x^2+1}{x+3} \right)}{x-2} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{\cancel{x+3} \cdot \frac{2x(x+3) - (x^2+1)}{(x+3)^2}}{1} =$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + 6x - 1}{(x^2+1)(x+3)} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{x^2}{x^3} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x+3} \right)^{\frac{1}{x-2}} = e^0 = 1$$

$$62) \lim_{x \rightarrow 0} \frac{\operatorname{sen} x}{1 - \cos x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\cos x}{\operatorname{sen} x} = \left[\frac{1}{0} \right]$$

$$\rightarrow \begin{cases} \lim_{x \rightarrow 0^-} \frac{\cos x}{\operatorname{sen} x} = -\infty \\ \lim_{x \rightarrow 0^+} \frac{\cos x}{\operatorname{sen} x} = +\infty \end{cases}$$

$$\Rightarrow \nexists \lim_{x \rightarrow 0} \frac{\operatorname{sen} x}{1 - \cos x}$$

$$63) \lim_{x \rightarrow 0} \frac{2\sec x}{x^2 - 3x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{2\cos x}{2x - 3} = \frac{2}{-3} = -\frac{2}{3}$$

$$64) \lim_{x \rightarrow 0} \frac{x - \sec x}{x \cdot \sec x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sec x + x \cdot \cos x} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\sec x}{\cos x + \cos x - x \sec x} = \frac{0}{2} = 0$$

$$65) \lim_{x \rightarrow 0} \frac{x^4 - \left(\frac{1}{3}\right)x^3}{x - \operatorname{tg} x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{4x^3 - x^2}{1 - 1 - \operatorname{tg}^2 x} =$$

$$= \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{12x^2 - 2x}{-2 \operatorname{tg} x (1 + \operatorname{tg}^2 x)} = \lim_{x \rightarrow 0} \frac{x - 6x^2}{\operatorname{tg} x (1 + \operatorname{tg}^2 x)} =$$

$$= \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{1 - 12x}{(1 + \operatorname{tg}^2 x)^2 + 2 \operatorname{tg}^2 x (1 + \operatorname{tg}^2 x)} = 1$$

$$66) \lim_{x \rightarrow 0} \frac{x - 2\sec x}{\operatorname{tg} x - 2\sec x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{1 - 2\cos x}{1 + \operatorname{tg}^2 x - 2\cos x} = 1$$

$$67) \lim_{x \rightarrow 0} \frac{\ln(e^x + x^3)}{x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{e^x + 3x^2}{e^x + x^3} = 1$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x}}{\frac{3}{4} \cdot \frac{1}{\sqrt[4]{x}}} = \lim_{x \rightarrow 0^+} \frac{\sqrt[4]{x}}{3(1+x)} = \frac{0}{3} = 0$$

$$= \lim_{x \rightarrow 0} \frac{[1 + 4x^2(3x)]^3}{2} = \frac{3}{2}$$

$$\underline{\underline{\text{L'H}}}\lim_{x \rightarrow 0} \frac{e^x - (e^{\sin x} \cdot \cos^2 x - \sin x \cdot e^{\sin x})}{\cos x} = \frac{0}{1} = 0$$

$$= \lim_{x \rightarrow 0} \frac{-e^x}{2 \cos^2 x - 2 \sin^2 x} = -\frac{1}{2}$$

$$72) \lim_{x \rightarrow 2} \frac{2^x - x^2}{9 - 3^x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 2} \frac{2^x \ln 2 - 2x}{9 - 3^x \ln 3} =$$

$$= \frac{4(\ln 2 - 1)}{9(1 - \ln 3)} \approx 1.383$$

$$43) \lim_{x \rightarrow 0} (e^x - x)^{1/x} = [1^\infty]; \quad A = \lim_{x \rightarrow 0} (e^x - x)^{1/x}$$

$$\ln A = \ln \lim_{x \rightarrow 0} (e^x - x)^{1/x} = \lim_{x \rightarrow 0} \ln (e^x - x)^{1/x} = \lim_{x \rightarrow 0} \frac{1}{x} \ln(e^x - x)$$

$$\lim_{x \rightarrow 0} \frac{\ln(e^x - x)}{x} = \left[\frac{0}{0} \right] \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{e^x - x} = \frac{0}{1} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow 0} (e^x - x)^{1/x} = e^0 = 1$$

$$74) \lim_{x \rightarrow 0} (\cot x)^{\sec x} = [\infty^0]; \quad A = \lim_{x \rightarrow 0} (\cot x)^{\sec x}$$

