

Derivar las siguientes funciones:

a) $y = e^{-3x}$ b) $y = \sqrt{x-5}$ c) $y = 2\operatorname{tg}(3x-2)$ d) $y = \operatorname{cose}^x$ e) $y = 5\operatorname{sen}\frac{x}{2}$ f) $y = \sqrt[4]{3x}$

g) $y = \ln(\operatorname{sen}x)$ h) $y = \operatorname{tg}\frac{3}{x}$ i) $f(x) = e^{3x}$ j) $y = (\ln x)^2$ k) $y = \ln x^2$ l) $y = \operatorname{sen}(\cos x)$

m) $f(t) = \cos\left(\frac{t+8}{3}\right)$ n) $f(t) = \frac{\operatorname{sen}(6t)}{3}$ o) $f(t) = e^{t^2}$ p) $f(t) = \ln\left(\frac{1}{t}\right)$ q) $f(x) = 8\operatorname{tg}(xe^x)$

a) $y' = -3 \cdot e^{-3x}$ b) $y' = \frac{1}{2\sqrt{x-5}}$ c) $y' = 2 \cdot \frac{1}{\cos^2(3x-2)} \cdot 3 = \frac{6}{\cos^2(3x-2)}$

d) $y' = -\operatorname{sen}(e^x) \cdot e^x = -e^x \cdot \operatorname{sen}(e^x)$ e) $y' = 5 \cdot \cos\frac{x}{2} \cdot \frac{1}{2} = \frac{5}{2} \cdot \cos\frac{x}{2}$

f) $y' = \frac{3}{4 \cdot \sqrt[4]{(3x)^3}}$ g) $y' = \frac{1}{\operatorname{sen}x} \cdot \cos x = \frac{\cos x}{\operatorname{sen}x} = \cot g x$ h) $y' = \frac{1}{\cos^2 \frac{3}{x}} \cdot \left(\frac{-3}{x^2}\right) = \frac{-3}{x^2 \cdot \cos^2 \frac{3}{x}}$

i) $f'(x) = e^{3x} \cdot 3 = 3e^{3x}$ j) $y' = 2\ln x \cdot \frac{1}{x} = \frac{2\ln x}{x}$

k) $y' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$ l) $y' = \cos(\cos x) \cdot (-\operatorname{sen}x) = -\operatorname{sen}x \cdot \cos(\cos x)$

m) $f'(t) = -\operatorname{sen}\frac{t+8}{3} \cdot \frac{1}{3} = -\frac{1}{3} \cdot \operatorname{sen}\frac{t+8}{3}$ n) $f'(t) = \frac{1}{3} \cdot \cos(6t) \cdot 6 = 2\cos(6t)$

o) $f'(t) = e^{t^2} \cdot 2t = 2t \cdot e^{t^2}$ p) $f'(t) = \frac{1}{\frac{1}{t}} \cdot \left(-\frac{1}{t^2}\right) = -\frac{1}{t}$

q) $f'(x) = 8 \cdot \frac{1}{\cos^2(xe^x)} \cdot (e^x + x \cdot e^x) = \frac{8e^x + 8xe^x}{\cos^2(xe^x)}$

Derivar las siguientes funciones:

a) $y = \ln(\cos x^3)$ b) $y = \frac{\operatorname{sen}^3(6x-1)}{8}$ c) $y = \frac{x}{2} + \sqrt[3]{3x^2+5}$ d) $y = \operatorname{sen}^2 x + \cos^2 x$

e) $y = \frac{1-e^x}{1+e^x}$ f) $f(x) = x \cdot \operatorname{arctg}x$ g) $f(x) = \ln \frac{1}{x+\sqrt{x^2+1}}$ h) $y = \operatorname{sen}^2 x \cdot \operatorname{sen} x^2$

i) $f(x) = \sqrt{x} \cdot (x^3 - \sqrt{x} + 1)$ j) $y = (\sqrt[3]{x} + 2x) \cdot (1 + \sqrt[3]{x^2} + 3x)$ k) $y = \frac{1-x^3}{\sqrt{\pi}}$ l) $f(x) = \frac{1-\ln x}{1+\ln x}$

$$a) y' = \frac{1}{\cos x^3} \cdot (-\sin x^3) \cdot 3x^2 = \frac{-3x^2 \sin x^3}{\cos x^3} = -3x^2 \cdot \operatorname{tg} x^3$$

