1.4.2. Sistemas de Ecuaciones Exponenciales:

Problema 113 Resolver el sistema de ecuaciones exponenciales:

$$\left\{\begin{array}{rrr} 4^{x+1}- & 6^y = & 40 \\ 2\cdot 4^x- & 6^y = & -88 \end{array}\right.$$

Solución

$$\left\{\begin{array}{rrr} 4^{x+1}- & 6^y = & 40 \\ 2\cdot 4^x- & 6^y = & -88 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrr} 4\cdot 4^{x}- & 6^y = & 40 \\ 2\cdot 4^x- & 6^y = & -88 \end{array}\right.$$

Haciendo el cambio de variables $4^x = u$ y $6^y = v$ el sistema quedará de la siguiente forma:

$$\left\{\begin{array}{rrr} 4u- & v = & 40 \\ 2u- & v = & -88 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} u = 64 \\ v = 216 \end{array}\right.$$

Deshaciendo el cambio de variables nos quedaría:

$$\left\{\begin{array}{l} 4^x = u = 64 \\ 6^y = v = 216 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} 4^x = 4^3 \\ 6^y = 6^3 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x = 3 \\ y = 3 \end{array}\right.$$

Problema 114 Resolver el sistema de ecuaciones exponenciales:

$$\left\{\begin{array}{rrr} 2^{x}+ & 5^y = & 9 \\ 2^{x+2}+ & 5^{y+1} = & 41 \end{array}\right.$$

Solución

$$\left\{\begin{array}{rrr} 2^{x}+ & 5^y = & 9 \\ 2^{x+2}+ & 5^{y+1} = & 41 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrr} 2^{x}+ & 5^y = & 9 \\ 4\cdot 2^{x}+ & 5\cdot 5^{y} = & 41 \end{array}\right.$$

Haciendo el cambio de variables $2^x = u$ y $5^y = v$ el sistema quedará de la siguiente forma:

$$\left\{\begin{array}{rrr} u+ & v = & 9 \\ 4u+ & 5v = & 41 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} u = 4 \\ v = 5 \end{array}\right.$$

Deshaciendo el cambio de variables nos quedaría:

$$\left\{\begin{array}{l} 2^x = u = 4 \\ 5^y = v = 5 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} 2^x = 2^2 \\ 5^y = 5^1 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x = 2 \\ y = 1 \end{array}\right.$$

Problema 115 Resolver el sistema de ecuaciones exponenciales:

$$\left\{\begin{array}{rrr} 2^{x-1}+ & 3^{y+1} = & 4 \\ 2^{x+1}- & 3^{y+1} = & 5 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrr} 2^{x-1}+ & 3^{y+1} = & 4 \\ 2^{x+1}- & 3^{y+1} = & 5 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrr} \frac{2^x}{2}+ & 3\cdot 3^y = & 4 \\ 2\cdot 2^x- & 3\cdot 3^y = & 5 \end{array}\right.$$

Haciendo el cambio de variables $2^x = u$ y $3^y = v$ el sistema quedará de la siguiente forma:

$$\left\{\begin{array}{rrr} \frac{u}{2}+ & 3v = & 4 \\ 2u- & 3v = & 5 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} u = \frac{18}{5} \\ v = \frac{11}{15} \end{array}\right.$$

Deshaciendo el cambio de variables nos quedaría:

$$\left\{\begin{array}{l} 2^x = u = \frac{18}{5} \\ 3^y = v = \frac{11}{15} \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x\log 2 = \log\frac{18}{5} \\ y\log 3 = \log\frac{11}{15} \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x = \frac{\log\frac{18}{5}}{\log 2} = 1,847996906 \\ y = \frac{\log\frac{11}{15}}{\log 3} = -0,2823151820 \end{array}\right.$$

Problema 116 Resolver el sistema de ecuaciones exponenciales:

$$\left\{\begin{array}{rrr} 2^{x+1}- & 3^{y-1} = & 4 \\ 2^{x+1}+ & 3^{y+1} = & 5 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrr} 2^{x+1}- & 3^{y-1} = & 4 \\ 2^{x+1}+ & 3^{y+1} = & 5 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrr} 2\cdot 2^x- & \frac{3^y}{3} = & 4 \\ 2\cdot 2^x+ & 3\cdot 3^y = & 5 \end{array}\right.$$

Haciendo el cambio de variables $2^x = u$ y $3^y = v$ el sistema quedará de la siguiente forma:

$$\left\{\begin{array}{rrr} 2u- & \frac{v}{3} = & 4 \\ 2u+ & 3v = & 5 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} u = \frac{41}{20} = 2,05 \\ v = \frac{3}{10} = 0,3 \end{array}\right.$$

Deshaciendo el cambio de variables nos quedaría:

$$\left\{\begin{array}{l} 2^x = u = \frac{41}{20} \\ 3^y = v = \frac{3}{10} \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x\log 2 = \log\frac{41}{20} \\ y\log 3 = \log\frac{3}{10} \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x = \frac{\log\frac{41}{20}}{\log 2} = 1,035623909 \\ y = \frac{\log\frac{3}{10}}{\log 3} = -1,095903274 \end{array}\right.$$

