

1) Cálculo y reducción

Introduce los factores dentro del radical:

$$a) 2\sqrt[4]{3} \qquad 2\sqrt[4]{3} = \sqrt[4]{2^4 \cdot 3} = \sqrt[4]{16 \cdot 3} = \sqrt[4]{48}$$

$$b) x^2\sqrt[7]{x^3} \qquad x^2\sqrt[7]{x^3} = \sqrt[7]{(x^2)^7 \cdot x^3} = \sqrt[7]{x^{14} \cdot x^3} = \sqrt[7]{x^{17}}$$

Extrae los factores del radical:

$$a) \sqrt[4]{128} \qquad \sqrt[4]{128} = \sqrt[4]{2^7} = 2\sqrt[4]{2^3} = 2\sqrt[4]{8}$$

$$b) \sqrt[7]{x^{30}} \qquad \sqrt[7]{x^{30}} = \sqrt[7]{x^{28+2}} = \sqrt[7]{x^{28} \cdot x^2} = x^4\sqrt[7]{x^2}$$

Calcular las siguientes raíces:

$$a) \sqrt[5]{1024} \qquad \sqrt[5]{1024} = \sqrt[5]{2^{10}} = 2^2 = 4$$

$$b) \sqrt[7]{x^{84}} \qquad \sqrt[7]{x^{84}} = \sqrt[7]{x^{12 \cdot 7}} = \sqrt[7]{(x^{12})^7} = x^{12}$$

Reduce a índice común

$$a) \sqrt{3}; \sqrt[3]{5} \qquad \sqrt{2} = \sqrt[6]{2^3} = \sqrt[6]{8} \quad ; \quad \sqrt[3]{5} = \sqrt[6]{5^2} = \sqrt[6]{25}$$

$$b) \sqrt[4]{x^3}; \sqrt[6]{x^5} \qquad \sqrt[4]{x^3} = \sqrt[12]{x^9} \quad ; \quad \sqrt[6]{x^5} = \sqrt[12]{x^{10}}$$

2) Raíz de un cociente, raíz de un producto, raíz de una potencia, raíz de una raíz, simplificación y racionalización.

Escribe con una sola raíz:

$$a) \sqrt[5]{\sqrt{3}} \qquad \sqrt[5]{\sqrt{3}} = \sqrt[10]{3}$$

$$b) \sqrt[7]{x^4 \sqrt{x}} \qquad \sqrt[7]{x^4 \sqrt{x}} = \sqrt[7]{x^4 \cdot x^{1/2}} = \sqrt[14]{x^9}$$

Escribe con una sola raíz:

$$a) \sqrt[4]{3 \cdot \sqrt[4]{27}} \qquad \sqrt[4]{3 \cdot \sqrt[4]{27}} = \sqrt[4]{3 \cdot 3} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$$

$$b) \sqrt[5]{x \cdot \sqrt[5]{x^2}} \qquad \sqrt[5]{x \cdot \sqrt[5]{x^2}} = \sqrt[5]{x^3}$$

Escribe con una sola raíz:

$$a) \frac{\sqrt[3]{16}}{\sqrt[3]{2}} \qquad \frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \sqrt[3]{\frac{16}{2}} = \sqrt[3]{8} = 2$$

$$b) \frac{\sqrt[5]{x^4}}{\sqrt[5]{x^3}} \qquad \frac{\sqrt[5]{x^4}}{\sqrt[5]{x^3}} = \sqrt[5]{\frac{x^4}{x^3}} = \sqrt[5]{x}$$

Racionaliza.

