Calcula estas integrales por partes.

a) $\int x^3 \ln x\, dx$
b) $\int \ln(2x+1)\, dx$
c) $\int e^{-x}\, sen\, 2x\, dx$
d) $\int arc\, tg\, x\, dx$
e) $\int \frac{\ln x}{x}\, dx$
f) $\int x\, sen\, 2x\, dx$
g) $\int x^2\, sen\, 2x\, dx$
h) $\int (2x+3)e^{2x}\, dx$
i) $\int \frac{x}{e^x}\, dx$
j) $\int (x^2-5)\cos x\, dx$
k) $\int (2x^2+x-2)e^{3x}\, dx$
l) $\int (2+e^{2x})\cos(x+1)\, dx$

a) $$\int x^3 \ln x\, dx = \frac{x^4 \ln x}{4} - \int \frac{x^3}{4}\, dx = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + k = \frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + k$$

$u = \ln x \rightarrow du = \frac{1}{x}\, dx$

$dv = x^3\, dx \rightarrow v = \frac{x^4}{4}$

b) $$\int \ln(2x+1)\, dx = x \ln|2x+1| - \int \frac{2x}{2x+1}\, dx = x \ln|2x+1| - \int \left(1 - \frac{1}{2x+1}\right) dx =$$

$u = \ln(2x+1) \rightarrow du = \frac{2}{2x+1}\, dx$

$dv = dx \rightarrow v = x$

$$= x \ln|2x+1| - x + \frac{\ln|2x+1|}{2} + k = \left(x + \frac{1}{2}\right)\ln|2x+1| - x + k$$

c) $$\int e^{-x} sen\, 2x\, dx = -e^{-x}\frac{\cos 2x}{2} - \frac{1}{2}\int e^{-x}\cos 2x\, dx =$$

$u = e^{-x} \rightarrow du = -e^{-x}dx$

$dv = sen\, 2x\, dx \rightarrow v = \frac{-\cos 2x}{2}$

$u = e^{-x} \rightarrow du = -e^{-x}dx$

$dv = \cos 2x\, dx \rightarrow v = \frac{sen\, 2x}{2}$

$$= -e^{-x}\frac{\cos 2x}{2} - \frac{1}{2}\left(e^{-x}\frac{sen\, 2x}{2} + \frac{1}{2}\int e^{-x} sen\, 2x\, dx\right)\int e^{-x} sen\, 2x\, dx =$$

$$= -e^{-x}\frac{\cos 2x}{2} - e^{-x}\frac{sen\, 2x}{4} - \frac{1}{4}\int e^{-x} sen\, 2x\, dx$$

$$\frac{5}{4}\int e^{-x} sen\, 2x\, dx = -e^{-x}\left(\frac{\cos 2x}{2} + \frac{sen\, 2x}{4}\right)$$

$$\rightarrow \int e^{-x} sen\, 2x\, dx = \frac{-e^{-x}}{5}(2\cos 2x + sen\, 2x) + k$$

d) $$\int arc\,tg\,x\,dx = x \cdot arc\,tg\,x - \int \frac{x}{x^2+1}\,dx = x \cdot arc\,tg\,x - \frac{\ln|x^2+1|}{2} + k$$

$u = arc\,tg\,x \rightarrow du = \frac{1}{x^2+1}\,dx$

$dv = dx \rightarrow v = x$

e) $$\int \frac{\ln x}{x}\,dx = \ln^2 x - \int \frac{\ln x}{x}\,dx \rightarrow 2\int \frac{\ln x}{x}\,dx = \ln^2 x \rightarrow \int \frac{\ln x}{x}\,dx = \frac{\ln^2 x}{2} + k$$

$u = \ln x \rightarrow du = \frac{1}{x}\,dx$

$dv = \frac{1}{x}\,dx \rightarrow v = \ln x$

f) $$\int x\,sen\,2x\,dx = \frac{-x\cos 2x}{2} + \int \frac{\cos 2x}{2}\,dx = -\frac{x\cos 2x}{2} + \frac{sen\,2x}{4} + k$$

$u = x \rightarrow du = dx$

$dv = sen\,2x\,dx \rightarrow v = \frac{-\cos 2x}{2}$

g) $$\int x^2\,sen\,2x\,dx = \frac{-x^2\cos 2x}{2} + \int x\cos 2x\,dx =$$

$u = x^2 \rightarrow du = 2x\,dx$

$dv = sen\,2x\,dx \rightarrow v = \frac{-\cos 2x}{2}$

$u = x \rightarrow du = dx$

$dv = \cos 2x\,dx \rightarrow v = \frac{sen\,2x}{2}$

$$= \frac{-x^2\cos 2x}{2} + \frac{x\,sen\,2x}{2} - \int \frac{sen\,2x}{2}\,dx = \frac{-x^2\cos 2x}{2} + \frac{x\,sen\,2x}{2} + \frac{\cos 2x}{4} + k$$

h) $$\int (2x+3)e^{2x}\,dx = \frac{(2x+3)e^{2x}}{2} - \int e^{2x}\,dx = \frac{(2x+3)e^{2x}}{2} - \frac{e^{2x}}{2} + k = (x+1)e^{2x} + k$$