$$b) y' = \frac{1}{8} \cdot 3 \sin^2(6x-1) \cdot \cos(6x-1) \cdot 6 = \frac{9}{4} \cdot \sin^2(6x-1) \cdot \cos(6x-1)$$

$$c) y' = \frac{1}{2} + \frac{1}{3} \cdot (3x^2 + 5)^{\frac{2}{3}} \cdot 6x = \frac{1}{2} + \frac{2x}{\sqrt[3]{(3x^2 + 5)^2}}$$

$$d) y' = 0 \text{ (ya que } \sin^2 x + \cos^2 x = 1) \quad e) y' = \frac{-e^x \cdot (1 + e^x) - (1 - e^x) \cdot e^x}{(1 + e^x)^2} = \frac{-2e^x}{(1 + e^x)^2}$$

$$f) f'(x) = \operatorname{arctg} x + x \cdot \frac{1}{1+x^2} = \operatorname{arctg} x + \frac{x}{1+x^2}$$

$$g) f(x) = \ln 1 - \ln(x + \sqrt{x^2 + 1}) = 0 - \ln(x + \sqrt{x^2 + 1})$$

$$f'(x) = -\frac{1}{(x + \sqrt{x^2 + 1})} \cdot \left(1 + \frac{2x}{2\sqrt{x^2 + 1}}\right) = \frac{-\left(\sqrt{x^2 + 1} + x\right)}{\sqrt{x^2 + 1} \cdot (x + \sqrt{x^2 + 1})} = -\frac{1}{\sqrt{x^2 + 1}}$$

$$h) y' = 2 \sin x \cdot \cos x \cdot \sin x^2 + \sin^2 x \cdot \cos x^2 \cdot 2x$$

$$i) f(x) = \sqrt{x^7} - x + \sqrt{x} \quad f'(x) = \frac{7x^6}{2\sqrt{x^7}} - 1 + \frac{1}{2\sqrt{x}} = \frac{7x^3}{2\sqrt{x}} - 1 + \frac{1}{2\sqrt{x}}$$

$$j) y = \sqrt[3]{x} + 3x + 3 \cdot \sqrt[3]{x^4} + 2 \cdot \sqrt[3]{x^5} + 6x^2 \quad y' = \frac{1}{3 \cdot \sqrt[3]{x^2}} + 3 + 4 \cdot \sqrt[3]{x} + \frac{10}{3} \cdot \sqrt[3]{x^2} + 12x$$

$$k) y' = \frac{1}{\sqrt{\pi}} (-3x^2) = \frac{-3x^2}{\sqrt{\pi}} \quad l) f'(x) = \frac{-\frac{1}{x} \cdot (1 + \ln x) - (1 - \ln x) \cdot \frac{1}{x}}{(1 + \ln x)^2} = \frac{-2}{x \cdot (1 + \ln x)^2}$$

$$a) y = x \cdot \arcsen x + \sqrt{1-x^2} \quad b) y = \operatorname{arctg}[\ln(ax+b)] \quad c) y = \ln \operatorname{arctg} \sqrt{1+x^2}$$

$$d) y = \arccos \frac{2x-1}{\sqrt{3}} \quad e) y = 2^{\frac{x}{\ln x}} \quad f) y = x^3 - 3^x \quad g) y = e^{\sqrt{x-1}} \quad h) f(x) = e^{-x^2} \cdot \ln x$$

$$i) y = x \cdot 10^{\sqrt{x}} \quad j) y = \ln \frac{1-e^x}{e^x}$$

$$a) y' = \arcsen x + x \cdot \frac{1}{\sqrt{1-x^2}} + \frac{1}{2 \cdot \sqrt{1-x^2}} \cdot (-2x) = \arcsen x$$

$$b) y' = \frac{1}{1 + [\ln(ax+b)]^2} \cdot \frac{1}{ax+b} \cdot a = \frac{a}{(ax+b) \cdot (1 + [\ln(ax+b)]^2)}$$

$$c) y' = \frac{1}{\operatorname{arctg} \sqrt{1+x^2}} \cdot \frac{1}{1 + (\sqrt{1+x^2})^2} \cdot \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x}{\operatorname{arctg} \sqrt{1+x^2} \cdot (2+x^2) \cdot \sqrt{1+x^2}}$$

