

EJERCICIOS

1) Extrae factores fuera de la raíz:

$$a) \sqrt{2^3 \cdot 3 \cdot 5^2} = 2 \cdot 5 \sqrt{2 \cdot 3} = 10\sqrt{6}$$

$$b) \sqrt{120} = \sqrt{2^3 \cdot 3 \cdot 5} = 2 \cdot \sqrt{30}$$

$$c) \sqrt[3]{144} = \sqrt[3]{2^4 \cdot 3^2} = 2 \sqrt[3]{2 \cdot 3^2} = 2 \sqrt[3]{18}$$

$$d) \sqrt[4]{64 \cdot a^3 \cdot b^4} = \sqrt[4]{2^6 \cdot a^3 \cdot b^4} = 2b \sqrt[4]{2^2 a^3} = 2b \sqrt[4]{4a^3}$$

$$e) \sqrt{72 \cdot a^5 \cdot b^3 \cdot c} = \sqrt{2^3 \cdot 3^2 a^5 \cdot b^3 \cdot c} = 2 \cdot 3a^2 \cdot b \sqrt{2ac}$$

$$f) \sqrt{45 \cdot x \cdot y^6} = \sqrt{3^2 \cdot 5xy^6} = 3y^3 \sqrt{5x}$$

2) Efectúa:

$$a) \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{3} = \sqrt{2 \cdot 5 \cdot 3} = \sqrt{30}$$

$$b) \sqrt[3]{7} \cdot \sqrt[3]{5} = \sqrt[3]{35}$$

$$c) \sqrt[a]{b} \cdot \sqrt[a]{x} = \sqrt[a]{bx}$$

$$d) (\sqrt{2} + \sqrt{3})^2 = (\sqrt{2})^2 + 2 \cdot \sqrt{2} \cdot \sqrt{3} + (\sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$$

$$e) \sqrt{5}(\sqrt{3} - \sqrt{2})^2 = \sqrt{5} \left[(\sqrt{3})^2 - 2\sqrt{3} \cdot \sqrt{2} + (\sqrt{2})^2 \right] = \sqrt{5}(3 - 2\sqrt{6} + 2) = 5\sqrt{5} - 2\sqrt{30}$$

$$f) (\sqrt{5} - 2\sqrt{3})(3\sqrt{5} + \sqrt{2}) = 3 \cdot \sqrt{5} \cdot \sqrt{5} + \sqrt{5} \cdot \sqrt{2} - 2 \cdot 3 \cdot \sqrt{3} \cdot \sqrt{5} - 2 \cdot \sqrt{3} \cdot \sqrt{2} =$$

$$= 3 \cdot 5 + \sqrt{10} - 6\sqrt{15} - 2\sqrt{6} = 15 + \sqrt{10} - 6\sqrt{15} - 2\sqrt{6}$$

$$g) \sqrt{2}(2\sqrt{2} - 3\sqrt{3})(\sqrt{5} - \sqrt{2}) = \sqrt{2}(2\sqrt{10} - 2 \cdot 2 - 2\sqrt{15} + 3\sqrt{6}) = 2\sqrt{20} - 4\sqrt{2} - 3\sqrt{30} + 3\sqrt{18} =$$

$$= 2\sqrt{4 \cdot 5} - 4\sqrt{2} - 3\sqrt{30} + 3\sqrt{9 \cdot 2} = 4\sqrt{5} - 4\sqrt{2} - 3\sqrt{30} + 9\sqrt{2}$$

$$h) \sqrt{3}(\sqrt{3} - 2\sqrt{2})^2 = \sqrt{3} \left[(\sqrt{3})^2 - 2 \cdot \sqrt{3} \cdot 2\sqrt{2} + (2\sqrt{2})^2 \right] = \sqrt{3}(3 - 4\sqrt{6} + 4 \cdot 2) = 11\sqrt{3} - 4\sqrt{18} =$$

$$= 11\sqrt{3} - 4\sqrt{9 \cdot 2} = 11\sqrt{3} - 12\sqrt{2}$$

$$i) \left(\frac{3}{2\sqrt{3}} - \frac{2\sqrt{2}}{5} \right) \cdot \left(\frac{3}{2\sqrt{3}} + \frac{2\sqrt{2}}{5} \right) = \left(\frac{3}{2\sqrt{3}} \right)^2 - \left(\frac{2\sqrt{2}}{5} \right)^2 = \frac{9}{4 \cdot 3} - \frac{4 \cdot 2}{25} = \frac{3}{4} - \frac{8}{25} = \frac{75 - 32}{100} = \frac{43}{100}$$

