

$$\begin{aligned}
 1) \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x^2+1}}{\sqrt{x^2-x}} \right)^x &= 1^\infty = \lim_{x \rightarrow +\infty} \left[\sqrt{\frac{x^2+1}{x^2-x}} \right]^x = \lim_{x \rightarrow +\infty} \left(\frac{x^2+1}{x^2-x} \right)^{x/2} = L \\
 \ln L &= \ln \lim_{x \rightarrow +\infty} \left(\frac{x^2+1}{x^2-x} \right)^{x/2} = \lim_{x \rightarrow +\infty} \left[\ln \left(\frac{x^2+1}{x^2-x} \right)^{x/2} \right] = \\
 &= \lim_{x \rightarrow +\infty} \left[\frac{x}{2} \ln \left(\frac{x^2+1}{x^2-x} \right) \right] = \infty \cdot 0 \text{ Ind} = \lim_{x \rightarrow +\infty} \frac{\ln \left(\frac{x^2+1}{x^2-x} \right)}{\frac{2}{x}} = \frac{0}{0} \text{ Ind} = \\
 &= \lim_{x \rightarrow +\infty} \frac{\ln(x^2+1) - \ln(x^2-x)}{\frac{2}{x}} = \frac{0}{0} \text{ Ind (L'Hop)} = \\
 &= \lim_{x \rightarrow +\infty} \frac{\frac{2x}{x^2+1} - \frac{2x-1}{x^2-x}}{-\frac{2}{x^2}} = \lim_{x \rightarrow +\infty} \frac{-x^2[2x^2 - 2x^2 - 2x^3 - 2x + x^2 + 1]}{2(x^2+1)(x^2-x)} = \\
 &= \lim_{x \rightarrow +\infty} \frac{x^4}{2x^4} = \frac{1}{2} \Rightarrow \ln L = \frac{1}{2} \Rightarrow L = e^{1/2} = \sqrt{e}
 \end{aligned}$$

otra forma de hacerlo:

$$\begin{aligned}
 \bullet \lim_{x \rightarrow +\infty} \left(\frac{x^2+1}{x^2-x} \right)^{x/2} &= \lim_{x \rightarrow +\infty} \left(1 + \frac{x^2+1}{x^2-x} - 1 \right)^{x/2} = \lim_{x \rightarrow +\infty} \left(1 + \frac{x+1}{x^2-x} \right)^{x/2} = \\
 &= \lim_{x \rightarrow +\infty} \left[\left(1 + \frac{1}{\frac{x^2-x}{x+1}} \right)^{\frac{x^2-x}{x+1}} \right]^{\frac{x}{2} \cdot \frac{x+1}{x^2-x}} = e^{\lim_{x \rightarrow +\infty} \frac{x(x+1)}{2(x^2-x)}} = e^{1/2} = \underline{\underline{\sqrt{e}}}
 \end{aligned}$$

$$\begin{aligned}
 2) \lim_{x \rightarrow +\infty} (\sqrt{x^2-3x+2} - \sqrt{x^2-x}) &= \infty - \infty \text{ Ind} = \\
 &= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2-3x+2} - \sqrt{x^2-x})(\sqrt{x^2-3x+2} + \sqrt{x^2-x})}{\sqrt{x^2-3x+2} + \sqrt{x^2-x}} = \\
 &= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2-3x+2})^2 - (\sqrt{x^2-x})^2}{\sqrt{x^2-3x+2} + \sqrt{x^2-x}} = \lim_{x \rightarrow +\infty} \frac{-2x+2}{\sqrt{x^2-3x+2} + \sqrt{x^2-x}} = \\
 &= \lim_{x \rightarrow +\infty} \frac{-2x}{\sqrt{x^2} + \sqrt{x^2}} = \lim_{x \rightarrow +\infty} \frac{-2x}{2x} = \underline{\underline{-1}}
 \end{aligned}$$

$$\begin{aligned}
 3) \lim_{x \rightarrow 2\pi} \frac{\cos^2 x + 2\cos x - 3}{6\cos x - 6} &= \frac{0}{0} \text{ Ind (L'Hop)} = \\
 &= \lim_{x \rightarrow 2\pi} \frac{-2\sin x \cos x - 2\sin x}{-6\sin x} = \lim_{x \rightarrow 2\pi} \frac{-2\sin x (\cos x + 1)}{-6\sin x} = \frac{2 \cdot 2}{6} = \underline{\underline{\frac{2}{3}}}
 \end{aligned}$$

* Se pueden aplicar infinitésimos equivalentes, aunque al sacar factor común es innecesario.

Otra forma de hacerlo:

$$\lim_{x \rightarrow 2\pi} \frac{\cos^2 x + 2 \cos x - 3}{6 \cos x - 6} = L \quad \cos^2 x + 2 \cos x - 3 = 0 \Rightarrow \cos x = \frac{-2 \pm \sqrt{4+12}}{2} \Rightarrow$$

$$\Rightarrow \cos x = \frac{-2 \pm 4}{2} \quad \left\{ \begin{array}{l} \cos x = 1 \\ \cos x = -3 \end{array} \right.$$

$$L = \lim_{x \rightarrow 2\pi} \frac{(\cancel{\cos x - 1})(\cos x + 3)}{6(\cancel{\cos x - 1})} = \frac{1+3}{6} = \frac{4}{6} = \underline{\underline{\frac{2}{3}}}$$

$$4) \lim_{x \rightarrow +\infty} \left(\frac{x^2 - x + 2}{x^2 + 2x + 5} \right)^{\sqrt{2x+1}} = 1^\infty \text{ Ind}$$

Por L'Hopital puede resultar muy largo.

