

ECUACIONES TRIGONOMÉTRICAS

1. Resolver las siguientes ecuaciones:

- a) $\operatorname{sen} x = \operatorname{tg} x$
- b) $6 \cos^2 \frac{x}{2} + \cos x + 1 = 0$
- c) $2 \cdot \operatorname{sen} x + 2 \cdot \cos x = \sqrt{2}$
- d) $\cos 2x = \operatorname{sen} x$
- e) $\operatorname{sen} x - \cos x = 0$
- f) $\cos 2x = 2 \cdot \operatorname{sen} 2x$
- g) $2 \cdot \operatorname{sen}^2 x + 3 \cdot \cos x = 0$
- h) $\operatorname{sen} x - \sqrt{3} \cdot \cos x = 0$
- i) $\cos 2x + 1 = \cos x$
- j) $\cos 2x + \operatorname{sen} x = 4 \cdot \operatorname{sen} 2x$
- k) $2 \cos^2 x + 3 \cdot \cos x = 2$
- l) $\cos^2 x - 3 \operatorname{sen}^2 x = 0$
- m) $\operatorname{sen}^2 x - 3 \cdot \operatorname{sen} x \cdot \cos x + 2 \cdot \cos^2 x = 0$
- n) $\operatorname{tg} 2x = -\operatorname{tag} x$
- o) $\operatorname{sen}^2 x - \cos^2 x = \frac{1}{2}$
- p) $\operatorname{tg} x \cdot \sec x = \sqrt{2}$
- q)
- r) $2 \cdot \operatorname{tg} x - 3 \cdot \operatorname{ctg} x - 1 = 0$
- s) $3 \cdot \cos x = 2 \cdot \sec x - 5$
- t) $\cos 2x = 5 - 6 \cdot \cos^2 x$
- u) $\operatorname{cosec} \alpha \cdot \sec \alpha \cdot \cos^2 \alpha + \operatorname{tg} \alpha = \operatorname{ctg} \alpha$
- v) $2 \cdot \operatorname{sen} x + 1 = \operatorname{cosec} x$
- w) $\sec x + \operatorname{tg} x = 2$
- x) $\operatorname{sen} x + \cos x = \frac{13}{6}$
- y) $\cos 2x = \operatorname{sen} x$
- z) $\operatorname{tg}^2 x + 3 = 4 \cdot \operatorname{tg} x$
- aa) $\cos x - 2 \cdot \operatorname{sen} x \cdot \cos x = 0$
- bb)
- cc) $2 \cdot \cos^2 x + 4 \cdot \operatorname{sen}^2 x = 3$
- dd) $\sec x + 4 \cdot \cos x = 5$

ECUACIONES TRIGONOMÉTRICAS

1) Resolver las siguientes ecuaciones:

a) $\operatorname{sen} x = \operatorname{tg} x$

Solución.

$$\operatorname{sen} x = \frac{\operatorname{sen} x}{\cos x}$$

$$\operatorname{sen} x \cdot \cos x = \operatorname{sen} x : \operatorname{sen} x \cdot \cos x - \operatorname{sen} x = 0 : \operatorname{sen} x \cdot (\cos x - 1) = 0 : \begin{cases} \operatorname{sen} x = 0 \\ \cos x - 1 = 0 \end{cases}$$

Casos:

- i) Si $\operatorname{sen} x = 0 \Rightarrow x = 0; x = \pi = 180^\circ$ son soluciones de la ecuación.
- ii) Si $\operatorname{sen} x \neq 0 \Rightarrow \cos x = 1 \Rightarrow x = 0$ es la única solución de la ecuación, que implicaría además $\operatorname{sen} x = 0$, luego no puede ser.

b) $6 \cos^2\left(\frac{x}{2}\right) + \cos x + 1 = 0$

Solución.

$$6 \cos^2\left(\frac{x}{2}\right) + \cos x + 1 = 6 \frac{\cos x + 1}{2} + \cos x + 1 = 0$$

$$3 \cos x + 3 + \cos x + 1 = 0$$

$$4 \cos x + 4 = 0$$

$$\cos x = -1 \Rightarrow x = \pi + 2\pi k = 180^\circ + 360^\circ k$$

c) $2 \operatorname{sen} x + 2 \cos x = \sqrt{2}$

Solución.