3) Extrae factores fuera de la raíz:

$$a) \sqrt[5]{\frac{a^7 \cdot b^{12}}{c^6}} = \sqrt[5]{\frac{a^5 \cdot a^2 \cdot (b^2)^5 \cdot b^2}{c^5 \cdot c}} = \frac{a \cdot b^2}{c} \sqrt[5]{\frac{a^2 \cdot b^2}{c}}$$

$$b) \sqrt[3]{a^5 \sqrt{a^6}} = \sqrt[3]{a^5 \cdot a^3} = \sqrt[3]{a^8} = a^2 \cdot \sqrt[3]{a^2}$$

$$c) \sqrt{\frac{81a^8}{32b^3}} = \sqrt{\frac{81(a^4)^2}{16 \cdot 2b^2 \cdot b}} = \frac{9a^4}{4b} \sqrt{\frac{1}{2b}}$$

$$d) \sqrt[5]{\frac{a^9 \sqrt[3]{a^{18}}}{64b^{15}}} = \sqrt[5]{\frac{a^9 \cdot a^6}{2^5 \cdot 2(b^3)^5}} = \sqrt[5]{\frac{a^{15}}{2^5 \cdot 2(b^3)^5}} = \frac{a^3}{2b^3} \sqrt[5]{\frac{1}{2}}$$

$$e) \sqrt[3]{\frac{125 \sqrt{a^7}}{48b^8}} = \sqrt[3]{\frac{5^3 \cdot a^3 \sqrt{a}}{2^4 \cdot 3 \cdot b^6 \cdot b^2}} = \frac{5a}{2b^2} \sqrt[3]{\frac{\sqrt{a}}{6b^2}}$$

4) Introduce dentro de la raíz y simplifica:

$$a) 7\sqrt{a} = \sqrt{49a}$$

$$b) 2a\sqrt{3a} = \sqrt{(2a)^2 \cdot 3a} = \sqrt{4a^2 \cdot 3a} = \sqrt{12a^3}$$

$$c) x\sqrt{\frac{1}{x}} = \sqrt{\frac{x^2}{x}} = \sqrt{x}$$

$$d) x^3y\sqrt{\frac{x}{y}} = \sqrt{(x^3y)^2 \frac{x}{y}} = \sqrt{\frac{x^6 \cdot y^2 \cdot x}{y}} = \sqrt{x^7y}$$

$$e) \frac{1}{3}\sqrt[4]{\frac{27}{2}} = \sqrt[4]{\frac{27}{3^4 \cdot 2}} = \sqrt[4]{\frac{3^3}{3^4 \cdot 2}} = \sqrt[4]{\frac{1}{6}}$$

$$f) \frac{3}{2}\sqrt{\frac{2}{3}} = \sqrt{\frac{9 \cdot 2}{4 \cdot 3}} = \sqrt{\frac{3}{2}}$$

$$g) \frac{2}{3}\sqrt[3]{\frac{81}{4}} = \sqrt[3]{\frac{2^3 \cdot 3^4}{3^3 \cdot 2^2}} = \sqrt[3]{2 \cdot 3} = \sqrt[3]{6}$$

$$h) \left(1 + \frac{1}{2}\right)\sqrt[3]{\frac{4}{81}} = \frac{3}{2}\sqrt[3]{\frac{2^2}{3^4}} = \sqrt[3]{\frac{3^3 \cdot 2^2}{2^3 \cdot 3^4}} = \sqrt[3]{\frac{1}{6}}$$

$$i) \frac{a-b}{a+b}\sqrt{\frac{a+b}{a-b}} = \sqrt{\frac{(a-b)^2(a+b)}{(a+b)^2(a-b)}} = \sqrt{\frac{a-b}{a+b}}$$

$$j) (1-a)\sqrt{a-a^2} = \sqrt{(1-a)^2(a-a^2)} = \sqrt{(1-a)^2 a(1-a)} = \sqrt{a(1-a)^3}$$

$$k) (x+y)\sqrt{\frac{x-y}{x+y}} = \sqrt{\frac{(x+y)^2 \cdot (x-y)}{x+y}} = \sqrt{(x+y)(x-y)} = \sqrt{x^2 - y^2}$$

5) Efectúa las siguientes sumas de radicales:

$$a) \sqrt{18} - \sqrt{8} - \sqrt{2} = \sqrt{9 \cdot 2} - \sqrt{4 \cdot 2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} - \sqrt{2} = (3 - 2 - 1)\sqrt{2} = 0$$

$$b) \sqrt{27} + \sqrt{12} + \sqrt{3} = \sqrt{9 \cdot 3} + \sqrt{4 \cdot 3} + \sqrt{3} = 3\sqrt{3} + 2\sqrt{3} + \sqrt{3} = 6\sqrt{3}$$

