Radicales

49. Simplifica:

a) $\sqrt[5]{2\sqrt[3]{2\sqrt{2}}} = \sqrt[5]{2}\cdot\sqrt[15]{2}\cdot\sqrt[30]{2} = 2^{1/5}\cdot 2^{1/15}\cdot 2^{1/30} = 2^{\frac{1}{5}+\frac{1}{15}+\frac{1}{30}} =$

$= 2^{\frac{9}{30}} = 2^{\frac{3}{10}} = \sqrt[10]{2^3}$

Reducimos a índice común para multiplicar, m.c.m(2,6,30) = 30

b) $\sqrt{12\cdot\sqrt[3]{15\cdot\sqrt[5]{20}}} = \sqrt[2]{3\cdot 2^2}\cdot\sqrt[6]{3\cdot 5}\cdot\sqrt[30]{2^2\cdot 5} = \sqrt[30]{3^{15}\cdot 2^{30}\cdot 3^5\cdot 5^5\cdot 2^2\cdot 5} =$

$= \sqrt[30]{3^{20}\cdot 2^{32}\cdot 5^6} = 2\cdot\sqrt[30]{2^2\cdot 3^{20}\cdot 5^6} = 2\cdot\sqrt[15]{2\cdot 3^{10}\cdot 5^3}$

Simplificamos índice y exponentes : 2.

50 Reduce a un único radical:

a) $\sqrt{\sqrt[3]{729}} = \sqrt[6]{3^6} = 3$; b) $\sqrt[3]{\sqrt{4096\cdot x^{12}\cdot y^{18}}} = \sqrt[6]{2^{12}\cdot x^{12}\cdot y^{18}} = 2^2\cdot x^2\cdot y^3$; c) $\sqrt[3]{\sqrt[3]{512}} = \sqrt[9]{2^9} = 2$

51b. Simplifica:

$$\sqrt{\frac{\sqrt{x^3\cdot\sqrt{x^5}}}{\sqrt{x^5\cdot\sqrt{x^3}}}} = \frac{\sqrt[2]{\sqrt[2]{x^3}}\cdot\sqrt{\sqrt{\sqrt{x^5}}}}{\sqrt{\sqrt{x^5}}\cdot\sqrt[2]{\sqrt[2]{\sqrt[2]{x^3}}}} = \frac{\sqrt[4]{x^3}\cdot\sqrt[8]{x^5}}{\sqrt[4]{x^5}\cdot\sqrt[8]{x^3}} = \frac{x^{3/4}\cdot x^{5/8}}{x^{5/4}\cdot x^{3/8}} = \frac{x^{\frac{3}{4}+\frac{5}{8}}}{x^{\frac{5}{4}+\frac{3}{8}}} =$$

$2\cdot 2 = 4$; $2\cdot 2\cdot 2$

$$= \frac{x^{\frac{11}{8}}}{x^{\frac{13}{8}}} = x^{-2/8} = x^{-1/4} = \boxed{\frac{1}{\sqrt[4]{x}}}$$

$3\,|\,2 \rightarrow$ expon. sale
$1 \rightarrow$ expon. dentro

52b $2\cdot\sqrt{75} + 7\cdot\sqrt{27} - \sqrt{48} = 2\cdot\sqrt{5^2\cdot 3} + 7\cdot\sqrt[2]{3^3} - \sqrt{3\cdot 2^4} =$

$= 2\cdot 5\cdot\sqrt{3} + 7\cdot 3^1\cdot\sqrt{3^1} - 2^2\sqrt{3} = (10+21-4)\sqrt{3} = \boxed{27\sqrt{3}}$

d) $3\sqrt{32} - 5\sqrt{98} + \sqrt{18} = 3\cdot\sqrt{2^5} - 5\cdot\sqrt{2\cdot 7^2} + \sqrt{2\cdot 3^2} =$

$= 3\cdot 2^2\cdot\sqrt{2} - 5\cdot 7\cdot\sqrt{2} + 3\cdot\sqrt{2} = (12-35+3)\sqrt{2} = \boxed{-20\sqrt{2}}$