$$2 \operatorname{sen} x + 2 \cos x = \sqrt{2}$$

$$\operatorname{sen} x + \cos x = \frac{\sqrt{2}}{2}$$

elevando al cuadrado y desarrollando

$$(\operatorname{sen} x + \cos x)^2 = \frac{1}{2} : \operatorname{sen}^2 x + \cos^2 x + 2 \operatorname{sen} x \cos x = \frac{1}{2}$$

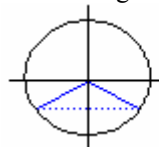
teniendo en cuenta que $\operatorname{sen}^2 x + \cos^2 x = 1$, y despejando

$$2 \operatorname{sen} x \cos x = -\frac{1}{2}$$

mediante la definición de ángulo doble

$$\operatorname{sen}(2x) = -\frac{1}{2}$$

teniendo en cuenta los ángulos asociados



$$\begin{cases} 2x = \frac{7\pi}{6} + 2\pi k = 210^\circ + 360^\circ k \Rightarrow x = \frac{7\pi}{12} + \pi k = 105^\circ + 180^\circ k \\ 2x = \frac{11\pi}{6} + 2\pi k = 330^\circ + 360^\circ k \Rightarrow x = \frac{11\pi}{12} + \pi k = 165^\circ + 180^\circ k \end{cases}$$

d) $\cos(2x) = \operatorname{sen} x$

Solución.

$$\cos(2x) = \operatorname{sen} x$$

por la definición de coseno del ángulo doble

$$\cos^2 x - \operatorname{sen}^2 x = \operatorname{sen} x$$

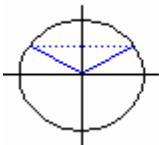
teniendo en cuenta la ecuación fundamental se despeja el $\cos^2 x$ en función del $\operatorname{sen}^2 x$

$$(1 - \operatorname{sen}^2 x) - \operatorname{sen}^2 x = \operatorname{sen} x$$

ordenando se obtiene una ecuación de segundo grado en función del $\operatorname{sen} x$

$$2\operatorname{sen}^2 x + \operatorname{sen} x - 1 = 0 \Rightarrow \operatorname{sen} x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} : \begin{cases} \operatorname{sen} x = \frac{1}{2} \\ \operatorname{sen} x = -1 \end{cases}$$

Teniendo en cuenta los ángulos asociados



$$\begin{cases} \operatorname{sen} x = \frac{1}{2} \Rightarrow \begin{cases} x = \frac{\pi}{6} + 2\pi k = 30^\circ + 360^\circ k \\ x = \frac{5\pi}{6} + 2\pi k = 150^\circ + 360^\circ k \end{cases} \\ \operatorname{sen} x = -1 \Rightarrow x = \frac{3\pi}{2} + 2\pi k = 270^\circ + 360^\circ k \end{cases}$$

e) $\operatorname{sen} x - \cos x = 0$

Solución.

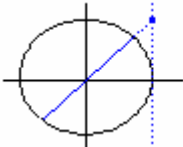
$$\operatorname{sen} x - \cos x = 0$$

$$\operatorname{sen} x = \cos x$$

dividiendo ambos miembros de la igualdad por $\cos x$

$$\operatorname{tg} x = 1$$

Teniendo en cuenta los ángulos asociados



$$\operatorname{tg} x = 1 \Rightarrow \begin{cases} x = \frac{\pi}{4} + 2\pi k = 45^\circ + 360^\circ k \\ x = \frac{5\pi}{4} + 2\pi k = 225^\circ + 360^\circ k \end{cases}$$

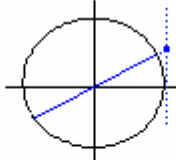
f) $\cos(2x) = 2\operatorname{sen}(2x)$

Solución.

$$\cos(2x) = 2\operatorname{sen}(2x)$$

dividiendo toda la ecuación por $2\cos(2x)$

$$\operatorname{tg}(2x) = \frac{1}{2}$$

$$2x = \arctan\left(\frac{1}{2}\right)$$


$$\begin{cases} 2x \cong 0,148\pi + 2\pi k = 26^\circ 33' 54'' + 360^\circ k \Rightarrow x \cong 0,074\pi + 2\pi k = 13^\circ 16' 57'' + 360^\circ k \\ 2x \cong 1,148\pi + 2\pi k = 206^\circ 33' 54'' + 360^\circ k \Rightarrow x \cong 1,074\pi + 2\pi k = 103^\circ 16' 57'' + 360^\circ k \end{cases}$$

g) $2\operatorname{sen}^2 x + 3\cos x = 0$

Solución.