$$c) \sqrt{18} + \sqrt{32} - \sqrt{48} - \sqrt{12} = \sqrt{9 \cdot 2} + \sqrt{16 \cdot 2} - \sqrt{16 \cdot 3} - \sqrt{4 \cdot 3} = 3\sqrt{2} + 4\sqrt{2} - 4\sqrt{3} - 2\sqrt{3} =$$

$$= (3 + 4)\sqrt{2} - (4 + 2)\sqrt{3} = 7\sqrt{2} - 6\sqrt{3}$$

$$d) \sqrt[3]{24} + \sqrt[4]{32} - \sqrt[3]{81} + \sqrt[4]{162} = \sqrt[3]{8 \cdot 3} + \sqrt[4]{16 \cdot 2} - \sqrt[3]{27 \cdot 3} + \sqrt[4]{81 \cdot 2} = 2\sqrt[3]{3} + 2\sqrt[4]{2} - 3\sqrt[3]{3} + 3\sqrt[4]{2} =$$

$$= (2 - 3)\sqrt[3]{3} + (2 + 3)\sqrt[4]{2} = -\sqrt[3]{3} + 5\sqrt[4]{2}$$

6) Efectúa las siguientes sumas de radicales. Expresa el resultado simplificado y racionalizado.

$$\begin{aligned} a) \frac{1}{2}\sqrt{12} - \frac{2}{9}\sqrt{75} + \frac{1}{6}\sqrt{108} - \frac{2}{5}\sqrt{300} &= \frac{1}{2}\sqrt{4 \cdot 3} - \frac{2}{9}\sqrt{25 \cdot 3} + \frac{1}{6}\sqrt{36 \cdot 3} - \frac{2}{5}\sqrt{100 \cdot 3} = \\ &= \frac{2}{2}\sqrt{3} - \frac{2 \cdot 5}{9}\sqrt{3} + \frac{6}{6}\sqrt{3} - \frac{2 \cdot 10}{5}\sqrt{3} = \sqrt{3} - \frac{10}{9}\sqrt{3} + \sqrt{3} - 4\sqrt{3} = -2\sqrt{3} - \frac{10}{9}\sqrt{3} = \frac{-18-10}{9}\sqrt{3} = -\frac{28}{9}\sqrt{3} \end{aligned}$$

$$\begin{aligned} b) \sqrt{18a} - \frac{4}{a}\sqrt{\frac{a^3}{32}} + \sqrt{\frac{a}{8}} - 2\sqrt{\frac{a}{72}} &= \sqrt{9 \cdot 2a} - \frac{4\cancel{a}}{\cancel{a}}\sqrt{\frac{a}{16 \cdot 2}} + \sqrt{\frac{a}{4 \cdot 2}} - 2\sqrt{\frac{a}{36 \cdot 2}} = \\ &= 3\sqrt{2a} - \frac{4}{4}\sqrt{\frac{a}{2}} + \frac{1}{2}\sqrt{\frac{a}{2}} - \frac{2}{6}\sqrt{\frac{a}{2}} = 3\sqrt{2a} - \sqrt{\frac{a \cdot 2}{2 \cdot 2}} + \frac{1}{2}\sqrt{\frac{a \cdot 2}{2 \cdot 2}} - \frac{1}{3}\sqrt{\frac{a \cdot 2}{2 \cdot 2}} = \\ &= 3\sqrt{2a} - \frac{1}{2}\sqrt{2a} + \frac{1}{4}\sqrt{2a} - \frac{1}{6}\sqrt{2a} = \left(3 - \frac{1}{2} + \frac{1}{4} - \frac{1}{6}\right)\sqrt{2a} = \frac{36-6+3-2}{12}\sqrt{2a} = \frac{31}{12}\sqrt{2a} \end{aligned}$$

$$\begin{aligned} c) \frac{1}{2}\sqrt{\frac{4}{125}} - \frac{1}{5}\sqrt{500} + \sqrt{1 - \frac{16}{25}} - \frac{4}{3}\sqrt{45} + \frac{2}{7}\sqrt{147} &= \\ &= \frac{2}{2}\sqrt{\frac{1}{25 \cdot 5}} - \frac{1}{5}\sqrt{100 \cdot 5} + \sqrt{\frac{25-16}{25}} - \frac{4}{3}\sqrt{9 \cdot 5} + \frac{2}{7}\sqrt{49 \cdot 3} = \frac{1}{5}\sqrt{\frac{1}{5}} - \frac{10}{5}\sqrt{5} + \sqrt{\frac{9}{25}} - \frac{4 \cdot 3}{3}\sqrt{5} + \frac{2 \cdot 7}{7}\sqrt{3} = \\ &= \frac{1}{5}\sqrt{\frac{5}{25}} - 2\sqrt{5} + \frac{3}{5} - 4\sqrt{5} + 2\sqrt{3} = \frac{1}{25}\sqrt{5} - 6\sqrt{5} + \frac{3}{5} + 2\sqrt{3} = \frac{1-150}{25}\sqrt{5} + \frac{3}{5} + 2\sqrt{3} = \\ &= -\frac{149}{25}\sqrt{5} + \frac{3}{5} + 2\sqrt{3} \end{aligned}$$

