Integración por partes.

## Ejercicios resueltos

**Ejemplo 1** *: Integre* $\displaystyle\int x^3 \ln x \; dx$

**Solución :** Hacemos

$$u = \ln x \quad \xrightarrow{\text{Al derivar}} \quad du = \frac{1}{x}\, dx$$

$$dv = x^3 \; dx \quad \xrightarrow{\text{Al integrar}} \quad v = \frac{x^4}{4}$$

La integral se transforma en

$$\int x^3 \ln x \; dx = \frac{x^4}{4} \; \ln x - \int \frac{x^4}{4} \; \frac{1}{x} \; dx = \frac{x^4}{4} \; \ln x - \frac{1}{4} \int x^3 \; dx = \frac{x^4}{4} \; \ln x - \frac{x^4}{16} + C.$$

Por lo tanto,

$$\int x^3 \ln x \; dx = \frac{x^4}{4} \; \ln x - \frac{x^4}{16} + C.$$

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**Ejemplo 2** *: Integre* $\displaystyle\int x^2 \cos x \; dx$

**Solución :** Hacemos

$$u = x^2 \quad \xrightarrow{\text{Al derivar}} \quad du = 2x \; dx$$

$$dv = \cos x \; dx \quad \xrightarrow{\text{Al integrar}} \quad v = \operatorname{sen} x$$

La integral se transforma en

$$\int x^2 \cos x \; dx = x^2 \operatorname{sen} x - \int 2x \operatorname{sen} x \; dx = x^2 \operatorname{sen} x - 2 \int x \operatorname{sen} x \; dx$$

Resolvemos la nueva integral $\displaystyle\int x \operatorname{sen} x \; dx$, integramos otra vez por partes. Hacemos

$$u = x \quad \xrightarrow{\text{Al derivar}} \quad du = dx$$

$$dv = \operatorname{sen} x \; dx \quad \xrightarrow{\text{Al integrar}} \quad v = -\cos x$$

y obtenemos

$$\int x \operatorname{sen} x \; dx = -x \cos x - \int -\cos x \; dx = -x \cos x + \int \cos x \; dx = -x \cos x + \operatorname{sen} x + C_1,$$

entonces,

$$\int x^2 \cos x \; dx = x^2 \operatorname{sen} x - 2 \left(-x \cos x + \operatorname{sen} x + C_1\right) = x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + C,$$

así, la familia de primitivas es

$$\int x^2 \cos x \; dx = x^2 \operatorname{sen} x + 2x \cos x - 2 \operatorname{sen} x + C.$$

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**Ejemplo 3** *: Integre* $\displaystyle\int \operatorname{sen}^2 x \; dx$

**Solución :** Observemos que

$$\int \operatorname{sen}^2 x \; dx = \int \operatorname{sen} x \operatorname{sen} x \; dx.$$

Integramos por partes, hacemos

$$u = \operatorname{sen} x \quad \xrightarrow{\text{Al derivar}} \quad du = \cos x \; dx$$

$$dv = \operatorname{sen} x \; dx \quad \xrightarrow{\text{Al integrar}} \quad v = -\cos x$$

La integral se transforma en

$$\int \operatorname{sen}^2 x\ dx = -\operatorname{sen} x\cos x - \int -\cos x\cos x\ dx = -\operatorname{sen} x\cos x + \int \cos^2 x\ dx$$

$$= -\operatorname{sen} x\cos x + \int \left(1-\operatorname{sen}^2 x\right)\ dx = -\operatorname{sen} x\cos x + \int dx - \int \operatorname{sen}^2 x\ dx$$

como $\int dx = x + C_1$, tenemos que

$$\int \operatorname{sen}^2 x\ dx = -\operatorname{sen} x\cos x + x - \int \operatorname{sen}^2 x\ dx + C_1 \implies \int \operatorname{sen}^2 x\ dx + \int \operatorname{sen}^2 x\ dx = -\operatorname{sen} x\cos x + x + C_1,$$

de aquí,

$$2\int \operatorname{sen}^2 x\ dx = -\operatorname{sen} x\cos x + x + C_1 \qquad \implies \qquad \int \operatorname{sen}^2 x\ dx = \frac{1}{2}\left(-\operatorname{sen} x\cos x + x + C_1\right).$$