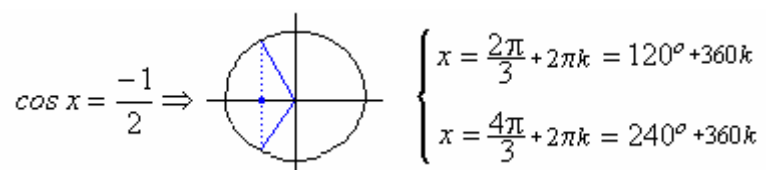
Se transforma en una ecuación de segundo grado en función del $\cos x$

$$2\operatorname{sen}^2 x + 3\cos x = 0$$

$$2(1 - \cos^2 x) + 3\cos x = 0$$

$$2\cos^2 x - 3\cos x - 2 = 0 : \cos x = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm 5}{4} : \begin{cases} \cos x = 2 \\ \cos x = -\frac{1}{2} \end{cases}$$

$\cos x = 2$ no tiene sentido ya que el coseno está acotado en $[-1, 1]$



h) $\operatorname{sen} x - \sqrt{3}\cos x = 0$

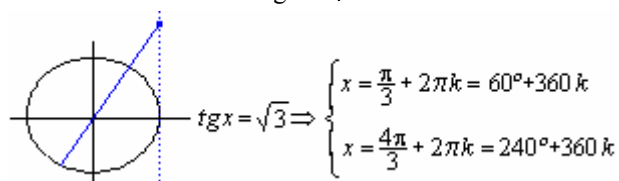
Solución.

Se transforma a $\operatorname{tg} x$

$$\operatorname{sen} x - \sqrt{3}\cos x = 0$$

$$\operatorname{sen} x = \sqrt{3}\cos x \xrightarrow{\div \cos x} \frac{\operatorname{sen} x}{\cos x} = \frac{\sqrt{3}}{\cos x}$$

$$\operatorname{tg} x = \sqrt{3}$$



i) $\cos(2x) + 1 = \cos x$

Solución.

Se transforma en una ecuación de segundo grado en función del $\cos x$

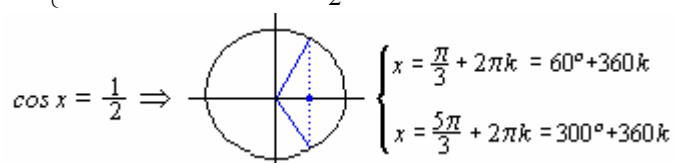
$$\cos(2x) + 1 = \cos x$$

$$\cos^2 x - \operatorname{sen}^2 x + 1 = \cos x$$

$$\cos^2 x - (1 - \cos^2 x) + 1 = \cos x$$

$$2\cos^2 x = \cos x ; 2\cos^2 x - \cos x = 0$$

$$\cos x \cdot (2\cos x - 1) = 0 : \begin{cases} \cos x = 0 \Rightarrow \begin{cases} x = 0 = 360^\circ k = 2\pi k \\ x = \frac{3\pi}{2} + 2\pi k = 270^\circ + 360^\circ k \end{cases} \\ 2\cos x - 1 = 0 \Rightarrow \cos x = \frac{1}{2} \end{cases}$$



j) $\cos(2x) + \operatorname{sen} x = \operatorname{sen}(3x)$

Solución.

Aplicando transformaciones de sumas en producto se obtiene una ecuación equivalente.

$$\cos(2x) + \operatorname{sen} x = \operatorname{sen}(3x)$$

$$\cos(2x) = \operatorname{sen}(3x) - \operatorname{sen} x$$

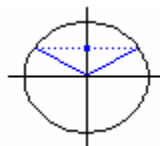
$$\cos(2x) = 2 \cos\left(\frac{3x+x}{2}\right) \cdot \operatorname{sen} \frac{3x-x}{2}$$

$$\cos(2x) = 2 \cos 2x \cdot \operatorname{sen} x$$

$$\cos(2x) - 2 \cos 2x \cdot \operatorname{sen} x = 0$$

$$\cos(2x) \cdot (1 - 2 \operatorname{sen} x) = 0 : \begin{cases} \cos(2x) = 0 \\ 1 - 2 \operatorname{sen} x = 0 \end{cases}$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + \pi k = 90^\circ + 180^\circ k$$

$$\operatorname{sen} x = \frac{1}{2} \Rightarrow \begin{cases} x = \frac{\pi}{6} + 2\pi k = 30^\circ + 360^\circ k \\ x = \frac{5\pi}{6} + 2\pi k = 150^\circ + 360^\circ k \end{cases}$$