$$\begin{aligned} d) \frac{1}{b}\sqrt{\frac{ab}{3}} - a\sqrt{\frac{48}{ab}} + \sqrt{\frac{27a}{b}} - \frac{a}{b}\sqrt{\frac{b}{12a}} &= \frac{1}{b}\sqrt{\frac{3ab}{3^2}} - a\sqrt{\frac{16 \cdot 3ab}{(ab)^2}} + \sqrt{\frac{9 \cdot 3ab}{b^2}} - \frac{a}{b}\sqrt{\frac{b \cdot 3a}{4 \cdot 3a \cdot 3a}} = \\ &= \frac{1}{3b}\sqrt{3ab} - \frac{4a}{ab}\sqrt{3ab} + \frac{3}{b}\sqrt{3ab} - \frac{a}{2b \cdot 3a}\sqrt{3a} = \left(\frac{1}{3b} - \frac{4}{b} + \frac{3}{b} - \frac{1}{6b}\right)\sqrt{3a} = \frac{2-24+18-1}{6b}\sqrt{3a} = \\ &= -\frac{5}{6b}\sqrt{3a} \end{aligned}$$

$$\begin{aligned} e) 3\sqrt{27x} - \frac{5}{2}\sqrt{\frac{x}{3}} + 5\sqrt{\frac{3x}{81}} - \frac{1}{4}\sqrt{48x} &= 3\sqrt{9 \cdot 3x} - \frac{5}{2}\sqrt{\frac{3x}{3^2}} + \frac{5}{9}\sqrt{3x} - \frac{1}{4}\sqrt{16 \cdot 3x} = \\ &= 9\sqrt{3x} - \frac{5}{6}\sqrt{3x} + \frac{5}{9}\sqrt{3x} - \frac{4}{4}\sqrt{3x} = \left(8 - \frac{5}{6} + \frac{5}{9}\right)\sqrt{3x} = \frac{144-15+10}{18}\sqrt{3x} = \frac{139}{18}\sqrt{3x} \end{aligned}$$

$$\begin{aligned} f) 3\sqrt{24x} - \sqrt{\frac{x}{6}} + 5\sqrt{\frac{6x}{100}} - \sqrt{54x} &= 3\sqrt{4 \cdot 6x} - \sqrt{\frac{6x}{6^2}} + \frac{5}{10}\sqrt{6x} - \sqrt{9 \cdot 6x} = \\ &= 6\sqrt{6x} - \frac{1}{6}\sqrt{6x} + \frac{1}{2}\sqrt{6x} - 3\sqrt{6x} = \left(3 - \frac{1}{6} + \frac{1}{2}\right)\sqrt{6x} = \frac{18-1+3}{6}\sqrt{6x} = \frac{20}{6}\sqrt{6x} = \frac{10}{3}\sqrt{6x} \end{aligned}$$

$$\begin{aligned} g) \sqrt{\frac{3a}{2}} + \sqrt{\frac{2a}{3}} - \sqrt{6a} + \sqrt{\frac{a}{6}} &= \sqrt{\frac{3a \cdot 2}{2^2}} + \sqrt{\frac{2a \cdot 3}{3^2}} - \sqrt{6a} + \sqrt{\frac{6a}{6^2}} = \frac{1}{2}\sqrt{6a} + \frac{1}{3}\sqrt{6a} - \sqrt{6a} + \frac{1}{6}\sqrt{6a} = \\ &= \left(\frac{1}{2} + \frac{1}{3} - 1 + \frac{1}{6}\right)\sqrt{6a} = \frac{3+2-6+1}{6}\sqrt{6a} = \frac{0}{6}\sqrt{6a} = 0 \end{aligned}$$