$$d) y' = \frac{-1}{\sqrt{1 - \left(\frac{2x-1}{\sqrt{3}}\right)^2}} \cdot \frac{2}{\sqrt{3}} = \frac{-2}{\sqrt{3 - 4x^2 + 4x - 1}} = \frac{-2}{\sqrt{-4x^2 + 4x + 2}} = \frac{-\sqrt{2}}{\sqrt{-2x^2 + 2x + 1}}$$

$$e) y' = 2^{\frac{x}{\ln x}} \cdot \frac{\ln x - x \cdot \frac{1}{x}}{(\ln x)^2} \cdot \ln 2 = \frac{2^{\frac{x}{\ln x}} \cdot \ln 2 \cdot (\ln x - 1)}{(\ln x)^2}$$

$$f) y' = 3x^2 - 3^x \cdot \ln 3 \quad g) y' = e^{\sqrt{x-1}} \cdot \frac{1}{2 \cdot \sqrt{x-1}} = \frac{e^{\sqrt{x-1}}}{2 \cdot \sqrt{x-1}}$$

$$h) f'(x) = e^{-x^2} \cdot (-2x) \cdot \ln x + e^{-x^2} \cdot \frac{1}{x} = -2x \cdot e^{-x^2} \cdot \ln x + \frac{e^{-x^2}}{x} = e^{-x^2} \cdot \left(-2x \cdot \ln x + \frac{1}{x}\right)$$

$$i) y' = 10^{\sqrt{x}} + x \cdot 10^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \cdot \ln 10 = 10^{\sqrt{x}} \cdot \left(1 + \frac{x \cdot \ln 10}{2\sqrt{x}}\right)$$

$$j) y' = \frac{1}{\frac{1 - e^x}{e^x}} \cdot \frac{-e^x \cdot e^x - (1 - e^x)e^x}{(e^x)^2} = \frac{-e^x}{(1 - e^x) \cdot e^x} = \frac{-1}{1 - e^x}$$

$$a) y = \cos^2(1-x) \quad b) y = \cos^2(\operatorname{arcsec} x^2) \quad c) y = \arcsen \frac{x+1}{x-1} \quad d) y = \operatorname{arctg} \frac{x+a}{1-ax}$$

$$e) y = \arcsen(\sen x^2) + \arccos(\cos x^2) \quad f) y = x \cdot \sqrt{a^2 - x^2} + a^2 \cdot \arcsen \frac{x}{a}$$

$$g) y = \frac{8x^4 - 3}{32} \cdot \arcsen x + \frac{2x^3 + 3x}{32} \cdot \sqrt{1 - x^2} \quad h) y = e^x \cdot (x-1) \cdot [(x+1) \cdot \cos x + (x-1) \cdot \sen x]$$

$$a) y = \frac{1}{\sen^2(1-x)} \quad y' = \frac{-2 \sen(1-x) \cdot \cos(1-x) \cdot (-1)}{\sen^4(1-x)} = \frac{2 \cdot \cos(1-x)}{\sen^3(1-x)}$$

b) Podemos sustituir $\operatorname{arcsec} x^2$ por una expresión más familiar a través de los siguientes pasos:

$$\operatorname{arcsec} x^2 = z \rightarrow \sec z = x^2 \rightarrow \frac{1}{\cos z} = x^2 \rightarrow \cos z = \frac{1}{x^2} \rightarrow z = \arccos \frac{1}{x^2}$$

$$\text{La función a derivar es entonces: } y = \cos^2\left(\arccos \frac{1}{x^2}\right) = \left(\cos\left(\arccos \frac{1}{x^2}\right)\right)^2 = \frac{1}{x^4}$$

$$y' = \frac{-4x^3}{x^8} = -\frac{4}{x^5}$$

$$c) \quad y' = \frac{1}{\sqrt{1 - \left(\frac{x+1}{x-1}\right)^2}} \cdot \frac{(x-1) - (x+1)}{(x-1)^2} = \frac{-2}{\sqrt{\frac{-4x}{(x-1)^2}} \cdot (x-1)^2} = \frac{-1}{(x-1) \cdot \sqrt{-x}}$$