Por lo tanto,

$$\int \operatorname{sen}^2 x\ dx = \frac{1}{2}\operatorname{sen} x\cos x + \frac{1}{2}x + C$$

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## Ejemplo 4 *: Integre* $\int e^{\sqrt{x}}\ dx$

**Solución :** Hacemos el cambio de variable

$$p = \sqrt{x}; \qquad\qquad dp = \frac{1}{2\sqrt{x}} \implies 2p\ dp = dx$$

y la integral nos queda

$$\int e^{\sqrt{x}}\ dx = \int 2pe^p\ dp = 2\int pe^p\ dp$$

integramos por partes, hacemos

$$u = p \xrightarrow{\text{Al derivar}} du = dp$$

$$dv = e^p\ dp \xrightarrow{\text{Al integrar}} v = e^p,$$

la integral se transforma,

$$\int pe^p\ dp = pe^p - \int e^p\ dp = pe^p - e^p + C_1,$$

como $p = \sqrt{x}$, se tiene

$$\int e^{\sqrt{x}}\ dx = 2\left(\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + C_1\right) = 2e^{\sqrt{x}}\left(\sqrt{x}-1\right) + C,$$

es decir,

$$\int e^{\sqrt{x}}\ dx = 2e^{\sqrt{x}}\left(\sqrt{x}-1\right) + C.$$

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## Ejemplo 5 *: Integre* $\int \csc^3 x\ dx$

**Solución :** Escribimos la integral como

$$\int \csc^3 x\ dx = \int \csc^2 x\csc x\ dx.$$

Integramos por partes, hacemos

$$u = \csc x \xrightarrow{\text{Al derivar}} du = -\csc x\cot x\ dx$$

$$dv = \csc^2 x\ dx \xrightarrow{\text{Al integrar}} v = -\cot x,$$

La integral se transforma en

$$\int \csc^3 x\ dx = \csc x\left(-\cot x\right) - \int \left(-\cot x\right)\left(-\csc x\cot x\right)\ dx = -\csc x\cot x - \int \csc x\cot^2 x\ dx,$$

es conocido que

$$\cot^2 x = \csc^2 x - 1,$$

así,

$$\int \csc^3 x\ dx = -\csc x \cot x - \int \csc x \cot^2 x\ dx = -\csc x\ \cot x - \int \csc x \left(\csc^2 x - 1\right)\ dx$$

$$= -\csc x \cot x - \int \csc^3 x\ dx + \int \csc x\ dx = -\csc x \cot x - \int \csc^3 x\ dx + \ln|\csc x - \cot x| + C,$$

es decir,

$$\int \csc^3 x\ dx = -\csc x \cot x - \int \csc^3 x\ dx + \ln|\csc x - \cot x| + C,$$

de aquí,

$$2\int \csc^3 x\ dx = -\csc x \cot x + \ln|\csc x - \cot x| + C,$$

con lo que,

$$\int \csc^3 x\ dx = -\frac{1}{2}\csc x \cot x + \frac{1}{2}\ln|\csc x - \cot x| + C.$$

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**Ejemplo 6** *: Demuestre que*

$$\int \frac{x^n\ dx}{\sqrt{1-x^2}} = -x^{n-1}\ \sqrt{1-x^2} + (n-1)\int x^{n-2}\ \sqrt{1-x^2}\ dx$$

**Demostración :** Escribimos la integral como

$$\int \frac{x^n\ dx}{\sqrt{1-x^2}} = \int x^{n-1}\ \frac{x}{\sqrt{1-x^2}}\ dx$$

Integramos por partes, hacemos

$$u = x^{n-1} \qquad \xrightarrow{\text{Al derivar}} \qquad du = (n-1)\,x^{n-2}\ dx$$

$$dv = \frac{x}{\sqrt{1-x^2}}\ dx \qquad \xrightarrow{\text{Al integrar}} \qquad v = -\sqrt{1-x^2}.$$

