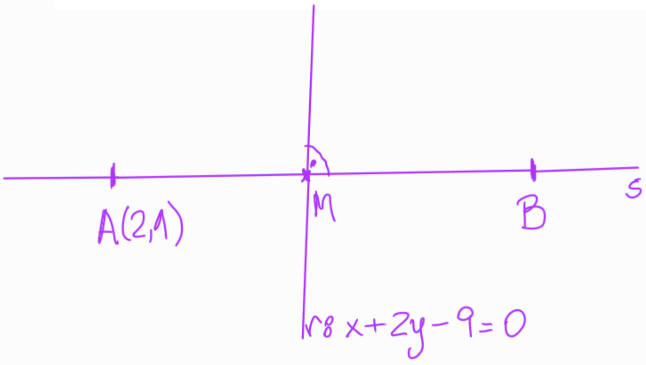
51. Halla el extremo B del segmento $\overline{AB}$ siendo $A(2, 1)$ y sabiendo que la mediatriz del segmento es $r: x+2y-9=0$



¿B? Necesito obligatoriamente hallar M

$M = r \cap S_{AB}$

$S_{AB}\begin{cases} A(2,1) \\ s \perp r \Rightarrow \vec{v}_s = \vec{n}_r = (1,2) \end{cases}$

$m = \frac{v_2}{v_1} = \frac{2}{1} = 2$

$S_{AB}: y = mx + n$

$y = 2x + n$ pero $A(2,1) \in S_{AB}$

$1 = 2\cdot 2 + n$

$1 = 4 + n$

$1 - 4 = n \Rightarrow n = -3$

$S_{AB}: y = 2x - 3$

$M = r \cap S_{AB}\begin{cases} r: x+2y-9=0 \\ s: y = 2x-3 \end{cases} \Rightarrow x + 2\cdot(2x-3) - 9 = 0$

$x + 4x - 6 - 9 = 0$

$5x - 15 = 0$

$5x = 15$

$x = \frac{15}{5} = 3$

$y = 2\cdot 3 - 3 = 6 - 3 = 3$

Luego $M = (3,3)$ y es el pto medio de $\overline{AB}$:

$\left.\begin{cases} 3 = \frac{2+x'}{2} \Rightarrow 6 = 2 + x' \Rightarrow \boxed{x' = 6 - 2 = 4} \\ 3 = \frac{1+y'}{2} \Rightarrow 6 = 1 + y' \Rightarrow \boxed{y' = 6 - 1 = 5} \end{cases}\right\} \boxed{B = (4,5)}$

117. Dada las rectas $r: x-4y+2=0$ y $s: 2x-3y=-4$:

a) Calcula su punto de corte.

b) Demuestra que el punto $P(1, 2)$ pertenece a s y calcula su simétrico respecto de la recta r.

a) $P = r \cap s = \begin{cases} x-4y+2=0 \\ 2x-3y+4=0 \end{cases} \xrightarrow{2E_1} \begin{cases} 2x-8y+4=0 \\ 2x-3y+4=0 \end{cases}$

$\ominus \quad \cancel{} \; -5y \; \cancel{} = 0$

$y = 0$

$x - \cancel{4\cdot 0} + 2 = 0$

$x = -2$

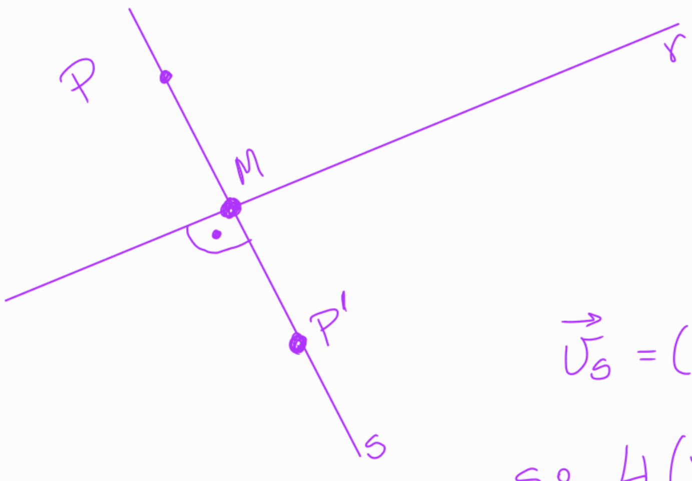
$\boxed{P(-2,0)}$

b) $¿P(1,2) \in s? \Rightarrow 2\cdot 1 - 3\cdot 2 + 4 = 0$

$2-6+4=0$

$0=0 \Rightarrow \boxed{P \in s}$

¿Simétrico de $P_r = P'$?



M pto medio de $\overline{PP'}$

$M = r \cap s$ tal que $s \begin{cases} P(1,2) \\ s \perp r \end{cases}$

$\vec{v_s} = \vec{n_r} = (1,-4)$

$r: x-4y+2$

$\vec{v_s} = (1,-4) \Rightarrow \vec{n_s} = (4,1)$

$s: 4(x-1) + 1(y-2) = 0$

$4x+y-6=0$

$M = r \cap s = \begin{cases} x-4y+2=0 \longrightarrow x = 4y-2 \\ 4x+y-6=0 \longrightarrow 4\cdot(4y-2)+y-6=0 \end{cases}$

$16y-8+y-6=0$

$17y-14=0$

$17y = 14 \rightarrow y = \frac{14}{17}$

Luego $x = 4y - 2$

$$x = 4 \cdot \frac{14}{17} - 2 = \frac{56}{17} - 2 = \frac{56-34}{17} = \frac{22}{17}$$

$M = \left(\frac{22}{17}, \frac{14}{17}\right)$ que es pto medio de $\overline{PP'}$:

$$\begin{cases} \frac{22}{17} = \frac{1 + x'}{2} \Rightarrow 44 = 17 + 17x' \Rightarrow 44 - 17 = 17x' \Rightarrow 27 = 17x' \\ \qquad x' = \frac{27}{17} \\ \frac{14}{17} = \frac{2 + y'}{2} \Rightarrow 28 = 34 + 17y' \Rightarrow 28 - 34 = 17y' \Rightarrow -6 = 17y' \\ \qquad y' = \frac{-6}{17} \end{cases}$$

Por lo tanto $\boxed{P' = \left(\frac{27}{17}, \frac{-6}{17}\right)}$