Problema 117 Resolver el sistema de ecuaciones exponenciales:

$$\left\{\begin{array}{rrr} 2^{x+1}- & 3^{y-1} = & 3 \\ 2^{x+1}+ & 3^{y+1} = & 4 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrr} 2^{x+1}- & 3^{y-1} = & 3 \\ 2^{x+1}+ & 3^{y+1} = & 4 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrr} 2\cdot 2^x- & \frac{3^y}{3} = & 3 \\ 2\cdot 2^x+ & 3\cdot 3^y = & 4 \end{array}\right.$$

Haciendo el cambio de variables $2^x = u$ y $3^y = v$ el sistema quedará de la siguiente forma:

$$\left\{\begin{array}{rrr} 2u- & \frac{v}{3} = & 3 \\ 2u+ & 3v = & 4 \end{array}\right. \Longrightarrow \left\{\begin{array}{l} u = \frac{31}{20} = 1,55 \\ v = \frac{3}{10} = 0,3 \end{array}\right.$$

Deshaciendo el cambio de variables nos quedaría:

$$\left\{\begin{array}{l} 2^x = u = \frac{31}{20} \\ 3^y = v = \frac{3}{10} \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x\log 2 = \log\frac{31}{20} \\ y\log 3 = \log\frac{3}{10} \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x = \frac{\log\frac{31}{20}}{\log 2} = 0,6322682154 \\ y = \frac{\log\frac{3}{10}}{\log 3} = -1,095903274 \end{array}\right.$$

Problema 118 Resolver el sistema de ecuaciones exponenciales:

$$\left\{\begin{array}{rrr} 2^{x+1}- & 3^{y-1} = & 1 \\ 2^{x-1}+ & 3^{y+1} = & 2 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrr} 2^{x+1}- & 3^{y-1} = & 1 \\ 2^{x-1}+ & 3^{y+1} = & 2 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrr} 2\cdot 2^x+ & \dfrac{3^y}{3} = & 1 \\ & & \\ \dfrac{2^x}{2}- & 3\cdot 3^y = & 2 \end{array}\right.$$

Haciendo el cambio de variables $2^x = u$ y $3^y = v$ el sistema quedará de la siguiente forma:

$$\left\{\begin{array}{rrr} 2u- & \dfrac{v}{3} = & 1 \\ & & \\ \dfrac{u}{2}+ & 3v = & 2 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrr} 6u- & v = & 3 \\ u+ & 6v = & 4 \end{array}\right. \left\{\begin{array}{l} u = \frac{22}{37} \\ v = \frac{21}{37} \end{array}\right.$$

Deshaciendo el cambio de variables nos quedaría:

$$\left\{\begin{array}{l} 2^x = u = \frac{22}{37} \\ 3^y = v = \frac{21}{37} \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x\log 2 = \log\frac{22}{37} \\ y\log 3 = \log\frac{21}{37} \end{array}\right. \Longrightarrow \left\{\begin{array}{l} x = \frac{\log\frac{22}{37}}{\log 2} = -0,7500217469 \\ y = \frac{\log\frac{21}{37}}{\log 3} = -0,5155553790 \end{array}\right.$$

Problema 119

$$\left\{\begin{array}{rrl} 3^{x-1}+ & 2^{y+1} & = 2 \\ 3^x- & 2^y & = 3 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrl} \dfrac{3^x}{3}+ & 2\cdot 2^y & = 2 \\ & & \\ 3^x- & 2^y & = 3 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrl} \dfrac{u}{3}+ & 2v & = 2 \\ & & \\ u- & v & = 3 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u = \dfrac{27}{7} = 3^x \Longrightarrow x = 1,22876 \\ \\ v = \dfrac{13}{7} = 2^y \Longrightarrow y = 0,89385 \end{array}\right.$$

Problema 120

$$\left\{\begin{array}{rrl} 2^{x-1}+ & 3^{y+1} & = 5 \\ 2^x- & 3^y & = 2 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrl} \dfrac{2^x}{2}+ & 3\cdot 3^y & = 5 \\ \\ 2^x- & 3^y & = 2 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrl} \dfrac{u}{2}+ & 3v & = 5 \\ \\ u- & v & = 2 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u = \dfrac{22}{7} = 2^x \Longrightarrow x = 1,65208 \\ \\ v = \dfrac{8}{7} = 3^y \Longrightarrow y = 0,12154 \end{array}\right.$$

Problema 121

$$\left\{\begin{array}{rrl} 3^{x-2}+ & 2^y & = 1 \\ 2^x+ & 3\cdot 2^y & = 5 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrl} \dfrac{3^x}{9}+ & 2^y & = 1 \\ \\ 3^x+ & 3\cdot 2^y & = 5 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrl} \dfrac{u}{9}+ & v & = 1 \\ \\ u+ & 3v & = 5 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u = 3 = 3^x \Longrightarrow x = 1 \\ \\ v = \dfrac{2}{3} = 2^y \Longrightarrow y = -0,585 \end{array}\right.$$

