42. Calcula el ángulo que forman las rectas:

a) r: $3x-4y=0$ y s: $2x+2y+3=0$

b) r: $y=x-5$ y s: $y=2x+2$

OBSERVACIÓN

$$\cos(\widehat{r,s}) = \left|\cos(\widehat{\vec{v}_r,\vec{v}_s})\right| = \left|\cos(\widehat{\vec{n}_r,\vec{n}_s})\right|$$

¡¡ $(\widehat{\vec{v}_r,\vec{v}_s}) = (\widehat{\vec{n}_r,\vec{n}_s})$!!

a) $r: 3x-4y=0 \longrightarrow \vec{n}_r=(3,-4)$

$s: 2x+2y+3=0 \longrightarrow \vec{n}_s=(2,2)$

$$\cos(\widehat{r,s}) = \left|\cos(\widehat{\vec{n}_r,\vec{n}_s})\right| = \frac{|\vec{n}_r\cdot\vec{n}_s|}{|\vec{n}_r|\cdot|\vec{n}_s|} = \frac{|(3,-4)\cdot(2,2)|}{\sqrt{3^2+(-4)^2}\cdot\sqrt{2^2+2^2}} =$$

$$= \frac{|3\cdot 2+(-4)\cdot 2|}{5\cdot\sqrt{8}} = \frac{|-2|}{5\cdot 2\sqrt{2}} = \frac{\cancel{2}\,1}{\cancel{10}_5\sqrt{2}} = \frac{1}{5\sqrt{2}}$$

$$\cos(\widehat{r,s}) = \frac{1}{5\sqrt{2}} \Rightarrow (\widehat{r,s}) = \arccos\frac{1}{5\sqrt{2}} = \boxed{81,87^\circ}$$

b) $r: y=x-5 \longrightarrow x-y-5=0 \longrightarrow \vec{n}_r=(1,-1)$

$s: y=2x+2 \longrightarrow 2x-y+2=0 \longrightarrow \vec{n}_s=(2,-1)$

$$\cos(\widehat{r,s}) = \frac{|(1,-1)\cdot(2,-1)|}{\sqrt{1^2+(-1)^2}\cdot\sqrt{2^2+(-1)^2}} = \frac{|2+1|}{\sqrt{2}\cdot\sqrt{5}} = \frac{3}{\sqrt{10}}$$

$$(\widehat{r,s}) = \arccos\frac{3}{\sqrt{10}} = 18,43^\circ$$

43. Calcula el ángulo formado por r: $4x + 2y - 7 = 0$ y el eje Y.

$r: 4x+2y-7=0 \longrightarrow \vec{n}_r = (4,2)$

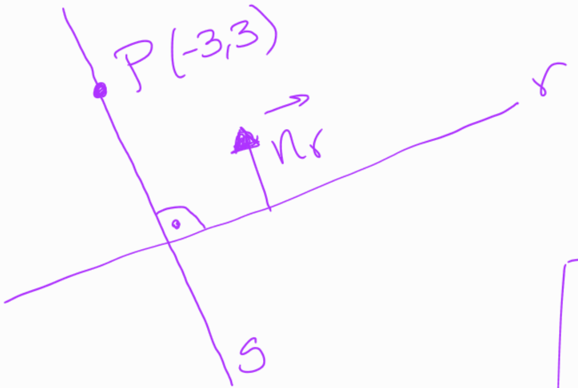
$s: OY \longrightarrow x=0 \longrightarrow \vec{n}_{OY} = (1,0)$

$$\cos(\widehat{r,OY}) = \left|\cos(\widehat{\vec{n}_r,\vec{n}_{OY}})\right| = \frac{|(4,2)\cdot(1,0)|}{\sqrt{4^2+2^2}\cdot\sqrt{1^2+0^2}} =$$

$$= \frac{|4+0|}{\sqrt{20}\cdot 1} = \frac{2\cancel{4}}{\cancel{2}\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$(\widehat{r,OY}) = \arccos\left(\frac{2}{\sqrt{5}}\right) = 26{,}56^\circ$$

44. Calcula la recta perpendicular a r: $x + y - 3 = 0$ y que pasa por el punto $P(-3, 3)$.



¿s?

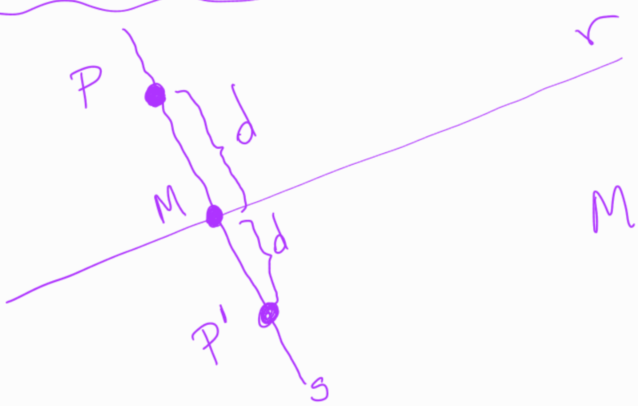
$r: x+y-3=0$

$\vec{n}_r = (1,1) = \vec{v}_s$

$$\boxed{s: \frac{x+3}{1} = \frac{y-3}{1}}$$

PUNTO SIMÉTRICO A UNA RECTA:

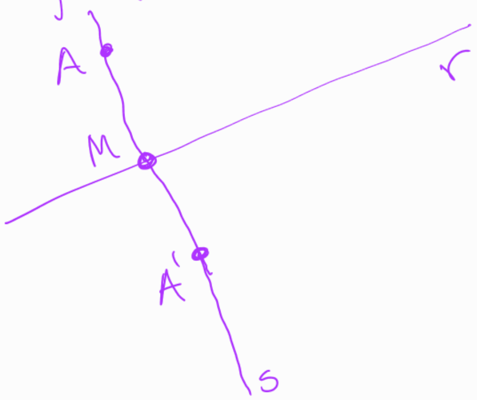
Nos dan P y r



¡¡ M es pto medio de PP' !!

$M = r \cap s$ tal que $s \begin{cases} P \in s \\ s \perp r \end{cases}$

Ejemplo: Sea $A=(3,2)$ respecto $r: 2x-y+1=0$



$$M = r \cap s \quad y \quad s\begin{cases} A(3,2) \\ s \perp r \end{cases}$$

$$\underbrace{\vec{v}_s = \vec{n}_r = (2,-1)}_{\vec{n}_s = (1,2)}$$

$$s: 1\cdot(x-3)+2\cdot(y-2)=0$$

$$x+2y-7=0$$

$$\underset{r\cap s}{\overset{\parallel}{M}} \begin{cases} 2x-y+1=0 \longrightarrow 2x+1=y \\ x+2y-7=0 \longrightarrow x+2(2x+1)-7=0 \end{cases}$$

$$x+4x+2-7=0$$

$$5x-5=0$$

$$5x=5$$

$$x=1 \Rightarrow y=2\cdot 1+1=3$$

$$\underbrace{M=(1,3)}_{\text{pto medio } AA'} \begin{cases} 1=\dfrac{3+x'}{2} \Rightarrow 2=3+x' \Rightarrow 2-3=x' \Rightarrow x'=-1 \\[2ex] 3=\dfrac{2+y'}{2} \Rightarrow 6=2+y' \Rightarrow 6-2=y' \Rightarrow y'=4 \end{cases}$$

$$\boxed{A'=(-1,4)}$$