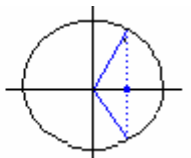
k) $2 \cos^2 x + 3 \cos x = 2$

Solución.

$$2 \cos^2 x + 3 \cos x = 2$$

$$2 \cos^2 x + 3 \cos x - 2 = 0$$

$$\cos x = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4} : \begin{cases} \cos x = -2 \notin [-1, 1] \text{ no tiene sentido} \\ \cos x = \frac{1}{2} \end{cases}$$

$$\cos x = \frac{1}{2} \Rightarrow \begin{cases} x = \frac{\pi}{3} + 2\pi k = 60^\circ + 360^\circ k \\ x = \frac{5\pi}{3} + 2\pi k = 300^\circ + 360^\circ k \end{cases}$$


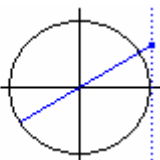
l) $\cos^2 x - 3 \operatorname{sen}^2 x = 0$

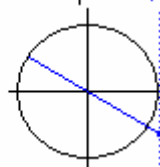
Solución.

$$\cos^2 x - 3 \operatorname{sen}^2 x = 0$$

$$\cos^2 x = 3 \operatorname{sen}^2 x \xrightarrow{+3 \cos^2 x} \frac{3 \operatorname{sen}^2 x}{3 \cos^2 x} = \frac{\cos^2 x}{3 \cos^2 x}$$

$$\operatorname{tg}^2 x = \frac{1}{3} \Rightarrow \operatorname{tg} x = \pm \frac{\sqrt{3}}{3}$$

$$\operatorname{tg} x = \frac{\sqrt{3}}{3} \Rightarrow \begin{cases} x = \frac{\pi}{6} + 2\pi k = 30^\circ + 360^\circ k \\ x = \frac{7\pi}{6} + 2\pi k = 210^\circ + 360^\circ k \end{cases}$$


$$\operatorname{tg} x = -\frac{\sqrt{3}}{3} \Rightarrow \begin{cases} x = \frac{5\pi}{6} + 2\pi k = 150^\circ + 360^\circ k \\ x = \frac{11\pi}{6} + 2\pi k = 330^\circ + 360^\circ k \end{cases}$$


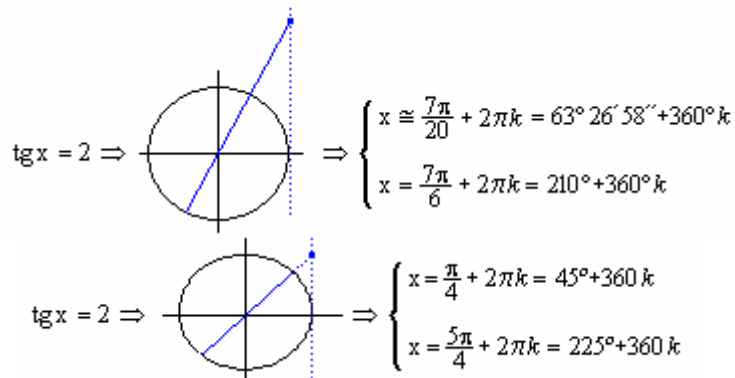
m) $\operatorname{sen}^2 x - 3 \operatorname{sen} x \cos x + 2 \cos^2 x = 0$

Solución.

Se transforma en una ecuación de segundo grado dividiendo todos los términos de la ecuación por $\cos^2 x$.

$$\begin{aligned}\frac{\sin^2 x}{\cos^2 x} - \frac{3 \sin x \cos x}{\cos^2 x} + \frac{2 \cos^2 x}{\cos^2 x} &= 0 \\ \tan^2 x - 3 \tan x + 2 &= 0\end{aligned}$$

$$\tan^2 x - 3 \tan x + 2 = 0 : \quad \tan x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{3 \pm 1}{2}$$



n) $\tan(2x) = -\tan x$

Solución.