$$a) \frac{1}{\sqrt[5]{9}} \qquad \frac{1}{\sqrt[5]{9}} = \frac{1}{\sqrt[5]{3^2}} = \frac{1 \cdot \sqrt[5]{3^3}}{\sqrt[5]{3^2} \cdot \sqrt[5]{3^3}} = \frac{\sqrt[5]{3^2}}{\sqrt[5]{3^5}} = \frac{\sqrt[5]{9}}{3}$$

$$b) \frac{2}{5\sqrt[3]{4}} \qquad \frac{2}{5\sqrt[3]{4}} = \frac{2}{5\sqrt[3]{2^2}} = \frac{2 \cdot \sqrt[3]{2}}{5\sqrt[3]{2^2} \cdot \sqrt[3]{2}} = \frac{2 \cdot \sqrt[3]{2}}{5\sqrt[3]{2^3}} = \frac{2 \cdot \sqrt[3]{2}}{5 \cdot 2} = \frac{\sqrt[3]{2}}{5}$$

Racionaliza:

$$a) \frac{1}{\sqrt[3]{x^4}} = \frac{1 \cdot \sqrt[3]{x^3}}{\sqrt[3]{x^4} \cdot \sqrt[3]{x^3}} = \frac{\sqrt[3]{x^3}}{\sqrt[3]{x^7}} = \frac{\sqrt[3]{x^3}}{x}$$

$$b) \frac{1}{x^2 \sqrt[3]{x^3}} = \frac{1 \cdot \sqrt[3]{x^4}}{x^2 \sqrt[3]{x^3} \cdot \sqrt[3]{x^4}} = \frac{\sqrt[3]{x^4}}{x^2 \sqrt[3]{x^7}} = \frac{\sqrt[3]{x^4}}{x^2 \cdot x} = \frac{\sqrt[3]{x^4}}{x^3}$$

Racionaliza:

$$a) \frac{1}{\sqrt{3} - \sqrt{2}} = \frac{1(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})} = \frac{(\sqrt{3} + \sqrt{2})}{3 - 2} = (\sqrt{3} + \sqrt{2})$$

$$b) \frac{\sqrt{2}}{\sqrt{5} + 2} = \frac{\sqrt{2}(\sqrt{5} - 2)}{(\sqrt{5} + 2)(\sqrt{5} - 2)} = \frac{\sqrt{10} - 2\sqrt{2}}{5 - 4} = \sqrt{10} - 2\sqrt{2}$$

$$c) \frac{1}{3 - \sqrt{x}} = \frac{1(3 + \sqrt{x})}{(3 - \sqrt{x})(3 + \sqrt{x})} = \frac{3 + \sqrt{x}}{9 - x}$$

3) Operaciones

Calcular la suma:

$$a) \sqrt{40} + \sqrt{90} = \sqrt{4 \cdot 10} + \sqrt{9 \cdot 10} = 2\sqrt{10} + 3\sqrt{10} = 5\sqrt{10}$$

$$b) 2\sqrt{32} - \sqrt{8} = 2\sqrt{2^5} - \sqrt{2^3} = 2 \cdot 2^2\sqrt{2} - 2\sqrt{2} = 8\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$$

$$c) \sqrt[3]{4} + \sqrt[5]{16} = \sqrt[3]{4} + \sqrt[5]{4^2} = \sqrt[3]{4} + \sqrt[3]{4} = 2\sqrt[3]{4}$$

$$d) 2\sqrt{\frac{1}{2}} + 5\sqrt{8} = \sqrt{\frac{4 \cdot 1}{2}} + 5\sqrt{2^3} = \sqrt{2} + 10\sqrt{2} = 12\sqrt{2}$$

Calcular y simplificar:

$$a) \sqrt[3]{3} \cdot \sqrt[4]{27} = \sqrt[3]{3} \cdot \sqrt[4]{3^3} = \sqrt[3]{3} \cdot \sqrt[3]{3} = 3$$

$$b) \sqrt[3]{x} \cdot \sqrt[5]{x^2} = \sqrt[15]{x^3}$$

$$c) \sqrt[5]{x^3} \sqrt{x} \cdot \sqrt{x} = \sqrt[5]{x^3 \cdot x \cdot x} = \sqrt[5]{x^5} = x$$

$$d) \sqrt[3]{2} \cdot \sqrt{2} \cdot \sqrt[4]{8} = \sqrt[3]{2} \cdot \sqrt{2} \cdot \sqrt[4]{2^3} = \sqrt[12]{2^4 \cdot 2^6 \cdot 2^9} = \sqrt[12]{2^{19}} = 2^{\frac{19}{12}}$$