53b) $\dfrac{2\cdot\sqrt[5]{\sqrt[3]{2}+\sqrt[3]{16}}}{3\cdot\sqrt[5]{128}\cdot\sqrt[3]{4}} = \dfrac{2\cdot\sqrt[5]{\sqrt[3]{2}+2\cdot\sqrt[3]{2}}}{3\cdot\sqrt[5]{2^7}\cdot\sqrt[3]{2^2}} = \dfrac{2\cdot\sqrt[5]{3\cdot\sqrt[3]{2}}}{3\cdot 2\sqrt[5]{2^2}\cdot\sqrt[3]{2^2}} =$

$\sqrt[3]{2^4}=2\cdot\sqrt[3]{2}$

$\begin{array}{c|c} 7 & 5 \\ 2 & 1 \end{array}$

$= \dfrac{\sqrt[5]{\sqrt[3]{3^3\cdot 2}}}{3\cdot\sqrt[15]{2^6\cdot 2^{10}}} = \dfrac{\sqrt[15]{3^3\cdot 2}}{3\cdot\sqrt[15]{2^{16}}} = \dfrac{1}{3}\cdot\sqrt[15]{\dfrac{3^3\cdot 2}{2^{16}}} = \dfrac{1}{3}\cdot\sqrt[15]{\dfrac{3^3}{2^{15}}} = \dfrac{1}{3\cdot 2}\cdot\sqrt[15]{3^3} = \boxed{\dfrac{\sqrt[5]{3}}{6}}$

$5\cdot 3$

53c) $\sqrt[5]{\dfrac{\sqrt{5}+3\sqrt{20}}{7\sqrt[4]{5}}} = \sqrt[5]{\dfrac{\sqrt{5}+3\sqrt{2^2\cdot 5}}{7\sqrt[4]{5}}} = \sqrt[5]{\dfrac{\sqrt{5}+3\cdot 2\sqrt{5}}{7\sqrt[4]{5}}} = \sqrt[5]{\dfrac{\cancel{7}\sqrt{5}}{\cancel{7}\sqrt[4]{5}}} =$

$= \sqrt[5]{\sqrt[4]{\dfrac{5^2}{5}}} = \boxed{\sqrt[20]{5}}$

54d) Racionaliza: $\dfrac{7}{5\cdot\sqrt[7]{3^4}} = \dfrac{7}{5\cdot\sqrt[7]{3^4}}\cdot\dfrac{\sqrt[7]{3^3}}{\sqrt[7]{3^3}} = \dfrac{7\cdot\sqrt[7]{3^3}}{5\cdot\sqrt[7]{\cancel{3^7}}} = \dfrac{7\sqrt[7]{3^3}}{15}$

$7-4$

$3^4\cdot 3^3 = 3^{4+3} = 3^7$

57c) Opera y racionaliza: $\dfrac{7\cdot\sqrt{2}}{3\sqrt{2}\cdot\sqrt{2}} - \dfrac{3\cdot\sqrt{5}}{2\sqrt{5}\cdot\sqrt{5}} = \dfrac{7\sqrt{2}}{3\cancel{\sqrt{2^2}}} - \dfrac{3\sqrt{5}}{2\cancel{\sqrt{5^2}}} =$

$= \dfrac{7\sqrt{2}}{6} - \dfrac{3\cdot\sqrt{5}}{10} = \dfrac{35\sqrt{2}-9\sqrt{5}}{30}$

58a) Racionaliza $\dfrac{\sqrt{2}-\sqrt{3}}{-\sqrt{2}-\sqrt{3}} = \dfrac{\sqrt{2}-\sqrt{3}}{\underbrace{-\sqrt{2}-\sqrt{3}}}\cdot\dfrac{\overbrace{-\sqrt{2}+\sqrt{3}}^{-(\sqrt{2}-\sqrt{3})}}{\underbrace{-\sqrt{2}+\sqrt{3}}} = \dfrac{(\sqrt{2}-\sqrt{3})\cdot(\sqrt{2}-\sqrt{3})\cdot(-1)}{(-\sqrt{2})^2-\sqrt{3}^2} =$