La integral se transforma en

$$\int \frac{x^n\ dx}{\sqrt{1-x^2}} = x^{n-1}\left(-\sqrt{1-x^2}\right) - \int \left(-\sqrt{1-x^2}\right)(n-1)\,x^{n-2}\ dx = -x^{n-1}\ \sqrt{1-x^2} + (n-1)\int x^{n-2}\ \sqrt{1-x^2}\ dx$$

entonces,

$$\int \frac{x^n\ dx}{\sqrt{1-x^2}} = -x^{n-1}\ \sqrt{1-x^2} + (n-1)\int x^{n-2}\ \sqrt{1-x^2}\ dx.$$

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## Ejercicios

1. Calcular las siguientes integrales

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|---|---|---|---|---|
| 1. $\int xe^x\ dx$ | 2. $\int \frac{x}{e^x}\ dx$ | 3. $\int x\ 2^{-x}\ dx$ | 4. $\int x \operatorname{sen} x\ dx$ | 5. $\int t\cos t\ dt$ |
| 6. $\int xe^{2x}\ dx$ | 7. $\int \frac{x^2}{e^{3x}}\ dx$ | 8. $\int x^2\ 3^x\ dx$ | 9. $\int x^2 \operatorname{sen} x dx$ | 10. $\int t^3 \operatorname{sen} t\ dt$ |
| 11. $\int \ln x\ dx$ | 12. $\int \arctan x\ dx$ | 13. $\int \operatorname{arcsen} x\ dx$ | 14. $\int 4x \ln 2x\ dx$ | |
| 15. $\int \sqrt{x}\ln x\ dx$ | 16. $\int x \arctan x\ dx$ | 17. $\int x \operatorname{arcsen} x\ dx$ | 18. $\int x^3 e^{x^2}\ dx$ | |
| 19. $\int \cos^2 x\ dx$ | 20. $\int \theta\cos 3\theta\ d\theta$ | 21. $\int x^5 \cos\left(x^3\right) dx$ | 22. $\int \left(t^2+5t+6\right)\cos 2t\ dt$ | |
| 23. $\int \sec^3\theta\ d\theta$ | 24. $\int e^x \operatorname{sen} x\ dx$ | 25. $\int \operatorname{sen} 3x \cos 5x\ dx$ | 26. $\int x \operatorname{sen} x \cos x\ dx$ | |

27. $\int x^2 \ln x \, dx$ 28. $\int \frac{\ln x}{\sqrt{x}} \, dx$ 29. $\int e^{5x} \cos 2x \, dx$ 30. $\int \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{3}\right) \, dx$

31. $\int z^2 e^{3z} \, dz$ 32. $\int t^2 e^{-t/2} \, dt$ 33. $\int e^{at} \cos bt \, dt$ 34. $\int \left(x^2 - 2x + 5\right) e^{-x} \, dx$

35. $\int \frac{x \, dx}{\operatorname{sen}^2 x}$ 36. $\int x \ln\left(\frac{1-x}{1+x}\right) \, dx$ 37. $\int x^2 \arctan 3x \, dx$ 38. $\int 5^x \operatorname{sen} 5x \, dx$

39. $\int \ln^2 x \, dx$ 40. $\int e^{\sqrt{x}} \, dx$ 41. $\int e^{ax} \operatorname{sen} bx \, dx$ 42. $\int \ln\left(x\sqrt{1+x^2}\right) \, dx$

43. $\int \operatorname{sen}(\ln x) \, dx$ 44. $\int y^3 e^{-y^2} \, dy$ 45. $\int \frac{x \cos x}{\operatorname{sen}^2 x} \, dx$ 46. $\int 3^x \cos x \, dx$

47. $\int x^5 e^{x^2} \, dx$ 48. $\int \frac{\ln^2 t}{t^2} \, dt$ 49. $\int \frac{\ln(\ln x)}{x} \, dx$ 50. $\int \left(x^2 - 2x + 3\right) \ln x \, dx$

51. $\int t^3 e^t \, dt$ 52. $\int \sqrt{x^2+1} \, dx$ 53. $\int x \tan^2 2x \, dx$ 54. $\int x \left(\arctan x\right)^2 \, dx$

55. $\int \frac{\ln x}{x^3} \, dx$ 56. $\int \frac{\operatorname{arcsen} \sqrt{\theta}}{\sqrt{1-\theta}} \, d\theta$ 57. $\int \frac{\operatorname{sen}^2 x}{e^x} \, dx$ 58. $\int \cos x \cos^2(3x) \, dx$