Problema 122

$$\left\{\begin{array}{rrl} 2^x- & 3^y & = 1 \\ 2^x+ & 3^y & = 3 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrl} 2^x- & 3^y & = 1 \\ 2^x+ & 3^y & = 3 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrl} u- & v & = 1 \\ \\ u+ & v & = 3 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u = 2 = 2^x \Longrightarrow x = 1 \\ \\ v = 1 = 3^y \Longrightarrow y = 0 \end{array}\right.$$

Problema 123

$$\left\{\begin{array}{rrl} 2^{x+2}- & 3^y & =1 \\ 2^x+ & 2\cdot 3^y & =3 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrl} 2\cdot 2^x- & 3^y & =1 \\ 2^x+ & 2\cdot 3^y & =3 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrl} 2u- & v & =1 \\ \\ u+ & 2v & =3 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u=1=2^x \Longrightarrow x=0 \\ \\ v=1=3^y \Longrightarrow y=0 \end{array}\right.$$

Problema 124

$$\left\{\begin{array}{rrl} 2^x+ & 3^y & =2 \\ 2^{x+1}- & 3^y & =1 \end{array}\right.$$

Solución:

$$\left\{\begin{array}{rrl} 2^x+ & 3^y & =2 \\ 2\cdot 2^x- & 3^y & =1 \end{array}\right. \Longrightarrow \left\{\begin{array}{rrl} u+ & v & =2 \\ \\ 2u- & v & =1 \end{array}\right. \Longrightarrow$$

$$\left\{\begin{array}{l} u=1=2^x \Longrightarrow x=0 \\ \\ v=1=3^y \Longrightarrow y=0 \end{array}\right.$$

Problema 125 Unos problemas para ejercitarse:

1.

$$\left\{\begin{array}{rrr} 3\cdot 2^x- & 4\cdot 7^y = & -172 \\ 7\cdot 2^x+ & 2\cdot 7^y = & 154 \end{array}\right.$$

Sol: $x=3;\quad y=2$

2.

$$\left\{\begin{array}{rrr} 4^{x+1}- & 6^y = & 40 \\ 2\cdot 4^x- & 6^y = & -88 \end{array}\right.$$

Sol: $x=3;\quad y=3$

3.

$$\left\{\begin{array}{rrr} 2\cdot 3^{x+1}- & 5^{y+2} = & -2639 \\ 4\cdot 3^x+ & 5^y = & 449 \end{array}\right.$$

Sol: $x=4;\quad y=4$

4.

$$\left\{\begin{array}{rrl} 3^x+ & 2^y = & 31 \\ 3^{x+1}- & 2^{y+2} = & 65 \end{array}\right.$$

Sol: $x = 3; \quad y = 2$

5.

$$\left\{\begin{array}{lcl} 5^{x+y} & = & 25^3 \\ 3^{x-y} & = & 25 \end{array}\right.$$

Sol: $x = 4; \quad y = 2$

6.

$$\left\{\begin{array}{rlcrlr} 15\cdot & 5^{x-1} & - & & 6^y = & 339 \\ 3\cdot & 5^x & + & 2\cdot & 6^{y+1} = & 807 \end{array}\right.$$

Sol: $x = 3; \quad y = 2$

7.

$$\left\{\begin{array}{lcl} a^{x+y} & = & a^4 \\ a^{x-y} & = & a^2 \end{array}\right.$$

Sol: $x = 3; \quad y = 1$

8.

$$\left\{\begin{array}{lclcr} 8^y & \cdot & 2^{2x} & = & 128 \\ 3^{2y} & \cdot & 3^{x-1} & = & 27 \end{array}\right.$$

Sol: $x = -70; \quad y = 49$

9.

$$\left\{\begin{array}{lcr} 3^{3x-y} & = & \sqrt{3^{10}} \\ 3^{2x+y} & = & 3 \end{array}\right.$$

Sol: $x = \frac{6}{5}; \quad y = -\frac{7}{5}$

10.

$$\left\{\begin{array}{rrr} 3 \cdot 2^x- & 2 \cdot 3^y = & -6 \\ 4 \cdot 2^x- & 3 \cdot 3^y = & -11 \end{array}\right.$$

Sol: $x = 2; \quad y = 2$

11.

$$\left\{\begin{array}{rrr} 3 \cdot 2^x- & 5 \cdot 3^y = & 3 \\ 2^{x+1}+ & 3^{y+1} = & 59 \end{array}\right.$$

Sol: $x = 4; \quad y = 2$

12.

$$\left\{ \begin{array}{rrr} 2^x- & 3^{y-1} = & 5 \\ 2^{x+1}+ & 8 \cdot 3^y = & 712 \end{array} \right.$$

Sol: $x = 5; \quad y = 4$

13.

$$\left\{ \begin{array}{rlcr} 2 \cdot 3^x+ & 2^{y+3} & = & 86 \\ 3^x- & 2^y & = & 23 \end{array} \right.$$

Sol: $x = 3; \quad y = 2$

14.

$$\left\{ \begin{array}{lcr} 2^{x+2y} & = & 32 \\ 5^{2x-y} & = & 1 \end{array} \right.$$

Sol: $x = 1; \quad y = 2$