$$d) \quad y' = \frac{1}{1 + \left(\frac{x+a}{1-ax}\right)^2} \cdot \frac{(1-ax) - (x+a) \cdot (-a)}{(1-ax)^2} = \frac{1+a^2}{(1-ax)^2 + (x+a)^2} = \frac{1+a^2}{1+a^2x^2 + x^2 + a^2} =$$

$$\frac{1+a^2}{1+a^2+x^2(1+a^2)} = \frac{1+a^2}{(1+a^2)(1+x^2)} = \frac{1}{1+x^2}$$

$$e) \quad y = \arcsen(\sen x^2) + \arccos(\cos x^2) = x^2 + x^2 = 2x^2 \quad y' = 4x$$

$$f) \quad y' = \sqrt{a^2 - x^2} + x \cdot \frac{1}{2\sqrt{a^2 - x^2}} \cdot (-2x) + a^2 \cdot \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \cdot \frac{1}{a} = \sqrt{a^2 - x^2} - \frac{2x^2}{2\sqrt{a^2 - x^2}} +$$

$$\frac{a}{\sqrt{\frac{a^2 - x^2}{a^2}}} = \sqrt{a^2 - x^2} - \frac{x^2}{\sqrt{a^2 - x^2}} + \frac{a^2}{\sqrt{a^2 - x^2}} = \frac{2(a^2 - x^2)}{\sqrt{a^2 - x^2}} = \sqrt{a^2 - x^2}$$

$$g) \quad y' = x^3 \cdot \arcsen x + \frac{8x^4 - 3}{32} \cdot \frac{1}{\sqrt{1-x^2}} + \frac{6x^2 + 3}{32} \cdot \sqrt{1-x^2} + \frac{2x^3 + 3x}{32} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) =$$

$$x^3 \cdot \arcsen x + \frac{8x^4 - 3}{32\sqrt{1-x^2}} + \frac{(6x^2 + 3) \cdot \sqrt{1-x^2}}{32} - \frac{4x^4 + 6x^2}{64\sqrt{1-x^2}} =$$

$$x^3 \cdot \arcsen x + \frac{16x^4 - 6 - 4x^4 - 6x^2}{64 \cdot \sqrt{1-x^2}} + \frac{(6x^2 + 3) \cdot \sqrt{1-x^2}}{32} =$$

$$x^3 \cdot \arcsen x + \frac{3(2x^4 - 1 - x^2)}{32\sqrt{1-x^2}} + \frac{3(2x^2 + 1) \cdot \sqrt{1-x^2}}{32}$$

$$h) \quad y = e^x \cdot (x^2 - 1) \cdot \cos x + e^x \cdot (x-1)^2 \cdot \sen x$$

$$y' = e^x \cdot (x^2 - 1) \cdot \cos x + e^x \cdot 2x \cdot \cos x + e^x \cdot (x^2 - 1) \cdot (-\sin x) + e^x \cdot (x - 1)^2 \cdot \sin x +$$

$$e^x \cdot 2 \cdot (x - 1) \cdot \sin x + e^x \cdot (x - 1)^2 \cdot \cos x = 2x^2 \cdot e^x \cdot \cos x$$

$$\text{a a) } y = \ln \sqrt{\frac{1 + 2\sin x}{1 - 2\sin x}}$$

$$\text{b) } y = \frac{1}{2} \cdot \ln \left(\operatorname{tg} \frac{x}{2} \right) - \frac{\cos x}{2 \sin^2 x}$$

$$\text{c) } y = \ln \frac{1 + \operatorname{tg} \frac{x}{2}}{1 - \operatorname{tg} \frac{x}{2}}$$

$$\text{d) } f(x) = \frac{1}{3} \cdot \operatorname{arcsen} \frac{3 + 4\sin x}{4 + 3\sin x}$$

$$\text{e) } f(x) = \frac{x}{a + \sqrt{a^2 - x^2}}$$

$$\text{f) } y = \sqrt{x^2 + 1} - \ln \left(\frac{1}{x} + \sqrt{\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}} \right)$$

$$\text{a) } y = \ln \left(\frac{1 + 2 \sin x}{1 - 2 \sin x} \right)^{\frac{1}{2}} = \frac{1}{2} \cdot \ln \frac{1 + 2 \sin x}{1 - 2 \sin x}$$