59. $\int x \csc^2 x \, dx$ 60. $\int x \tan^{-1} x \, dx$ 61. $\int \cos^2(\ln x) \, dx$ 62. $\int \cos t \ln(\operatorname{sen} t) \, dt$

63. $\int (\ln x)^2 \, dx$ 64. $\int \operatorname{sen} \sqrt{x} \, dx$ 65. $\int x^2 \cos 3x \, dx$ 66. $\int x \cos^2 x \operatorname{sen} x \, dx$

67. $\int \sec^5 \theta \, d\theta$ 68. $\int \frac{x \, dx}{\cos^3(x^2)}$ 69. $\int \frac{xe^x}{(x+1)^2} \, dx$ 70. $\int (\operatorname{arcsen} x)^2 \, dx$

71. $\int x^3 \ln x \, dx$ 72. $\int t \operatorname{sen} 4t \, dt$ 73. $\int x^2 \operatorname{sen} 2x \, dx$ 74. $\int \sec^5(ax+b) \, dx$

75. $\int x 5^x \, dx$ 76. $\int \theta \sec^2 \theta \, d\theta$ 78. $\int \sec^3(ax+b) \, dx$

79. $\int z \cos 2z \, dz$ 80. $\int x \operatorname{sen}^2 x \, dx$ 81. $\int e^{-\theta} \cos 3\theta \, d\theta$ 82. $\int xa^x \, dx$

83. $\int \frac{\ln x \, dx}{\sqrt{1-x}}$ 84. $\int \arccos z \, dz$ 85. $\int \operatorname{sen} 2t \operatorname{sen} 4t \, dt$ 86. $\int \operatorname{sen} 2t \ln\left(\cos^7 t\right) dt$

87. $\int \frac{xe^{2x} \, dx}{\sqrt{1-e^{2x}}}$ 88. $\int t^3 \arctan 2t \, dt$ 89. $\int \frac{x \operatorname{arcsen} x}{\sqrt{1-x^2}} \, dx$ 90. $\int \frac{x \operatorname{arcsen} x}{\sqrt{(1-x^2)^3}} \, dx$

91. $\int \frac{x \ln x \, dx}{\sqrt{1-x^2}}$ 92. $\int \operatorname{sen} 2x \ln\left(\operatorname{sen}^4 x \cos^5 x\right) \, dx$ 93. $\int \operatorname{sen} 2x \ln\left(\frac{\cos^{1/2} x}{\operatorname{sen}^{1/3} x}\right) \, dx$

94. $\int \operatorname{sen}(2ax) \ln(\tan ax) \, dx$ 95. $\int \operatorname{sen} 2x \ln\left(\operatorname{sen}^5 x\right) \, dx$ 96. $\int \operatorname{sen} x \ln\left(\cot \frac{x}{2}\right) \, dx$

97. $\int \cos 2x \ln(\operatorname{sen} x + \cos x) \, dx$ 98. $\int \cos x \ln\left(\operatorname{sen}^{-2} x \cos^3 x\right) \, dx$

99. $\int \operatorname{sen} x \ln\left(\operatorname{sen}^4 x \cos^5 x\right) \, dx$ 100. $\int \frac{\operatorname{arcsen}^4 t \ln\left(\operatorname{arcsen}^3 t\right)}{\sqrt{1-t^2}} \, dt$

101. $\displaystyle\int \cos bx \ln\left(\operatorname{sen}^n bx \cos^m bx\right)\ dx$ 102. $\displaystyle\int \operatorname{sen} bx \ln\left(\operatorname{sen}^n bx \cos^m bx\right)\ dx$

103. $\displaystyle\int x \operatorname{senh}^2\left(x^2\right)\ dx$ 104. $\displaystyle\int_{-1}^{1} \cosh^2 x\ dx$ 105. $\displaystyle\int \operatorname{senh}\sqrt{x}\ dx$

106. $\displaystyle\int 3^t \operatorname{senh} 3t\ dt$ 107. $\displaystyle\int e^{at} \cosh bt\ dt$ 108. $\displaystyle\int x^5 \cosh\left(x^3\right)\ dx$