$$\ln A = \ln \lim_{x \rightarrow 0} (\cot x)^{\sec x} = \lim_{x \rightarrow 0} \ln (\cot x)^{\sec x} = \lim_{x \rightarrow 0} \sec x \cdot \ln(\cot x)$$

$$\lim_{x \rightarrow 0} \sec x \cdot \ln(\cot x) = [0 \cdot \infty] = \lim_{x \rightarrow 0} \frac{\ln(\cot x)}{1/\sec x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} =$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{\lg(x)} \cdot \left(\frac{+(1+\lg^2 x)}{\lg^2 x} \right)}{\frac{+\cos x}{\sec^2 x}} = \lim_{x \rightarrow 0} \frac{\cancel{\sec^2 x} \cdot (1+\lg^2 x)}{\cancel{\cos x} \cdot \frac{\cancel{\sec x}}{\cancel{\cos x}}} =$$

$$= \lim_{x \rightarrow 0} \operatorname{sen} x \cdot (1 + \operatorname{tg}^2 x) = 0 \cdot 1 = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow 0} [\cot x]^{\operatorname{sen} x} = e^0 = 1$$

$$75) \lim_{x \rightarrow 0} \left(\frac{2}{\ln(1+x)} - \frac{2}{x} \right) = [\infty - \infty] =$$

$$= \lim_{x \rightarrow 0} \frac{2x - 2\ln(1+x)}{x \cdot \ln(1+x)} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{=} \frac{2+x-2}{1+x}$$

$$= \lim_{x \rightarrow 0} \frac{2 - \frac{2}{1+x}}{\ln(1+x) + \frac{x}{1+x}} = \lim_{x \rightarrow 0} \frac{\frac{2+x-2}{1+x}}{\frac{(1+x)\ln(1+x) + x}{1+x}} =$$

$$= \lim_{x \rightarrow 0} \frac{x}{(1+x)\ln(1+x) + x} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{=} \frac{1}{\ln(1+x) + 1 + 1}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\ln(1+x) + 1 + 1} = \frac{1}{2}$$

$$76) \lim_{x \rightarrow \pi/2} (x - \pi/2) \cdot \operatorname{tg} x = [0 \cdot \infty] = \lim_{x \rightarrow \pi/2} \frac{(x - \pi/2) \cdot \operatorname{sen} x}{\cos x} = \left[\frac{0}{0} \right] =$$

$$\stackrel{\text{L'H}}{=} \lim_{x \rightarrow \pi/2} \frac{\operatorname{sen} x + (x - \pi/2) \cdot \cos x}{-\operatorname{sen} x} = -1$$

$$77) \lim_{x \rightarrow \infty} x \cdot e^{2/x} = \infty$$

$$78) \lim_{x \rightarrow 0} (\operatorname{arcsen} x \cdot \cotg x) = [0 \cdot \infty] = \lim_{x \rightarrow 0} \frac{\operatorname{arcsen} x}{\tg(x)} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{1}{1+x^2} = \frac{1}{1} = 1$$

$$79) \lim_{x \rightarrow \infty} x \cdot \operatorname{arctg} \left(\frac{2}{x} \right) = [\infty \cdot 0] = \lim_{x \rightarrow \infty} \frac{\operatorname{arctg} \left(\frac{2}{x} \right)}{1/x} = \left[\frac{0}{0} \right]$$

$$\stackrel{L'H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \left(\frac{2}{x} \right)^2} \cdot \left(-\frac{2}{x^2} \right)}{+ \frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{2}{1 + \left(\frac{2}{x} \right)^2} = 2$$

$$80) \lim_{x \rightarrow \infty} (\ln x)^{e^{-x}} = [\infty^0]; \quad A = \lim_{x \rightarrow \infty} (\ln x)^{e^{-x}}$$

$$\ln A = \ln \lim_{x \rightarrow \infty} (\ln x)^{e^{-x}} = \lim_{x \rightarrow \infty} \ln (\ln x)^{e^{-x}} = \lim_{x \rightarrow \infty} e^{-x} \cdot \ln(\ln x)$$

$$\lim_{x \rightarrow \infty} e^{-x} \cdot \ln(\ln x) = [0 \cdot \infty] = \lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{e^x} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{x \cdot e^x \cdot \ln x} = \frac{1}{\infty} = 0$$

$$\Rightarrow \ln A = 0 \Rightarrow A = \lim_{x \rightarrow \infty} (\ln x)^{e^{-x}} = e^0 = 1$$

0 (orden magnitud)