$$\begin{aligned}
 h) \frac{2}{3}\sqrt{24} - \sqrt{\frac{432}{2}} - \frac{2}{5}\sqrt{50} + \frac{12}{\sqrt{6}} &= \frac{2}{3}\sqrt{4 \cdot 6} - \sqrt{216} - \frac{2}{5}\sqrt{25 \cdot 2} + \frac{12\sqrt{6}}{\sqrt{6 \cdot \sqrt{6}}} = \\
 &= \frac{4}{3}\sqrt{6} - \sqrt{36 \cdot 6} - \frac{2 \cdot 5}{5}\sqrt{2} + \frac{12}{6}\sqrt{6} = \frac{4}{3}\sqrt{6} - 6\sqrt{6} - 2\sqrt{2} + 2\sqrt{6} = \left(\frac{4}{3} - 4\right)\sqrt{6} - 2\sqrt{2} = \\
 &= \frac{4-12}{3}\sqrt{6} - 2\sqrt{2} = -\frac{8}{3}\sqrt{6} - 2\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 i) 2\sqrt{80} + \frac{14}{5}\sqrt{1+\frac{1}{49}} - \sqrt{8} - \frac{9}{4}\sqrt{1-\frac{1}{81}} &= 2\sqrt{16 \cdot 5} + \frac{14}{5}\sqrt{\frac{49+1}{49}} - \sqrt{4 \cdot 2} - \frac{9}{4}\sqrt{\frac{81-1}{81}} = \\
 &= 8\sqrt{5} + \frac{14}{5 \cdot 7}\sqrt{50} - 2\sqrt{2} - \frac{9}{4 \cdot 9}\sqrt{80} = 8\sqrt{5} + \frac{2}{5}\sqrt{25 \cdot 2} - 2\sqrt{2} - \frac{1}{4}\sqrt{16 \cdot 5} = \\
 &= 8\sqrt{5} + \frac{2 \cdot 5}{5}\sqrt{2} - 2\sqrt{2} - \frac{4}{4}\sqrt{5} = 7\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 j) \sqrt{4a-8b} - \sqrt{9a-18b} + 2\sqrt{16a-32b} &= \sqrt{4(a-2b)} - \sqrt{9(a-2b)} + 2\sqrt{16(a-2b)} = \\
 &= 2\sqrt{a-2b} - 3\sqrt{a-2b} + 8\sqrt{a-2b} = 7\sqrt{a-2b}
 \end{aligned}$$

$$\begin{aligned}
 k) \sqrt{\frac{2a}{b}} - \sqrt{\frac{2b}{a}} + \sqrt{\frac{a}{2b}} + \sqrt{\frac{2}{ab}} - \sqrt{\frac{ab}{2}} &= \sqrt{\frac{2a \cdot b}{b^2}} - \sqrt{\frac{2b \cdot a}{a^2}} + \sqrt{\frac{a \cdot 2b}{(2b)^2}} + \sqrt{\frac{2ab}{(ab)^2}} - \sqrt{\frac{2ab}{2^2}} = \\
 &= \frac{1}{b}\sqrt{2ab} - \frac{1}{a}\sqrt{2ab} + \frac{1}{2b}\sqrt{2ab} + \frac{1}{ab}\sqrt{2ab} - \frac{1}{2}\sqrt{2ab} = \left(\frac{1}{b} - \frac{1}{a} + \frac{1}{2b} + \frac{1}{ab} - \frac{1}{2}\right)\sqrt{2ab} = \\
 &= \frac{2a - 2b + a + 2 - ab}{2ab}\sqrt{2ab} = \frac{3a - 2b + a + 2 - ab}{2ab}\sqrt{2ab}
 \end{aligned}$$

$$\begin{aligned}
 l) \frac{cd}{a}\sqrt{\frac{a^6}{cd}} - \frac{b^2d}{a}\sqrt{\frac{4a^4c}{b^2d}} + \frac{d^2}{c}\sqrt{\frac{b^4c^3}{d^3}} &= \frac{cd \cdot a^3}{a}\sqrt{\frac{1}{cd}} - \frac{b^2d \cdot 2a^2}{ab}\sqrt{\frac{c}{d}} + \frac{d^2b^2c}{cd}\sqrt{\frac{c}{d}} = \\
 &= a^2cd\sqrt{\frac{cd}{(cd)^2}} - 2abd\sqrt{\frac{cd}{d^2}} + b^2d\sqrt{\frac{cd}{d^2}} = a^2\sqrt{cd} - 2ab\sqrt{cd} + b^2\sqrt{cd} = (a^2 - 2ab + b^2)\sqrt{cd} = \\
 &= (a-b)^2\sqrt{cd}
 \end{aligned}$$

7) Racionaliza y simplifica

$$a) \frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$$

$$b) \frac{2\sqrt{6}}{\sqrt{2}} = \frac{2\sqrt{6} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{2\sqrt{12}}{2} = \sqrt{4 \cdot 3} = 2\sqrt{3} \quad ; \quad b) \frac{2\sqrt{6}}{\sqrt{2}} = 2\sqrt{\frac{6}{2}} = 2\sqrt{3}$$

$$c) \sqrt{\frac{2}{3}} = \sqrt{\frac{2 \cdot 3}{3^2}} = \frac{1}{3}\sqrt{6} \quad ; \quad c) \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$d) \sqrt{\frac{5}{2}} = \sqrt{\frac{5 \cdot 2}{2^2}} = \frac{1}{2}\sqrt{10}$$