109. $\displaystyle\int \csc^3 x\ dx$ 110. $\displaystyle\int \frac{x^3\ dx}{\sqrt{1-x^2}}$ 111. $\displaystyle\int \frac{x^5\ dx}{\sqrt{1-x^2}}$ 112. $\displaystyle\int \frac{x^7\ dx}{\sqrt{1-x^2}}$

2. Demostrar la fórmula de reducción

$$\int \cos^n x\ dx = \frac{1}{n}\cos^{n-1} x \operatorname{sen} x + \frac{n-1}{n}\int \cos^{n-2} x\ dx$$

3. Demostrar la fórmula de reducción $\displaystyle\int (\ln x)^n\ dx = x(\ln x)^n - n\int (\ln x)^{n-1}\ dx$

4. Demostrar la fórmula de reducción $\displaystyle\int x^n e^x\ dx = x^n e^x - n\int x^{n-1} e^x\ dx$

5. Demostrar la fórmula de reducción

$$\int \left(x^2+a^2\right)^n\ dx = \frac{x\left(x^2+a^2\right)^n}{2n+1} + \frac{2na^2}{2n+1}\int \left(x^2+a^2\right)^{n-1}\ dx,$$

con $n \neq -\dfrac{1}{2}$.

6. Demostrar la fórmula de reducción

con $n \neq 1$. $$\int \sec^n x\ dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\ dx,$$

## Respuestas: Ejercicios

1.1. $(x-1)e^x + C$; 1.2. $-(x+1)e^{-x} + C$; 1.3. $-(1+x\ln 2)\frac{2^{-x}}{\ln^2 2} + C$; 1.4. $\operatorname{sen} x - x\cos x + C$;

1.5. $\cos t + t \operatorname{sen} t + C$; 1.6. $(2x-1)\frac{e^{2x}}{4} + C$; 1.7. $-\left(\frac{2}{27} + \frac{2}{9}x + \frac{1}{3}x^2\right)e^{-3x} + C$; 1.8. $3^x\left(\frac{2}{\ln^3 3} - \frac{2x}{\ln^2 3} + \frac{x^2}{\ln 3}\right) + C$;

1.9. $2\cos x + 2x\operatorname{sen} x - x^2\cos x + C$; 1.10. $6t\cos t - 6\operatorname{sen} t - t^3\cos t + 3t^2\operatorname{sen} t + C$; 1.11. $x(\ln x - 1) + C$;

1.12. $x\arctan x - \frac{1}{2}\ln\left(x^2+1\right) + C$; 1.13. $x\operatorname{arcsen} x + \sqrt{1-x^2} + C$; 1.14. $(2\ln x + 2\ln 2 - 1)x^2 + C$;

1.15. $\frac{2}{3}(\ln x - 1)x^{3/2} + C$; 1.16. $\frac{1}{2}\arctan x - \frac{1}{2}x + \frac{1}{2}x^2\arctan x + C$; 1.17. $\frac{1}{2}x^2\operatorname{arcsen} x - \frac{1}{4}\operatorname{arcsen} x + \frac{1}{4}x\sqrt{1-x^2} + C$;

1.18. $\frac{1}{2}e^{x^2}\left(x^2-1\right) + C$; 1.19. $\frac{1}{2}\cos x\operatorname{sen} x + \frac{1}{2}x + C$; 1.20. $\frac{1}{9}\cos 3\theta + \frac{1}{3}\theta\operatorname{sen} 3\theta + C$;

1.21. $\frac{1}{3}\cos\left(x^3\right) + \frac{1}{3}x^3\operatorname{sen}\left(x^3\right) + C$; 1.22. $\frac{5}{4}\cos 2t + \frac{11}{4}\operatorname{sen} 2t + \frac{1}{2}t\cos 2t + \frac{5}{2}t\operatorname{sen} 2t + \frac{1}{2}t^2\operatorname{sen} 2t + C$;

1.23. $\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$; 1.24. $\frac{e^x}{2}(\operatorname{sen} x - \cos x) + C$; 1.25. $\frac{5}{16}\operatorname{sen} 3x\operatorname{sen} 5x + \frac{3}{16}\cos 3x\cos 5x + C$;