Se transforma la igualdad en función de $\sin x$ y $\cos x$.

$$\tan(2x) = -\tan x$$

$$\frac{\sin(2x)}{\cos(2x)} = -\frac{\sin x}{\cos x}$$

$$\frac{2 \sin x \cos x}{\cos^2 x - \sin^2 x} = -\frac{\sin x}{\cos x}$$

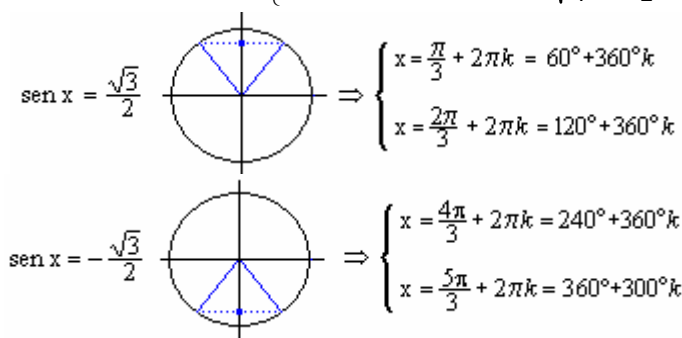
$$2 \sin x \cdot \cos^2 x = -\sin x \cdot \cos^2 x + \sin^3 x$$

$$\sin^3 x - 3 \sin x \cdot \cos^2 x = 0$$

$$\sin^3 x - 3 \sin x \cdot (1 - \sin^2 x) = 0$$

$$4 \sin^3 x - 3 \sin x = 0$$

$$\sin x \cdot (4 \sin^2 x - 3) = 0 : \begin{cases} \sin x = 0 : x = 0 = \pi k = 180^\circ k \\ 4 \sin^2 x - 3 = 0 : \sin x = \pm \sqrt{\frac{3}{4}} = \frac{\pm \sqrt{3}}{2} \end{cases}$$



$$o) \quad \text{sen}^2 x - \cos^2 x = \frac{1}{2}$$

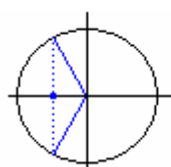
Solución.

Teniendo en cuenta la definición de coseno del ángulo doble, se transforma la expresión

$$\text{sen}^2 x - \cos^2 x = \frac{1}{2}$$

$$-\cos(2x) = \frac{1}{2}$$

$$\cos(2x) = -\frac{1}{2}$$

$$\cos(2x) = -\frac{1}{2} \quad \left\{ \begin{array}{l} 2x = \frac{2\pi}{3} + 2\pi k = 120^\circ + 360^\circ k \implies x = \frac{\pi}{3} + \pi k = 60^\circ + 180^\circ k \\ 2x = \frac{4\pi}{3} + 2\pi k = 240^\circ + 360^\circ k \implies x = \frac{2\pi}{3} + \pi k = 120^\circ + 180^\circ k \end{array} \right.$$


$$p) \quad \text{tg} x \cdot \sec x = \sqrt{2}$$

Solución.

$$\text{tg} x \sec x = \sqrt{2}$$

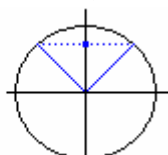
$$\frac{\text{sen} x}{\cos x} \cdot \frac{1}{\cos x} = \sqrt{2}$$

$$\frac{\text{sen} x}{1 - \text{sen}^2 x} = \sqrt{2}$$

$$\sqrt{2} \text{sen}^2 x + \text{sen} x - \sqrt{2} = 0$$

Ecuación de segundo grado en función de sen x

$$\text{sen} x = \frac{-1 \pm \sqrt{1+8}}{2\sqrt{2}} = \frac{-1 \pm 3}{2\sqrt{2}} \quad \left\{ \begin{array}{l} \text{sen} x = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \text{sen} x = \frac{-4}{2\sqrt{2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2} \notin [-1, 1] \Rightarrow \text{No válida} \end{array} \right.$$

$$\text{sen} x = \frac{\sqrt{2}}{2} \quad \Rightarrow \quad \left\{ \begin{array}{l} x = \frac{\pi}{4} + 2\pi k = 45^\circ + 360^\circ k \\ x = \frac{3\pi}{4} + 2\pi k = 135^\circ + 360^\circ k \end{array} \right.$$


r) $2\operatorname{tg} x - 3\cot x - 1 = 0$

Solución.