$$e) \sqrt{\frac{a}{b}} = \sqrt{\frac{ab}{b^2}} = \frac{1}{b} \sqrt{ab}$$

$$f) \sqrt[3]{\frac{3}{2}} = \sqrt[3]{\frac{3 \cdot 2^2}{2 \cdot 2^2}} = \sqrt[3]{\frac{12}{2^3}} = \frac{1}{2} \sqrt[3]{12}$$

$$g) \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{3\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3}}{3 \cdot 3} = \frac{2\sqrt{3}}{9}$$

$$h) \frac{3y}{x\sqrt{y}} = \frac{3y\sqrt{y}}{x\sqrt{y} \cdot \sqrt{y}} = \frac{3y\sqrt{y}}{xy} = \frac{3\sqrt{y}}{x}$$

$$i) \frac{x\sqrt{y}}{y\sqrt{x}} = \frac{x\sqrt{y}}{y\sqrt{x} \cdot \sqrt{x}} = \frac{x\sqrt{y}}{yx} = \frac{\sqrt{y}}{y}$$

$$j) \frac{2\sqrt{12}}{5\sqrt{3}} = \frac{2}{5} \sqrt{\frac{12}{3}} = \frac{2}{5} \sqrt{4} = \frac{4}{5}$$

$$k) \frac{3}{\sqrt{2-x}} = \frac{3\sqrt{2-x}}{\sqrt{2-x} \cdot \sqrt{2-x}} = \frac{3\sqrt{2-x}}{2-x}$$

$$l) \frac{\sqrt{2-x}}{\sqrt{2+x}} = \frac{\sqrt{2-x} \cdot \sqrt{2+x}}{\sqrt{2+x} \cdot \sqrt{2+x}} = \frac{\sqrt{(2-x) \cdot (2+x)}}{\sqrt{(2+x)^2}} = \frac{\sqrt{2^2 - x^2}}{2+x} = \frac{\sqrt{4-x^2}}{2+x}$$

$$m) \frac{3xy^2}{\sqrt[3]{x^2y}} = \frac{3xy^2 \cdot \sqrt[3]{xy^2}}{\sqrt[3]{x^2y} \cdot \sqrt[3]{xy^2}} = \frac{3xy^2 \cdot \sqrt[3]{xy^2}}{\sqrt[3]{x^3y^3}} = \frac{3xy^2 \cdot \sqrt[3]{xy^2}}{xy} = 3y\sqrt[3]{xy^2}$$

$$n) \frac{a}{\sqrt{a}} = \frac{a\sqrt{a}}{\sqrt{a} \cdot \sqrt{a}} = \frac{a\sqrt{a}}{a} = \sqrt{a}$$

$$\tilde{n}) \frac{2+x}{\sqrt{2+x}} = \frac{(2+x) \cdot \sqrt{2+x}}{\sqrt{2+x} \cdot \sqrt{2+x}} = \frac{(2+x) \cdot \sqrt{2+x}}{2+x} = \sqrt{2+x}$$

$$o) \frac{\sqrt{b}}{\sqrt{ab}} = \frac{\sqrt{b} \cdot \sqrt{ab}}{\sqrt{ab} \cdot \sqrt{ab}} = \frac{\sqrt{ab^2}}{ab} = \frac{b\sqrt{a}}{ab} = \frac{\sqrt{a}}{a} \quad ; \quad o) \frac{\sqrt{b}}{\sqrt{ab}} = \sqrt{\frac{b}{ab}} = \frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{\sqrt{a} \cdot \sqrt{a}} = \frac{\sqrt{a}}{a}$$

$$p) \frac{\sqrt{x} - \sqrt{y}}{\sqrt{x}} = \frac{(\sqrt{x} - \sqrt{y})\sqrt{x}}{\sqrt{x} \cdot \sqrt{x}} = \frac{x - \sqrt{xy}}{x}$$

$$q) \frac{x}{1-\sqrt{x}} = \frac{x(1+\sqrt{x})}{(1-\sqrt{x})(1+\sqrt{x})} = \frac{x+x\sqrt{x}}{1^2 - (\sqrt{x})^2} = \frac{x+x\sqrt{x}}{1-x}$$