1.26. $\frac{1}{2}x\operatorname{sen}^2 x + \frac{1}{4}\cos x\operatorname{sen} x - \frac{1}{4}x + C$; 1.27. $\frac{1}{3}x^3\left(\ln x - \frac{1}{3}\right) + C$; 1.28. $2\sqrt{x}(\ln x - 2) + C$;

1.29. $\frac{e^{5x}}{29}(5\cos 2x + \operatorname{sen} 2x) + C$; 1.30. $-\frac{12}{5}\cos\left(\frac{x}{2}\right)\operatorname{sen}\left(\frac{x}{3}\right) + \frac{18}{5}\operatorname{sen}\left(\frac{x}{2}\right)\cos\left(\frac{x}{3}\right) + C$; 1.31. $\frac{1}{3}e^{3z}\left(\frac{2}{9} - \frac{2}{3}z + z^2\right) + C$;

1.32. $-2e^{-t/2}\left(8 + 4t + t^2\right) + C$; 1.33. $\frac{e^{at}}{a^2+b^2}(a\cos bt + b\operatorname{sen} bt) + C$; 1.34. $-e^{-x}\left(x^2+5\right) + C$;

1.35. $-x\cot x + \ln|\operatorname{sen} x| + C$; 1.36. $\frac{1}{2}x^2\ln\left(\frac{1-x}{x+1}\right) - x + \tanh^{-1} x + C$; 1.37. $\frac{1}{3}x^3\arctan 3x - \frac{1}{18}x^2 + \frac{1}{162}\ln\left(x^2 + \frac{1}{9}\right) + C$;

1.38. $\frac{5^x}{\ln^2 5 + 25}((\ln 5)\operatorname{sen} 5x - 5\cos 5x) + C$; 1.39. $x\left(\ln^2 x - 2\ln x + 2\right) + C$; 1.40. $2e^{\sqrt{x}}\left(\sqrt{x} - 1\right) + C$;

1.41. $\frac{e^{ax}}{a^2+b^2}\left(a\operatorname{sen} bx - b\cos bx\right)+C$; 1.42. $\arctan x - 2x + x\ln\left(x\sqrt{x^2+1}\right)+C$; 1.43. $\frac{1}{2}x\left(\operatorname{sen}\left(\ln x\right)-\cos\left(\ln x\right)\right)+C$;

1.44. $-\frac{1}{2}e^{-y^2}\left(1+y^2\right)+C$; 1.45. $-x\csc x+\ln\left|\csc x-\cot x\right|+C$; 1.46. $\frac{3^x}{\ln^2 3+1}\left(\operatorname{sen} x+\left(\ln 3\right)\cos x\right)+C$;

1.47. $\left(1-x^2+\frac{1}{2}x^4\right)e^{x^2}+C$; 1.48. $-\frac{1}{t}\left(2+2\ln t+\ln^2 t\right)+C$; 1.49. $\left(\ln\left(\ln x\right)-1\right)\ln x+C$;

1.50. $\frac{1}{2}x^2-3x-\frac{1}{9}x^3+\left(3x-x^2+\frac{1}{3}x^3\right)\ln x+C$; 1.51. $\left(6t-6-3t^2+t^3\right)e^t+C$; 1.52. $\frac{1}{2}x\sqrt{x^2+1}+\frac{1}{2}\operatorname{senh}^{-1}x+C$;

1.53. $\frac{x}{2}\tan 2x-\frac{1}{4}\ln\left|\sec 2x\right|-\frac{x^2}{2}+C$; 1.54. $\frac{x^2+1}{2}\arctan^2 x - x\arctan x+\frac{1}{2}\ln\left(x^2+1\right)+C$; 1.55. $-\left(\frac{1}{2}+\ln x\right)\frac{1}{2x^2}+C$;

1.56. $2\sqrt{\theta}-2\sqrt{1-\theta}\operatorname{arcsen}\sqrt{\theta}+C$; 1.57. $e^{-x}\left(\frac{1}{10}\cos 2x-\frac{1}{5}\operatorname{sen} 2x-\frac{1}{2}\right)+C$;

1.58. $\frac{1}{2}\operatorname{sen} x+\frac{3}{35}\cos x\operatorname{sen} 6x-\frac{1}{70}\operatorname{sen} x\cos 6x+C$; 1.59. $-x\cot x+\ln\left|\operatorname{sen} x\right|+C$; 1.60. $\frac{1}{2}\arctan x-\frac{x}{2}+\frac{x^2}{2}\arctan x+C$;