$$y' = \frac{1}{2} \cdot \frac{1}{\frac{1 + 2 \sin x}{1 - 2 \sin x}} \cdot \frac{2 \cos x \cdot (1 - 2 \sin x) - (1 + 2 \sin x) \cdot (-2 \cos x)}{(1 - 2 \sin x)^2} = \frac{1}{2} \cdot \frac{1 - 2 \sin x}{1 + 2 \sin x} \cdot \frac{4 \cos x}{(1 - 2 \sin x)^2} =$$

$$\frac{4 \cos x}{2 - 8 \sin^2 x} = \frac{2 \cos x}{1 - 4 \sin^2 x}$$

$$\text{b) } y' = \frac{1}{2} \cdot \frac{1}{\operatorname{tg} \frac{x}{2}} \cdot \frac{1}{\cos^2 \frac{x}{2}} \cdot \frac{1}{2} - \frac{-\sin x \cdot 2 \cdot \sin^2 x - \cos x \cdot (2 \cdot 2 \sin x \cdot \cos x)}{4 \cdot \sin^4 x} =$$

$$\frac{1}{4 \cdot \sin \frac{x}{2} \cdot \cos \frac{x}{2}} + \frac{2 \sin^3 x + 4 \sin x \cdot \cos^2 x}{4 \cdot \sin^4 x} = \frac{1}{2 \cdot \sin x} + \frac{\sin^2 x + 2 \cdot \cos^2 x}{2 \sin^3 x} =$$

$$\frac{2 \sin^2 x + 2 \cos^2 x}{2 \sin^3 x} = \frac{1}{\sin^3 x}$$

$$c) y' = \frac{1}{\frac{1 + \operatorname{tg} \frac{x}{2}}{1 - \operatorname{tg} \frac{x}{2}}} \cdot \frac{\frac{1}{\cos^2 \frac{x}{2}} \cdot \frac{1}{2} \cdot \left(1 - \operatorname{tg} \frac{x}{2}\right) - \left(1 + \operatorname{tg} \frac{x}{2}\right) \cdot \left(-\frac{1}{\cos^2 \frac{x}{2}}\right) \cdot \frac{1}{2}}{\left(1 - \operatorname{tg} \frac{x}{2}\right)^2} =$$

$$\frac{\frac{1}{\cos^2 \frac{x}{2}}}{1 - \operatorname{tg}^2 \frac{x}{2}} = \frac{1}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} = \frac{1}{\cos x}$$

$$d) f'(x) = \frac{1}{3} \cdot \frac{1}{\sqrt{1 - \left(\frac{3 + 4 \sin x}{4 + 3 \sin x}\right)^2}} \cdot \frac{4 \cos x \cdot (4 + 3 \sin x) - (3 + 4 \sin x) \cdot (3 \cos x)}{(4 + 3 \sin x)^2} =$$

$$\frac{16 \cos x + 12 \sin x \cdot \cos x - 9 \cos x - 12 \sin x \cdot \cos x}{3 \cdot \sqrt{\frac{16 + 24 \sin x + 9 \sin^2 x - 9 - 24 \sin x - 16 \sin^2 x}{(4 + 3 \sin x)^2}}} \cdot (4 + 3 \sin x)^2 =$$

$$\frac{7 \cos x}{3 \cdot \sqrt{7 - 7 \sin^2 x} \cdot (4 + 3 \sin x)} = \frac{7 \cos x}{3 \sqrt{7} \cos x \cdot (4 + 3 \sin x)} = \frac{\sqrt{7}}{3(4 + 3 \sin x)}$$

$$e) f'(x) = \frac{\left(a + \sqrt{a^2 - x^2}\right) - x \cdot \left(\frac{1}{2 \sqrt{a^2 - x^2}} \cdot (-2x)\right)}{\left(a + \sqrt{a^2 - x^2}\right)^2} = \frac{a + \sqrt{a^2 - x^2} + \frac{x^2}{\sqrt{a^2 - x^2}}}{\left(a + \sqrt{a^2 - x^2}\right)^2} =$$

$$\frac{a \cdot \sqrt{a^2 - x^2} + a^2 - x^2 + x^2}{\sqrt{a^2 - x^2} \cdot \left(a + \sqrt{a^2 - x^2}\right)^2} = \frac{a \cdot \left(\sqrt{a^2 - x^2} + a\right)}{\sqrt{a^2 - x^2} \cdot \left(a + \sqrt{a^2 - x^2}\right)^2} = \frac{a}{\sqrt{a^2 - x^2} \cdot \left(a + \sqrt{a^2 - x^2}\right)}$$