8) Racionaliza y simplifica

$$a) \frac{6(3-y)}{\sqrt[3]{(3-y)^2}} = \frac{6(3-y)\sqrt[3]{3-y}}{\sqrt[3]{(3-y)^2} \cdot \sqrt[3]{3-y}} = \frac{6(3-y)\sqrt[3]{3-y}}{\sqrt[3]{(3-y)^3}} = \frac{6(3-y)\sqrt[3]{3-y}}{3-y} = 6\sqrt[3]{3-y}$$

$$b) \frac{5}{\sqrt[3]{a^2-1}} = \frac{5\sqrt[3]{(a^2-1)^2}}{\sqrt[3]{a^2-1} \cdot \sqrt[3]{(a^2-1)^2}} = \frac{5\sqrt[3]{(a^2-1)^2}}{\sqrt[3]{(a^2-1)^3}} = \frac{5\sqrt[3]{(a^2-1)^2}}{a^2-1}$$

$$c) \frac{1}{\sqrt[3]{a^2b^2}} = \frac{\sqrt[3]{ab}}{\sqrt[3]{a^2b^2} \cdot \sqrt[3]{ab}} = \frac{\sqrt[3]{ab}}{ab}$$

$$d) \frac{1}{\sqrt[3]{a^2+b^2}} = \frac{\sqrt[3]{(a^2+b^2)^2}}{\sqrt[3]{a^2+b^2} \cdot \sqrt[3]{(a^2+b^2)^2}} = \frac{\sqrt[3]{(a^2+b^2)^2}}{\sqrt[3]{(a^2+b^2)^3}} = \frac{\sqrt[3]{(a^2+b^2)^2}}{a^2+b^2}$$

$$e) \frac{1}{\sqrt[4]{a^2-b^3+x}} = \frac{\sqrt[4]{(a^2-b^3+x)^3}}{\sqrt[4]{a^2-b^3+x} \cdot \sqrt[4]{(a^2-b^3+x)^3}} = \frac{\sqrt[4]{(a^2-b^3+x)^3}}{\sqrt[4]{(a^2-b^3+x)^4}} = \frac{\sqrt[4]{(a^2-b^3+x)^3}}{a^2-b^3+x}$$

$$f) \frac{x^2p^3}{\sqrt[6]{z^2x^4p}} = \frac{x^2p^3\sqrt[6]{z^4x^2p^5}}{\sqrt[6]{z^2x^4p} \cdot \sqrt[6]{z^4x^2p^5}} = \frac{x^2p^3\sqrt[6]{z^4x^2p^5}}{\sqrt[6]{z^6x^6p^6}} = \frac{x^2p^3\sqrt[6]{z^4x^2p^5}}{zxp} = \frac{xp^2\sqrt[6]{z^4x^2p^5}}{z}$$

$$g) \frac{\sqrt{a}-a}{a\sqrt{a}} = \frac{(\sqrt{a}-a)\sqrt{a}}{a\sqrt{a}\sqrt{a}} = \frac{a-a\sqrt{a}}{a^2} = \frac{a(1-\sqrt{a})}{a^2} = \frac{1-\sqrt{a}}{a}$$

$$h) \frac{\sqrt{a}}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a}(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{a-\sqrt{ab}}{(\sqrt{a})^2-(\sqrt{b})^2} = \frac{a-\sqrt{ab}}{a-b}$$

$$i) \frac{a^2-b^2}{\sqrt[3]{a+b}} = \frac{(a^2-b^2) \cdot \sqrt[3]{(a+b)^2}}{\sqrt[3]{a+b} \cdot \sqrt[3]{(a+b)^2}} = \frac{(a^2-b^2) \cdot \sqrt[3]{(a+b)^2}}{\sqrt[3]{(a+b)^3}} = \frac{(a+b)(a-b)\sqrt[3]{(a+b)^2}}{a+b} = (a-b)\sqrt[3]{(a+b)^2}$$

$$j) \frac{\sqrt{p-h}}{\sqrt{p+h}} = \frac{\sqrt{p-h} \cdot \sqrt{p+h}}{\sqrt{p+h} \cdot \sqrt{p+h}} = \frac{\sqrt{p^2-h^2}}{\sqrt{(p+h)^2}} = \frac{\sqrt{p^2-h^2}}{p+h}$$

$$k) \frac{\sqrt{p}-\sqrt{h}}{\sqrt{p}+\sqrt{h}} = \frac{(\sqrt{p}-\sqrt{h}) \cdot (\sqrt{p}-\sqrt{h})}{(\sqrt{p}+\sqrt{h}) \cdot (\sqrt{p}-\sqrt{h})} = \frac{(\sqrt{p}-\sqrt{h})^2}{(\sqrt{p})^2-(\sqrt{h})^2} = \frac{(\sqrt{p})^2-2\sqrt{ph}+(\sqrt{h})^2}{p-h} = \frac{p-2\sqrt{ph}+h}{p-h}$$

$$l) \frac{b\sqrt{a}+a\sqrt{b}}{\sqrt{ab}} = \frac{(b\sqrt{a}+a\sqrt{b})\sqrt{ab}}{\sqrt{ab} \cdot \sqrt{ab}} = \frac{b\sqrt{a^2b}+a\sqrt{ab^2}}{ab} = \frac{ba\sqrt{b}+ab\sqrt{a}}{ab} = \frac{ab(\sqrt{b}+\sqrt{a})}{ab} = \sqrt{b}+\sqrt{a}$$