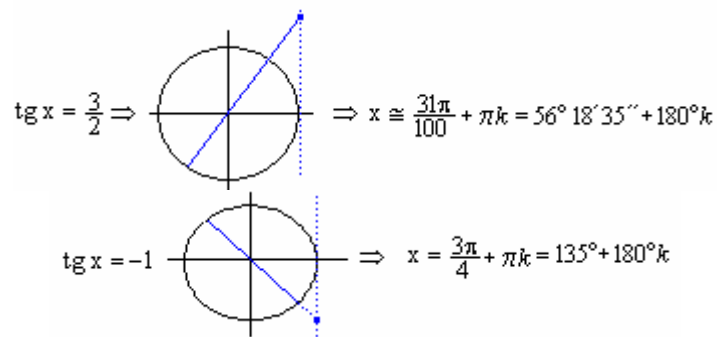
$$2\operatorname{tg} x - 3\cot x - 1 = 0$$

$$2\operatorname{tg} x - 3\frac{1}{\operatorname{tg} x} - 1 = 0$$

Multiplicando la igualdad por $\operatorname{tg} x$, se transforma en una ecuación de segundo grado en función de $\operatorname{tg} x$

$$2\operatorname{tg}^2 x - \operatorname{tg} x - 3 = 0$$

$$\operatorname{tg} x = \frac{1 \pm \sqrt{1+24}}{4} : \begin{cases} \operatorname{tg} x = \frac{3}{2} \\ \operatorname{tg} x = -1 \end{cases}$$



s) $3\cos x = 2\sec x - 5$

Solución.

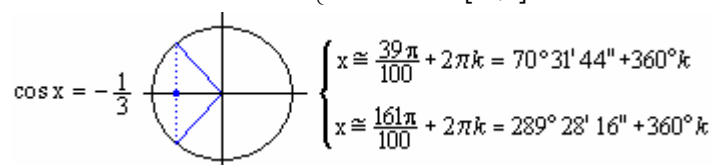
$$3\cos x = 2\sec x - 5$$

$$3\cos x = \frac{2}{\cos x} - 5$$

Multiplicando toda la ecuación por $\cos x$ se obtiene una ecuación de segundo grado

$$3\cos^2 x + 5\cos x - 2 = 0$$

$$\cos x = \frac{-5 \pm \sqrt{25+24}}{6} : \begin{cases} \cos x = \frac{1}{3} \\ \cos x = -2 \notin [-1, 1] \Rightarrow \text{No válida} \end{cases}$$



t) $\cos(2x) = 5 - 6\cos^2 x$

Solución.

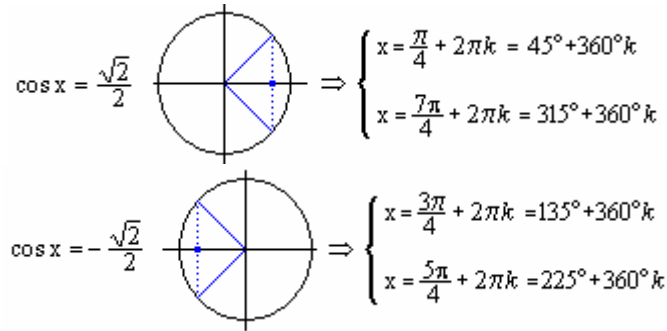
$$\cos(2x) = 5 - 6\cos^2 x$$

$$\cos^2 x - \sin^2 x = 5 - 6\cos^2 x$$

$$\cos^2 x - (1 - \cos^2 x) = 5 - 6\cos^2 x$$

$$8\cos^2 x = 4$$

$$\cos^2 x = \frac{1}{2} \Rightarrow \cos x = \pm \frac{\sqrt{2}}{2}$$



u) $\operatorname{cosec} \alpha \cdot \sec \alpha \cdot \cos^2 \alpha + \operatorname{tg} \alpha = \cot g \alpha$

Solución.