$$f) y' = \frac{1}{2 \sqrt{x^2 + 1}} \cdot 2x - \frac{1}{\frac{1}{x} + \sqrt{\frac{1}{x}} + \sqrt{1 + \frac{1}{x^2}}} \cdot \left(\left(-\frac{1}{x^2}\right) + \frac{1}{2 \sqrt{\frac{1}{x}}} \left(-\frac{1}{x^2}\right) + \frac{1}{2 \sqrt{\frac{x^2 + 1}{x^2}}} \cdot \left(-\frac{2}{x^3}\right) \right) =$$

$$\frac{x}{\sqrt{x^2+1}} - \frac{1}{\frac{1}{x} + \sqrt{\frac{1}{x}} + \sqrt{1+\frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2} + \frac{\sqrt{x}}{2} \left(-\frac{1}{x^2} \right) + \frac{x}{2\sqrt{x^2+1}} \cdot \left(-\frac{2}{x^3} \right) \right) =$$

$$\frac{x}{\sqrt{x^2+1}} - \frac{1}{\frac{1}{x} + \sqrt{\frac{1}{x}} + \sqrt{1+\frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2} \right) \cdot \left(1 + \frac{\sqrt{x}}{2} + \frac{1}{\sqrt{x^2+1}} \right)$$

Calcular las derivadas de las funciones

a) $y = \sin^2 x + \sin x^2$ b) $y = \ln \sqrt{\frac{1-x}{1+x}}$ c) $y = x^2 \arcsen x^2 + 3 \operatorname{cosec}^2(x+1)$

a) $y' = 2 \sin x \cdot \cos x + \cos x^2 \cdot 2x = 2 \cos x \cdot (\sin x + x \cdot \cos x)$

b) $y = \ln \left(\frac{1-x}{1+x} \right)^{\frac{1}{2}} = \frac{1}{2} \cdot \ln \left(\frac{1-x}{1+x} \right)$ $y' = \frac{1}{2} \cdot \frac{1}{\frac{1-x}{1+x}} \cdot \frac{-(1+x) - (1-x)}{(1+x)^2} = \frac{-2}{2(1-x^2)} = \frac{-1}{1-x^2}$

c) $y = x^2 \cdot \arcsen x^2 + \frac{3}{\sin^2(x+1)}$

$$y' = 2x \cdot \arcsen x^2 + x^2 \cdot \frac{1}{\sqrt{1-x^4}} \cdot 2x + \frac{-3 \cdot 2 \sin(x+1) \cdot \cos(x+1)}{\sin^4(x+1)} =$$

$$2x \cdot \arcsen x^2 + \frac{2x^3}{\sqrt{1-x^4}} - \frac{6 \cdot \cos(x+1)}{\sin^3(x+1)}$$

Calcular las derivadas de las funciones

a) $y = \sin^3 \sqrt{\operatorname{tg}^2 \frac{2\pi^2+5}{3}}$ b) $y = \sin^3 x \cdot \left(\sqrt{\operatorname{tg}^2 \frac{2\pi^2+5}{3}} \right)$ c) Calcular $y''(0)$ en $y = \frac{x^2-x}{e^x}$

a) $y' = 0$ por ser una constante

b) $y' = 3 \cdot \sin^2 x \cdot \cos x \cdot \sqrt{\operatorname{tg}^2 \frac{2\pi^2+5}{3}} + \sin^3 x \cdot 0 = 3 \cdot \sin^2 x \cdot \cos x \cdot \sqrt{\operatorname{tg}^2 \frac{2\pi^2+5}{3}}$

$$c) \ y' = \frac{(2x-1) \cdot e^x - (x^2 - x) \cdot e^x}{(e^x)^2} = \frac{e^x (2x-1-x^2+x)}{e^{2x}} = \frac{3x-1-x^2}{e^x}$$

$$y'' = \frac{(3-2x) \cdot e^x - (3x-1-x^2) \cdot e^x}{(e^x)^2} = \frac{4-5x+x^2}{e^x} \Rightarrow y''(0) = 4$$