$$\begin{aligned}
 m) \frac{b\sqrt{a} + a\sqrt{b}}{a\sqrt{b} - b\sqrt{a}} &= \frac{(b\sqrt{a} + a\sqrt{b})(a\sqrt{b} + b\sqrt{a})}{(a\sqrt{b} - b\sqrt{a})(a\sqrt{b} + b\sqrt{a})} = \frac{ba\sqrt{ab} + b^2 \cdot a + a^2 \cdot b + ab\sqrt{ba}}{(a\sqrt{b})^2 - (b\sqrt{a})^2} = \\
 &= \frac{2ab\sqrt{ab} + b^2 \cdot a + a^2 \cdot b}{a^2b - b^2a} = \frac{ab(2\sqrt{ab} + b + a)}{ab(a - b)} = \frac{2\sqrt{ab} + b + a}{a - b}
 \end{aligned}$$

9) Racionaliza y simplifica

$$a) \frac{\sqrt{2}}{\sqrt{2} + \sqrt{3}} = \frac{\sqrt{2}(\sqrt{2} - \sqrt{3})}{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})} = \frac{2 - \sqrt{6}}{(\sqrt{2})^2 - (\sqrt{3})^2} = \frac{2 - \sqrt{6}}{2 - 3} = -2 + \sqrt{6}$$

$$b) \frac{\sqrt{6}}{2\sqrt{2} - \sqrt{6}} = \frac{\sqrt{6}(2\sqrt{2} + \sqrt{6})}{(2\sqrt{2} - \sqrt{6})(2\sqrt{2} + \sqrt{6})} = \frac{2\sqrt{12} + 6}{(2\sqrt{2})^2 - (\sqrt{6})^2} = \frac{2\sqrt{4 \cdot 3} + 6}{4 \cdot 2 - 6} = \frac{4\sqrt{3} + 6}{2} = 2\sqrt{3} + 3$$

$$c) \frac{1 - \sqrt{5}}{\sqrt{5} - \sqrt{2}} = \frac{(1 - \sqrt{5})(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})} = \frac{\sqrt{5} + \sqrt{2} - 5 - \sqrt{10}}{(\sqrt{5})^2 - (\sqrt{2})^2} = \frac{\sqrt{5} + \sqrt{2} - 5 - \sqrt{10}}{5 - 2} = \frac{\sqrt{5} + \sqrt{2} - 5 - \sqrt{10}}{3}$$

$$\begin{aligned}
 d) \frac{\sqrt{3} - \sqrt{6}}{2\sqrt{3} - 1} &= \frac{(\sqrt{3} - \sqrt{6})(2\sqrt{3} + 1)}{(2\sqrt{3} - 1)(2\sqrt{3} + 1)} = \frac{2 \cdot 3 + \sqrt{3} - 2\sqrt{18} - \sqrt{6}}{(2\sqrt{3})^2 - 1^2} = \frac{6 + \sqrt{3} - 6\sqrt{2} - \sqrt{6}}{4 \cdot 3 - 1} = \\
 &= \frac{6 + \sqrt{3} - 6\sqrt{2} - \sqrt{6}}{11}
 \end{aligned}$$

$$\begin{aligned}
 e) \frac{\sqrt{10} - \sqrt{2}}{\sqrt{5} - 2\sqrt{2}} &= \frac{(\sqrt{10} - \sqrt{2})(\sqrt{5} + 2\sqrt{2})}{(\sqrt{5} - 2\sqrt{2})(\sqrt{5} + 2\sqrt{2})} = \frac{\sqrt{50} + 2\sqrt{20} - \sqrt{10} - 2 \cdot 2}{(\sqrt{5})^2 - (2\sqrt{2})^2} = \frac{\sqrt{25 \cdot 2} + 2\sqrt{4 \cdot 5} - \sqrt{10} - 4}{5 - 4 \cdot 2} = \\
 &= \frac{5\sqrt{2} + 4\sqrt{5} - \sqrt{10} - 4}{-3}
 \end{aligned}$$

$$f) \frac{2\sqrt{3} - 3\sqrt{2}}{2\sqrt{2} + 3\sqrt{3}} = \frac{(2\sqrt{3} - 3\sqrt{2})(2\sqrt{2} - 3\sqrt{3})}{(2\sqrt{2} + 3\sqrt{3})(2\sqrt{2} - 3\sqrt{3})} = \frac{4\sqrt{6} - 6 \cdot 3 - 6 \cdot 2 + 9\sqrt{6}}{(2\sqrt{2})^2 - (3\sqrt{3})^2} = \frac{13\sqrt{6} - 30}{4 \cdot 2 - 9 \cdot 3} = \frac{13\sqrt{6} - 30}{-19}$$