$u = 2x+3 \rightarrow du = 2\,dx$

$dv = e^{2x}dx \rightarrow v = \frac{e^{2x}}{2}$

i) $$\int \frac{x}{e^x}\,dx = \frac{x}{e^x} + \int e^{-x}\,dx = \frac{x}{e^x} - \frac{1}{e^x} + k = \frac{x-1}{e^x} + k$$

$u = x \rightarrow du = dx$

$dv = e^{-x}dx \rightarrow v = -e^{-x}$

j) $$\int (x^2-5)\cos x\,dx = (x^2-5)\,sen\,x - 2\int x\,sen\,x\,dx =$$

$u = x^2-5 \rightarrow du = 2x\,dx$

$dv = \cos x\,dx \rightarrow v = sen\,x$

$u = x \rightarrow du = dx$

$dv = sen\,x\,dx \rightarrow v = -\cos x$

$$= (x^2-5)\,sen\,x + 2x\cos x - 2\int \cos x\,dx = (x^2-5)\,sen\,x + 2x\cos x - 2\,sen\,x + k$$

k) $\displaystyle\int (2x^2+x-2)e^{3x}dx = \frac{(2x^2+x-2)e^{3x}}{3} - \int \frac{(4x+1)e^{3x}}{3}\,dx = \frac{(2x^2+x-2)e^{3x}}{3} -$

$u = 2x^2+x-2 \to du = (4x+1)\,dx$

$dv = e^{3x}dx \to v = \frac{e^{3x}}{3}$

$u = 4x+1 \to du = 4\,dx$

$dv = e^{3x}dx \to v = \frac{e^{3x}}{3}$

$$-\frac{1}{3}\left(\frac{(4x+1)e^{3x}}{3} - \int \frac{4e^{3x}}{3}\,dx\right) = \frac{(2x^2+x-2)e^{3x}}{3} - \frac{(4x+1)e^{3x}}{9} + \frac{4}{27}e^{3x} + k$$

l) $\displaystyle\int (2+e^{2x})\cos(x+1)\,dx = \int 2\cos(x+1)\,dx + \int e^{2x}\cos(x+1)\,dx =$

$$= 2\,\mathrm{sen}(x+1) + e^{2x}\mathrm{sen}(x+1) - \int 2e^{2x}\mathrm{sen}(x+1)\,dx =$$

$u = e^{2x} \to du = 2e^{2x}dx$

$dv = \mathrm{sen}\,(x+1)\,dx \to v = -\cos\,(x+1)$

$$= 2\,\mathrm{sen}\,(x+1) + e^{2x}\mathrm{sen}\,(x+1) + 2e^{2x}\cos\,(x+1) - 2\int 2e^{2x}\cos\,(x+1) =$$

$$= 2\,\mathrm{sen}\,(x+1) + \frac{e^{2x}\mathrm{sen}\,(x+1) + 2e^{2x}\cos\,(x+1)}{5} + k$$

Aplicar el método de integración por partes para calcular las siguientes primitivas.

a) $I = \displaystyle\int x\,e^{2x}\,dx$ b) $J = \displaystyle\int x \ln x\,dx$

a) $\displaystyle\int x e^{2x}dx = \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + k$

$u = x \to du = dx$

$dv = e^{2x}dx \to v = \frac{e^{2x}}{2}$

b) $\displaystyle\int x \ln x\,dx = \frac{x^2}{2}\cdot \ln x - \int \frac{x}{2}\,dx = \frac{x^2}{2}\cdot \ln x - \frac{x^2}{4} + k$

$u = \ln x \to du = \frac{1}{x}dx$

$dv = x\,dx \to v = \frac{x^2}{2}$

Utilizando el método de integración por partes, calcule $I = \int \ln x \, dx$.

$$I = \int \ln x \, dx = x \ln x - \int dx = x \ln x - x + k$$

$u = \ln x \rightarrow du = \frac{1}{x} dx$

$dv = dx \rightarrow v = x$

Calcula $\int (x^2 - 1)e^{-x} \, dx$.

$$\int (x^2 - 1)e^{x} dx = -(x^2 - 1)e^{-x} + \int 2x \, e^{-x} dx = -(x^2 - 1)e^{-x} - 2x \, e^{-x} + \int 2 \, e^{-x} dx =$$