1.61. $\frac{1}{2}x+\frac{1}{10}x\cos\left(2\ln x\right)+\frac{1}{5}x\operatorname{sen}\left(2\ln x\right)+C$; 1.62. $\left(\ln\left(\operatorname{sen} t\right)-1\right)\operatorname{sen} t+C$; 1.63. $x\left(\ln^2 x-2\ln x+2\right)+C$;

1.64. $2\operatorname{sen}\sqrt{x}-2\sqrt{x}\cos\sqrt{x}+C$; 1.65. $\frac{2}{9}x\cos 3x-\frac{2}{27}\operatorname{sen} 3x+\frac{1}{3}x^2\operatorname{sen} 3x+C$; 1.66. $\frac{1}{3}\operatorname{sen} x-\frac{1}{9}\operatorname{sen}^3 x-\frac{x}{3}\cos^3 x+C$;

1.67. $\frac{1}{4}\tan\theta\sec^3\theta+\frac{3}{8}\sec\theta\tan\theta+\frac{3}{8}\ln\left|\sec\theta+\tan\theta\right|+C$; 1.68. $\frac{1}{4}\sec\left(x^2\right)\tan\left(x^2\right)+\frac{1}{4}\ln\left|\sec\left(x^2\right)+\tan\left(x^2\right)\right|+C$;

1.69. $\frac{e^x}{x+1}+C$; 1.70. $x\operatorname{arcsen}^2 x-2x+2\sqrt{1-x^2}\operatorname{arcsen} x+C$; 1.71. $\left(4\ln x-1\right)\frac{x^4}{16}+C$;

1.72. $\frac{1}{16}\operatorname{sen} 4t-\frac{1}{4}t\cos 4t+C$; 1.73. $\frac{1}{4}\cos 2x+\frac{1}{2}x\operatorname{sen} 2x-\frac{1}{2}x^2\cos 2x+C$;

1.74. $\frac{1}{4a}\tan\left(ax+b\right)\sec^3\left(ax+b\right)+\frac{3}{8a}\sec\left(ax+b\right)\tan\left(ax+b\right)+\frac{3}{8a}\ln\left|\sec\left(ax+b\right)+\tan\left(ax+b\right)\right|+C$;

1.75. $5^x\left(\frac{x}{\ln 5}-\frac{1}{\ln^2 5}\right)+C$; 1.76. $\theta\tan\theta-\ln\left|\sec\theta\right|+C$; 1.77. ;

1.78. $\frac{1}{2a}\sec\left(ax+b\right)\tan\left(ax+b\right)+\frac{1}{2a}\ln\left|\sec\left(ax+b\right)+\tan\left(ax+b\right)\right|+C$; 1.79. $\frac{1}{4}\cos 2z+\frac{1}{2}z\operatorname{sen} 2z+C$;

1.80. $\frac{1}{4}x^2-\frac{1}{8}\cos 2x-\frac{1}{4}x\operatorname{sen} 2x+C$; 1.81. $\frac{e^{-\theta}}{10}\left(3\operatorname{sen} 3\theta-\cos 3\theta\right)+C$; 1.82. $a^x\left(\frac{x}{\ln a}-\frac{1}{\ln^2 a}\right)+C$;

1.83. $-2\sqrt{1-x}\left(\ln x\right)-4\tanh^{-1}\sqrt{1-x}+4\sqrt{1-x}+C$; 1.84. $z\arccos z-\sqrt{1-z^2}+C$;

1.85. $-\frac{1}{3}\operatorname{sen} 2t\cos 4t+\frac{1}{6}\cos 2t\operatorname{sen} 4t+C$; 1.86. $7\cos^2 t\left(\frac{1}{2}-\ln\left|\cos t\right|\right)+C$; 1.87. $\left(1-x\right)\sqrt{1-e^{2x}}+\frac{1}{2}\ln\left|\frac{\sqrt{1-e^{2x}}-1}{\sqrt{1-e^{2x}}+1}\right|+C$;