Calcular las derivadas de las funciones

$$a) \ y = \ln \sqrt{\frac{e^{3x-5}}{\operatorname{sen} \frac{\pi x}{3}}} \quad b) \ y = \log \left(\operatorname{tg}^2 \frac{\sqrt{2x-1}}{3x-1} \right) \quad c) \ y = \ln (\operatorname{sen}^2 x + \operatorname{arcsen}^2 x)$$

$$d) \ y = \log_a \frac{x^2 - 2\sqrt{x} + 3}{\operatorname{sen}^2 (3x+5)} \quad e) \ y = \ln \left(\frac{10^{3x+1}}{2x+8} \right)^x \quad f) \ y = \frac{\operatorname{sen} \frac{\pi}{6} \cdot e^x \cdot \cos \frac{\pi}{6} \cdot e^x}{\sqrt{x^2+1}}$$

$$a) \ y = \ln \sqrt{\frac{e^{3x+5}}{\operatorname{sen} \frac{\pi x}{3}}} = \ln \left(\frac{e^{3x+5}}{\operatorname{sen} \frac{\pi x}{3}} \right)^{\frac{1}{2}} = \frac{1}{2} \cdot \ln \frac{e^{3x+5}}{\operatorname{sen} \frac{\pi x}{3}}$$

$$y' = \frac{1}{2} \cdot \frac{1}{\frac{e^{3x-5}}{\operatorname{sen} \frac{\pi x}{3}}} \cdot \frac{e^{3x-5} \cdot 3 \cdot \operatorname{sen} \frac{\pi x}{3} - e^{3x-5} \cdot \cos \frac{\pi x}{3} \cdot \frac{\pi}{3}}{\operatorname{sen}^2 \frac{\pi x}{3}} = \frac{e^{3x-5} \left(3 \operatorname{sen} \frac{\pi x}{3} - \cos \frac{\pi x}{3} \cdot \frac{\pi}{3} \right)}{2 \left(\frac{e^{3x-5}}{\operatorname{sen} \frac{\pi x}{3}} \right) \cdot \operatorname{sen}^2 \frac{\pi x}{3}} =$$

$$\frac{e^{3x-5} \left(3 \operatorname{sen} \frac{\pi x}{3} - \cos \frac{\pi x}{3} \cdot \frac{\pi}{3} \right)}{2 e^{3x-5} \cdot \operatorname{sen} \frac{\pi x}{3}} = \frac{3}{2} - \frac{\pi}{6} \cdot \operatorname{cotg} \frac{\pi x}{3}$$

$$b) \ y = \log \left(\operatorname{tg} \frac{\sqrt{2x-1}}{3x-1} \right)^2 = 2 \log \left(\operatorname{tg} \frac{\sqrt{2x-1}}{3x-1} \right)$$

$$y' = 2 \cdot \frac{1}{\operatorname{tg} \frac{\sqrt{2x-1}}{3x-1}} \cdot \frac{1}{\cos^2 \frac{\sqrt{2x-1}}{3x-1}} \cdot \frac{1}{2\sqrt{2x-1}} \cdot \frac{2 \cdot (3x-1) - 3 \cdot \sqrt{2x-1}}{(3x-1)^2} \cdot \log e =$$

$$\frac{2 \cdot [3x-1-3(2x-1)] \cdot \log e}{\operatorname{tg} \frac{\sqrt{2x-1}}{3x-1} \cdot \cos^2 \frac{\sqrt{2x-1}}{3x-1} \cdot (3x-1)^2 \cdot \sqrt{2x-1}} = \frac{(-6x+4) \cdot \log e}{\operatorname{sen} \frac{\sqrt{2x-1}}{3x-1} \cdot \cos \frac{\sqrt{2x-1}}{3x-1} \cdot (3x-1)^2 \cdot \sqrt{2x-1}}$$

$$\text{c) } y' = \frac{1}{\operatorname{sen}^2 x + \arcsen^2 x} \cdot \left(2 \operatorname{sen} x \cdot \cos x + 2 \arcsen x \cdot \frac{1}{\sqrt{1-x^2}} \right) =$$

$$\frac{2 \left[\operatorname{sen} x \cdot \cos x \cdot \sqrt{1-x^2} + \arcsen x \right]}{\left[\operatorname{sen}^2 x + \arcsen^2 x \right] \cdot \sqrt{1-x^2}} = \frac{(1-x^2) \cdot \operatorname{sen} 2x + 2 \cdot \sqrt{1-x^2} \cdot \arcsen x}{(1-x^2) \cdot (\operatorname{sen}^2 x + \arcsen^2 x)}$$