$= \dfrac{-\left(\sqrt{2}^2+\sqrt{3}^2-2\cdot\sqrt{2}\cdot\sqrt{3}\right)}{2-3} = \dfrac{-\left(2+3-2\sqrt{6}\right)}{-1} = 5-2\sqrt{6}$

$$\text{(58b)}\quad \frac{a}{\sqrt{a+\sqrt{b}}}=\frac{a}{\sqrt{a+\sqrt{b}}}\cdot\frac{\sqrt{a+\sqrt{b}}}{\sqrt{a+\sqrt{b}}}=\frac{a\cdot\sqrt{a+\sqrt{b}}}{a+\sqrt{b}}\cdot\frac{a-\sqrt{b}}{a-\sqrt{b}}=$$

$$\cancel{\left(\sqrt{a+\sqrt{b}}\right)^{2}}$$

$$=\frac{a\cdot(a-\sqrt{b})\cdot\sqrt{a+\sqrt{b}}}{a^2-\sqrt{b}^2}=\frac{a\cdot(a-\sqrt{b})\cdot\sqrt{a+\sqrt{b}}}{a^2-b}$$

$$\text{(58d)}\quad \sqrt[3]{\frac{2\sqrt{5}}{\sqrt{5}+3\sqrt{20}}}=\sqrt[3]{\frac{2\sqrt{5}\cdot(\sqrt{5}-3\sqrt{20})}{(\sqrt{5}+3\sqrt{20})(\sqrt{5}-3\sqrt{20})}}=\sqrt[3]{\frac{2\cdot\sqrt{5}^2-6\sqrt{100}}{\sqrt{5}^2-3^2\sqrt{20}^2}}=$$

$$=\sqrt[3]{\frac{2\cdot5-6\cdot10}{5-9\cdot20}}=\sqrt[3]{\frac{-50}{-175}}=\sqrt[3]{\frac{2}{7}}=\frac{\sqrt[3]{2}}{\sqrt[3]{7^1}}\cdot\frac{\sqrt[3]{7^2}}{\sqrt[3]{7^{\boxed{2}}}}=\frac{\sqrt[3]{2\cdot7^2}}{7}$$

$$3-1 \qquad \sqrt[3]{7^{\cancel{3}}}$$

2ª forma:

$$\sqrt[3]{\frac{2\sqrt{5}}{\sqrt{5}+3\underset{\sqrt{2^2\cdot5}}{\underset{\parallel}{\sqrt{20}}}}}=\sqrt[3]{\frac{2\sqrt{5}}{\sqrt{5}+6\sqrt{5}}}=\sqrt[3]{\frac{2\cancel{\sqrt{5}}}{7\cancel{\sqrt{5}}}}=\sqrt[3]{\frac{2}{7}}=$$

1º Agrupo los nºs en un binomio para poder aplicar caso C.

$$\text{(125a) Racionaliza: }\frac{2}{3+\sqrt{5}-\sqrt{3}}=\frac{2}{(3+\sqrt{5})-\sqrt{3}}\cdot\frac{(3+\sqrt{5})+\sqrt{3}}{(3+\sqrt{5})+\sqrt{3}}=$$

Vuelvo a aplicar caso C

$$=\frac{2\cdot(3+\sqrt{5}+\sqrt{3})}{(3+\sqrt{5})^2-\sqrt{3}^2}=\frac{2\cdot(3+\sqrt{5}+\sqrt{3})}{9+5+6\sqrt{5}-3}=\frac{2\cdot(3+\sqrt{5}+\sqrt{3})}{11+6\sqrt{5}}\cdot\frac{11-6\sqrt{5}}{11-6\sqrt{5}}=$$

$$=\frac{(6+2\sqrt{5}+2\sqrt{3})(11-6\sqrt{5})}{11^2-6^2\cdot\sqrt{5}^2}=\frac{66\underbrace{-36\sqrt{5}+22\sqrt{5}}\overbrace{-12\cdot5}^{-60}+22\sqrt{3}-12\sqrt{15}}{121-36\cdot5}=$$

$$=\boxed{\frac{6-14\cdot\sqrt{5}+22\sqrt{3}-12\sqrt{15}}{-59}}$$