1.88. $\frac{1}{32}t-\frac{1}{24}t^3-\frac{1}{64}\arctan 2t+\frac{1}{4}t^4\arctan 2t+C$; 1.89. $-\sqrt{1-x^2}\operatorname{arcsen} x+x+C$; 1.90. $\frac{\operatorname{arcsen} x}{\sqrt{1-x^2}}-\tanh^{-1}x+C$;

1.91. $\left(1-\ln x\right)\sqrt{1-x^2}+\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right|+C$; 1.92. $2\operatorname{sen}^2 x\left(2\ln\left|\operatorname{sen} x\right|-1\right)+5\cos^2 x\left(\frac{1}{2}-\ln\left|\cos x\right|\right)+C$;

1.93. $\frac{1}{2}\cos^2 x\left(\frac{1}{2}-\ln\left|\cos x\right|\right)+\frac{1}{3}\operatorname{sen}^2 x\left(\frac{1}{2}-\ln\left|\operatorname{sen} x\right|\right)+C$; 1.94. $\frac{1}{a}\operatorname{sen}^2\left(ax\right)\ln\left|\tan\left(ax\right)\right|+\frac{1}{a}\ln\left|\cos\left(ax\right)\right|+C$;

1.95. $5\operatorname{sen}^2 x\left(\ln\left|\operatorname{sen} x\right|-\frac{1}{2}\right)+C$; 1.96. $-2\ln\left|\cos\left(\frac{x}{2}\right)\right|+2\operatorname{sen}^2\left(\frac{x}{2}\right)\ln\left|\cot\left(\frac{x}{2}\right)\right|+C$;

1.97. $\frac{1}{2}\left(\operatorname{sen} x+\cos x\right)^2\left(\ln\left(\operatorname{sen} x+\cos x\right)-\frac{1}{2}\right)+C$; 1.98. $\operatorname{sen} x\left(\ln\left|\frac{\cos^3 x}{\operatorname{sen}^2 x}\right|-1\right)-3\ln\left|\sec x-\tan x\right|+C$;

1.99. $4\ln\left|\csc x-\cot x\right|+9\cos x-5\cos x\ln\left(\operatorname{sen}^4 x\cos^5 x\right)+C$; 1.100. $\frac{3}{4}\operatorname{arcsen}^5 x\left(\ln\left|\operatorname{arcsen} x-\frac{1}{5}\right|\right)+C$;

1.101. $-\frac{m+n}{b}\operatorname{sen} bx+\frac{1}{b}\ln\left(\operatorname{sen}^n bx\cos^m bx\right)\operatorname{sen} bx-\frac{m}{b}\ln\left|\frac{1-\operatorname{sen} bx}{\cos bx}\right|+C$;

1.102. $\frac{m+n}{b}\cos bx-\frac{1}{b}\ln\left(\operatorname{sen}^n bx\cos^m bx\right)\cos bx+\frac{n}{b}\ln\left|\frac{1-\cos bx}{\operatorname{sen} bx}\right|+C$; 1.103. $\frac{1}{8}\operatorname{senh}\left(2x^2\right)-\frac{x^2}{4}+C$; 1.104. $\frac{\operatorname{senh} 2}{2}+1$;

1.105. $2\sqrt{x}\cosh\sqrt{x}-2\operatorname{senh}\sqrt{x}+C$; 1.106. $\frac{3^t}{9-\ln^2 3}\left(3\cosh 3t-\left(\ln 3\right)\operatorname{senh} 3t\right)+C$; 1.107. $\frac{e^{at}}{b^2-a^2}\left(b\operatorname{senh} bt-a\cosh bt\right)+C$;

1.108. $\frac{x^3}{3}\operatorname{senh}\left(x^3\right)-\frac{1}{3}\cosh\left(x^3\right)+C$; 1.109. $-\frac{1}{2}\csc x\cot x+\frac{1}{2}\ln\left|\csc x-\cot x\right|+C$; 1.110. $-\frac{1}{3}\sqrt{1-x^2}\left(2+x^2\right)+C$;

1.111. $-\frac{1}{15}\sqrt{1-x^2}\left(4x^2+3x^4+8\right)+C$; 1.112. $-\frac{1}{35}\sqrt{1-x^2}\left(8x^2+6x^4+5x^6+16\right)+C$;