$$\operatorname{cosec} x \cdot \sec x \cdot \cos^2 x + \operatorname{tg} x = \cot g x$$

$$\frac{1}{\operatorname{sen} x} \cdot \frac{1}{\cos x} \cdot \cos^2 x + \operatorname{tg} x = \cot g x$$

$$\frac{\cos x}{\operatorname{sen} x} + \operatorname{tg} x = \cot g x$$

$$\cot g x + \operatorname{tg} x = \cot g x$$

$$\operatorname{tg} x = 0 \Rightarrow x = 0 = 180^\circ k = \pi k$$

v) $2\operatorname{sen} x + 1 = \operatorname{cosec} x$

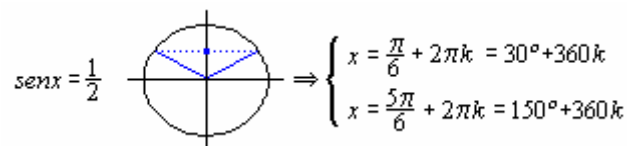
Solución.

$$2\operatorname{sen} x + 1 = \operatorname{cosec} x$$

$$2\operatorname{sen} x + 1 = \frac{1}{\operatorname{sen} x}$$

$$2\operatorname{sen}^2 x + \operatorname{sen} x - 1 = 0$$

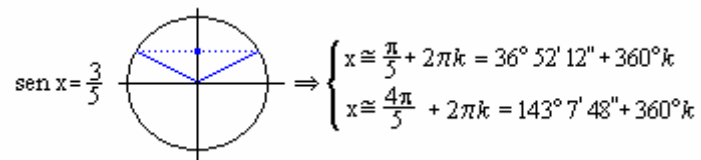
$$\operatorname{sen} x = \frac{-1 \pm \sqrt{1+8}}{4} : \begin{cases} \operatorname{sen} x = \frac{1}{2} \\ \operatorname{sen} x = -1 \Rightarrow x = \frac{3\pi}{2} + 2\pi k = 270^\circ + 360^\circ k \end{cases}$$



w) $\sec x + \operatorname{tg} x = 2$

Solución.

$$\begin{aligned}\sec x + \operatorname{tg} x &= 2 \\ \frac{1}{\cos x} + \frac{\operatorname{sen} x}{\cos x} &= 2 \\ \frac{1 + \operatorname{sen} x}{\cos x} &= 2 \\ 1 + \operatorname{sen} x &= 2 \cos x \\ 1 + 2 \operatorname{sen} x + \operatorname{sen}^2 x &= 4 \cos^2 x \\ 1 + 2 \operatorname{sen} x + \operatorname{sen}^2 x &= 4(1 - \operatorname{sen}^2 x) \\ 5 \operatorname{sen}^2 x + 2 \operatorname{sen} x - 3 &= 0 \\ \operatorname{sen} x = \frac{-2 \pm \sqrt{4 + 60}}{10} : \begin{cases} \operatorname{sen} x = \frac{6}{10} = \frac{3}{5} \\ \operatorname{sen} x = -1 \Rightarrow x = \frac{3\pi}{4} + 2\pi k = 270^\circ + 360^\circ k \end{cases}\end{aligned}$$

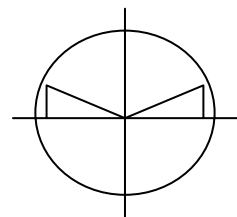


x) $\operatorname{sen} x + \operatorname{cosec} x = \frac{13}{6}$

Solución.

$$\begin{aligned}\operatorname{sen} x + \operatorname{cosec} x &= \frac{13}{6} \\ \operatorname{sen} x + \frac{1}{\operatorname{sen} x} &= \frac{13}{6} \\ \operatorname{sen}^2 x - \frac{13}{6} \operatorname{sen} x + 1 &= 0 \\ \operatorname{sen} x = \frac{\frac{13}{6} \pm \sqrt{\frac{169}{36} - 4}}{2} &= \frac{\frac{13}{6} \pm \sqrt{\frac{25}{36}}}{2} = \frac{\frac{13}{6} \pm \frac{5}{6}}{2} = \frac{13 \pm 5}{12}. \text{Casos :}\end{aligned}$$

$$\begin{cases} \operatorname{sen} x = \frac{18}{12} \Rightarrow \text{no tiene solución} \\ \operatorname{sen} x = \frac{2}{3} \Rightarrow x \cong \frac{23\pi}{100} = 41^\circ 48' 37''; x \cong \frac{77\pi}{100} = 138^\circ 11' 23'' \end{cases}$$



y) $\cos(2x) = \operatorname{sen} x$

Solución.