$u = x^2 - 1 \rightarrow du = 2x \, dx$

$dv = e^{-x} dx \rightarrow v = -e^{-x}$

$u = 2x \rightarrow du = 2 \, dx$

$dv = e^{-x} dx \rightarrow v = -e^{-x}$

$$= -(x^2 - 1)e^{-x} - 2x \, e^{-x} - 2e^{-x} + k = (-x^2 - 2x - 1)e^{-x} + k$$

Calcular la primitiva siguiente: $\int \ln (25 + x^2) \, dx$

$$\int \ln (25 + x^2) \, dx = x \ln (25 + x^2) - \int \frac{2x^2}{25 + x^2} dx =$$

$u = \ln (25 + x^2) \rightarrow du = \frac{2x}{25 + x^2} dx$

$dv = dx \rightarrow v = x$

$$= x \ln (25 + x^2) - \int \left(2 - \frac{50}{25 + x^2} \right) dx = x \ln (25 + x^2) - \int 2 \, dx + \int \frac{50}{25 + x^2} dx =$$

$$= x \ln (25 + x^2) - 2x + 10 \, arc \, tg \frac{x}{5} + k$$

Halla una función primitiva de:

$$f(x) = \frac{1}{x} + \ln x$$

que pase por el punto $P(e, 2)$.

$$F(x) = \int \left(\frac{1}{x} - \ln x \right) dx = \ln |x| - x \ln |x| + x + k$$

$$F(e) = 2 \rightarrow 1 - e + e + k = 2 \rightarrow k = 1 \rightarrow F(x) = \ln |x| - x \ln |x| + x + 1$$

En cada uno de los siguientes casos, obtén una función $f(x)$ que cumpla las condiciones que se señalan:

a) $f'(x) = x^2 \text{ sen } x,\ f(0) = -1$

b) $f'(x) = x \ln x,\ f(1) = \frac{1}{2}$

a) $$\int x^2 \text{sen}\, x\, dx = -x^2 \cos x + 2\int x \cos x\, dx = -x^2 \cos x + 2x\,\text{sen}\, x + 2\cos x + k =$$

$u = x^2 \rightarrow du = 2x\, dx$

$dv = \text{sen}\, x\, dx \rightarrow v = -\cos x$

$u = x \rightarrow du = dx$

$\cos x\, dv = dx \rightarrow v = \text{sen}\, x$

$$= (2 - x^2)\cos x + 2x\,\text{sen}\, x + k$$

$$f(0) = -1 \rightarrow 2 + k = -1 \rightarrow k = -3 \rightarrow f(x) = (1 - x^2)\cos x + x\,\text{sen}\, x - 3$$

b) $$\int x \ln x\, dx = \frac{x^2 \ln x}{2} - \int \frac{x}{2} dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + k$$

$u = \ln x \rightarrow du = \frac{1}{x} dx$

$dv = x\, dx \rightarrow v = \frac{x^2}{2}$

$$f(1) = \frac{1}{2} \rightarrow -\frac{1}{4} + k = \frac{1}{2} \rightarrow k = \frac{3}{4} \rightarrow f(x) = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + \frac{3}{4}$$

Sea $f\colon \mathbb{R} \rightarrow \mathbb{R}$ la función definida por $f'(x) = x^2 \text{ sen } 2x$. Calcula la primitiva de f cuya gráfica pasa por el punto (0, 1).

$$\int x^2 \text{sen}\, 2x\, dx = -\frac{x^2 \cos 2x}{2} + \int x \cos 2x\, dx = -\frac{x^2 \cos 2x}{2} + \frac{x\,\text{sen}\, 2x}{2} - \int \frac{\text{sen}\, 2x}{2} dx =$$

$u = x^2 \rightarrow du = 2x\, dx$

$dv = \text{sen}\, 2x\, dx \rightarrow v = -\frac{\cos 2x}{2}$

$u = x \rightarrow du = dx$

$dv = \cos 2x\, dx \rightarrow v = \frac{\text{sen}\, 2x}{2}$

$$= -\frac{x^2 \cos 2x}{2} + \frac{x\,\text{sen}\, 2x}{2} + \frac{\cos 2x}{4} = \frac{(1 - 2x^2)\cos 2x}{4} + \frac{x\,\text{sen}\, 2x}{2} + k$$

$$f(0) = 1 \rightarrow \frac{1}{4} + k = 1 \rightarrow k = \frac{3}{4} \rightarrow f(x) = \frac{(1 - 2x^2)\cos 2x}{4} + \frac{x\,\text{sen}\, 2x}{2} + \frac{3}{4}$$