$$\text{d) } y' = \frac{\left[\left(2x - 2 \cdot \frac{1}{2\sqrt{x}} \right) \cdot \operatorname{sen}^2 (3x+5) - (x^2 - 2\sqrt{x} + 3) \cdot 2 \operatorname{sen} (3x+5) \cdot \cos (3x+5) \cdot 3 \right] \cdot \log_a e}{\left(\frac{x^2 - 2\sqrt{x} + 3}{\operatorname{sen}^2 (3x+5)} \right) \cdot \operatorname{sen}^4 (3x+5)} =$$

$$\frac{\left[\left(2x - 2 \cdot \frac{1}{2\sqrt{x}} \right) \cdot \operatorname{sen}^2 (3x+5) - (x^2 - 2\sqrt{x} + 3) \cdot 2 \operatorname{sen} (3x+5) \cdot \cos (3x+5) \cdot 3 \right] \cdot \log_a e}{(x^2 - 2\sqrt{x} + 3) \cdot \operatorname{sen}^2 (3x+5)} =$$

$$\left[\frac{2 \cdot \sqrt{x^3} - 1}{\sqrt{x} \cdot (x^2 - 2\sqrt{x} + 3)} - 6 \cot g (3x+5) \right] \cdot \log_a e$$

$$\text{e) } y = \ln \left(\frac{10^{3x+1}}{2x+8} \right)^x = x \cdot \ln \left(\frac{10^{3x+1}}{2x+8} \right)$$

$$y' = \ln \left(\frac{10^{3x+1}}{2x+8} \right) + x \cdot \frac{1}{\frac{10^{3x+1}}{2x+8}} \cdot \frac{10^{3x+1} \cdot \ln 10 \cdot 3 \cdot (2x+8) - 2 \cdot 10^{3x+1}}{(2x+8)^2} =$$

$$\ln\left(\frac{10^{3x+1}}{2x+8}\right) + \frac{x \cdot \ln 10 \cdot 3 \cdot (2x+8) - 2x}{(2x+8)}$$

$$f) \quad y' = \operatorname{sen} \frac{\pi}{6} \cdot \cos \frac{\pi}{6} \cdot \frac{e^{2x} \cdot 2 \cdot \sqrt{x^2+1} - e^{2x} \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x}{x^2+1} =$$

$$\operatorname{sen} \frac{\pi}{6} \cdot \cos \frac{\pi}{6} \cdot \frac{e^{2x} \cdot 2 \cdot (x^2+1) - x \cdot e^{2x}}{(x^2+1) \cdot \sqrt{x^2+1}} = \frac{\sqrt{3} \cdot e^{2x} \cdot [2x^2 - x + 2]}{4 \cdot \sqrt{(x^2+1)^3}}$$

Calcular las derivadas de las funciones

$$a) \quad f(x) = \operatorname{arctg} \frac{x}{\sqrt{1-x^2}} \quad b) \quad f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}} \quad c) \quad f(x) = \sqrt{a^2 - x^2} + a \cdot \operatorname{arcsen} \left(\frac{x}{a} \right)$$

$$a) \quad f'(x) = \frac{1}{1 + \left(\frac{x}{\sqrt{1-x^2}} \right)^2} \cdot \frac{\sqrt{1-x^2} - x \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x)}{1-x^2} =$$

$$\frac{1-x^2+x^2}{\left(1 + \frac{x^2}{1-x^2}\right) \cdot (1-x^2) \cdot \sqrt{1-x^2}} = \frac{1}{\frac{1}{1-x^2} \cdot (1-x^2) \cdot \sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

$$b) \quad f'(x) = \frac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}} \cdot \left(1 + \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}} \right) \right)$$

$$c) \quad f'(x) = \frac{1}{2\sqrt{a^2 - x^2}} \cdot (-2x) + a \cdot \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \cdot \frac{1}{a} = \frac{-x}{\sqrt{a^2 - x^2}} + \frac{1}{\sqrt{\frac{a^2 - x^2}{a^2}}} = \frac{-x + a}{\sqrt{a^2 - x^2}}$$