$$\cos(2x) = \operatorname{sen} x$$

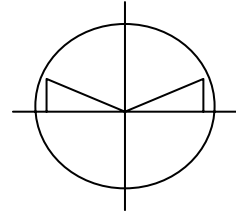
$$\cos^2 x - \operatorname{sen}^2 x = \operatorname{sen} x$$

$$(1 - \operatorname{sen}^2 x) - \operatorname{sen}^2 x = \operatorname{sen} x$$

$$2\operatorname{sen}^2 x + \operatorname{sen} x - 1 = 0$$

$$\operatorname{sen} x = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}. \text{Casos:}$$

$$\begin{cases} \operatorname{sen} x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} = 30^\circ; x = \frac{5\pi}{6} = 150^\circ \\ \operatorname{sen} x = -1 \Rightarrow x = \frac{3\pi}{2} = 270^\circ \end{cases}$$



z) $\operatorname{tg}^2 x + 3 = 4\operatorname{tg} x$

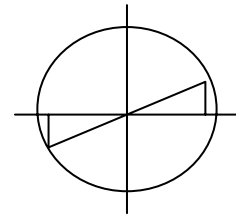
Solución.

$$\operatorname{tg}^2 x + 3 = 4\operatorname{tg} x$$

$$\operatorname{tg}^2 x - 4\operatorname{tg} x + 3 = 0$$

$$\operatorname{tg} x = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2}. \text{Casos:}$$

$$\begin{cases} \operatorname{tg} x = 3 \Rightarrow x \cong \frac{2\pi}{5} = 71^\circ 33' 54''; x \cong \frac{7\pi}{5} = 251^\circ 33' 54'' \\ \operatorname{tg} x = 1 \Rightarrow x = \frac{\pi}{4} = 45^\circ; x = \frac{5\pi}{4} = 225^\circ \end{cases}$$



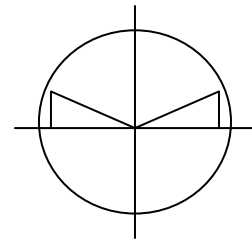
aa) $\cos x - 2\operatorname{sen} x \cdot \cos x = 0$

Solución.

$$\cos x - 2\operatorname{sen} x \cdot \cos x = 0$$

$$\cos x(1 - 2\operatorname{sen} x) = 0. \text{Casos:}$$

$$\begin{cases} \cos x = 0 \Rightarrow x = \frac{\pi}{2} = 90^\circ; x = \frac{3\pi}{2} = 270^\circ \\ 1 - 2\operatorname{sen} x = 0 \Rightarrow \operatorname{sen} x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} = 30^\circ; x = \frac{5\pi}{6} = 150^\circ \end{cases}$$



cc) $2\cos^2 x + 4\operatorname{sen}^2 x = 3$

SOLUCIÓN:

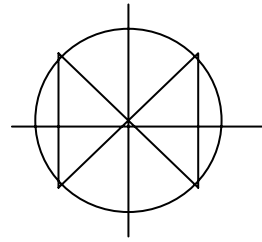
$$2\cos^2 x + 4\operatorname{sen}^2 x = 3$$

$$2(1 - \operatorname{sen}^2 x) + 4\operatorname{sen}^2 x = 3$$

$$2\operatorname{sen}^2 x = 1$$

$$\operatorname{sen}^2 x = \frac{1}{2}$$

$$\operatorname{sen} x = \pm \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4} = 45^\circ; x = \frac{3\pi}{4} = 135^\circ; x = \frac{5\pi}{4} = 225^\circ; x = \frac{7\pi}{4} = 315^\circ$$



dd) $\sec x + 4\cos x = 5$

Solución.

$$\sec x + 4\cos x = 5$$

$$\frac{1}{\cos x} + 4\cos x = 5$$

$$4\cos^2 x - 5\cos x + 1 = 0$$

$$\cos x = \frac{5 \pm \sqrt{25 - 16}}{8} = \frac{5 \pm 3}{8}. \text{Casos :}$$

$$\begin{cases} \cos x = 1 \Rightarrow x = 0 \\ \cos x = \frac{1}{4} \Rightarrow x \cong \frac{21\pi}{50} = 75^\circ 31' 21''; x \cong \frac{-21\pi}{50} = -75^\circ 31' 21'' \end{cases}$$