Calcular y simplificar:

$$a) \frac{\sqrt[3]{16}}{\sqrt[5]{2}} = \frac{\sqrt[3]{2^4}}{\sqrt[5]{2}} = \frac{\sqrt[15]{2^{20}}}{\sqrt[15]{2^3}} = \sqrt[15]{2^{17}} = 2^{\frac{17}{15}}$$

$$b) \frac{\sqrt[7]{x^4}}{\sqrt[14]{x^3}} = \frac{\sqrt[14]{x^8}}{\sqrt[14]{x^3}} = \sqrt[14]{x^5}$$

$$a) \frac{\sqrt[6]{8^4}}{\sqrt[8]{4^3}} = \frac{\sqrt[6]{(2^3)^4}}{\sqrt[8]{(2^2)^3}} = \frac{\sqrt[6]{2^{12}}}{\sqrt[8]{2^6}} = \frac{\sqrt[24]{(2^{12})^4}}{\sqrt[24]{(2^6)^3}} = \frac{\sqrt[24]{2^{48}}}{\sqrt[24]{2^{18}}} = \sqrt[24]{2^{30}} = \sqrt[4]{2^5} = 2\sqrt[4]{2}$$

$$b) \frac{\sqrt[3]{x^4} \sqrt{x}}{\sqrt[4]{x}} = \frac{\sqrt[3]{x^4 \cdot x}}{\sqrt[4]{x}} = \frac{\sqrt[3]{x^5}}{\sqrt[4]{x}} = \frac{\sqrt[12]{x^{20}}}{\sqrt[12]{x^3}} = \sqrt[12]{x^{17}} = x^{\frac{17}{12}}$$

$$5. \quad (\sqrt{5} + 2) \cdot (\sqrt{5} - 2) =$$

$$= (\sqrt{5})^2 - 2^2 = 5 - 4 = 1$$

$$6. \quad (2\sqrt{5} + 3\sqrt{2}) \cdot (2\sqrt{5} - 3\sqrt{2}) =$$

$$= (2\sqrt{5})^2 - (3\sqrt{2})^2 =$$

$$= 2^2 \cdot (\sqrt{5})^2 - 3^2 (\sqrt{2})^2 =$$

$$= 4 \cdot 5 - 9 \cdot 2 = 20 - 18 = 2$$

$$7. \quad \frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}} =$$

$$= \frac{\sqrt{2} \cdot (\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{3} + \sqrt{2})} =$$

$$= \frac{\sqrt{6} + \sqrt{2^2}}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{2 + \sqrt{6}}{3 - 2} = 2 + \sqrt{6}$$

$$8. \quad \frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} =$$

$$= \frac{(3\sqrt{2} - 2\sqrt{3}) \cdot (3\sqrt{2} - 2\sqrt{3})}{(3\sqrt{2} + 2\sqrt{3}) \cdot (3\sqrt{2} - 2\sqrt{3})} =$$

$$= \frac{(3\sqrt{2} - 2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2} =$$

$$= \frac{(3\sqrt{2})^2 - 2 \cdot 3 \cdot \sqrt{2} \cdot 2\sqrt{3} + (2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2} =$$

$$= \frac{9 \cdot 2 - 12\sqrt{6} + 4 \cdot 3}{9 \cdot 2 - 4 \cdot 3} =$$

$$= \frac{18 - 12\sqrt{6} + 12}{18 - 12} =$$

$$= \frac{30 - 12\sqrt{6}}{6} = 5 - 2\sqrt{6}$$