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{x^2 - x + 2}{x^2 + 2x + 5} - 1 \right)^{\sqrt{2x+1}} = \lim_{x \rightarrow +\infty} \left(1 + \frac{-3x - 3}{x^2 + 2x + 5} \right)^{\sqrt{2x+1}} =$$

$$= \lim_{x \rightarrow +\infty} \left[\left(1 + \frac{1}{\frac{x^2 + 2x + 5}{-3x - 3}} \right)^{\frac{-3x - 3}{-3x - 3}} \right]^{\sqrt{2x+1} \cdot \frac{(-3x - 3)}{-3x - 3}} = e^{\lim_{x \rightarrow +\infty} \frac{(-3x - 3)\sqrt{2x+1}}{x^2 + 2x + 5}} =$$

$$= e^0 = 1 \quad * \text{ grado del denominador} > \text{ grado del numerador.}$$

$$5) \lim_{x \rightarrow 0} \frac{1 - 2x - e^x + \sin(3x)}{x^2} = \frac{0}{0} \text{ Ind (L'Hopital)} =$$

$$= \lim_{x \rightarrow 0} \frac{-2 - e^x + 3 \cos(3x)}{2x} = \frac{0}{0} \text{ Ind (L'Hop)} = \lim_{x \rightarrow 0} \frac{-e^x - 9 \sin(3x)}{2} = \underline{\underline{\frac{-1}{2}}}$$

$$6) \lim_{x \rightarrow +\infty} \frac{(5x^2 + 2)(x - 6)}{(x^2 - 1)(2x - 1)} = \lim_{x \rightarrow +\infty} \frac{5x^3 + \dots}{2x^3 + \dots} = \underline{\underline{\frac{5}{2}}}$$

$$7) \lim_{x \rightarrow 3} \left(\frac{x}{x-3} - \frac{4x^2}{x^2-9} \right) = \infty - \infty \text{ Ind} = \lim_{x \rightarrow 3} \frac{x(x+3) - 4x^2}{(x-3)(x+3)} =$$

$$= \lim_{x \rightarrow 3} \frac{-3x^2 + 3x}{(x-3)(x+3)} = \cancel{X} \quad \left\{ \begin{array}{l} \lim_{x \rightarrow 3^-} f(x) = +\infty \\ \lim_{x \rightarrow 3^+} f(x) = -\infty \end{array} \right.$$

$$8) \lim_{x \rightarrow +\infty} \left(\frac{5x^2 + x}{5x^2 - 3} \right)^{\frac{2x^2 - 1}{x}} = 1^\infty \text{ Ind} = \lim_{x \rightarrow +\infty} \left(1 + \frac{5x^2 + x}{5x^2 - 3} - 1 \right)^{\frac{2x^2 - 1}{x}} =$$

$$= \lim_{x \rightarrow +\infty} \left(1 + \frac{x+3}{5x^2 - 3} \right)^{\frac{2x^2 - 1}{x}} = \lim_{x \rightarrow +\infty} \left[\left(1 + \frac{1}{\frac{5x^2 - 3}{x+3}} \right)^{\frac{5x^2 - 3}{x+3}} \right]^{\frac{2x^2 - 1}{x} \cdot \frac{x+3}{5x^2 - 3}} =$$

$$= e^{\lim_{x \rightarrow +\infty} \frac{(2x^2 - 1)(x+3)}{x(5x^2 - 3)}} = e^{2/5} = \underline{\underline{\sqrt[5]{e^2}}}$$

$$9) \lim_{x \rightarrow +\infty} \frac{x^2+3x}{\sqrt{2x^2+1}} = \lim_{x \rightarrow +\infty} \frac{x^2}{\sqrt{2}x} = +\infty$$

$$10) \lim_{x \rightarrow -\infty} \frac{x^2+3x}{\sqrt{2x^2+1}} = \lim_{x \rightarrow -\infty} \frac{(-x)^2+3(-x)}{\sqrt{2(-x)^2+1}} = \lim_{x \rightarrow +\infty} \frac{x^2-3x}{\sqrt{2x^2+1}} = +\infty$$

* Antes de operar el límite comprobamos el dominio de:

$$f(x) = \frac{x^2+3x}{\sqrt{2x^2+1}} \quad 2x^2+1 > 0 \rightarrow 2x^2+1 \text{ es siempre positivo. Dom } f = \mathbb{R}.$$

$$11) \lim_{x \rightarrow +\infty} \frac{x^2+3x}{\sqrt{2x^3+1}} = +\infty$$

$$12) \lim_{x \rightarrow -\infty} \frac{x^2+3x}{\sqrt{2x^3+1}} \text{ no límite.}$$

* Calculamos el dominio de $f(x) = \frac{x^2+3x}{\sqrt{2x^3+1}}$.

$$2x^3+1 > 0 \rightarrow 2x^3+1=0 \rightarrow x = \sqrt[3]{\frac{-1}{2}} = \frac{-1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{-\sqrt[3]{2^2}}{2} \rightarrow \text{Si estudiamos el signo}$$

$$\text{Dom } f = \left(\frac{-\sqrt[3]{2^2}}{2}, +\infty \right).$$

$$13) \lim_{x \rightarrow 0} [x \cdot \cotg(2x)] = 0 \cdot \infty \text{ Ind} = \lim_{x \rightarrow 0} \frac{x}{\tan(2x)} = \frac{0}{0} \text{ Ind (L'Hop)} =$$

$$= \lim_{x \rightarrow 0} \frac{1}{2(1+\tan^2(2x))} = \frac{1}{2(1+0)} = \frac{1}{2}$$

Otra forma:

$$\lim_{x \rightarrow 0} [x \cotg(2x)] = 0 \cdot \infty \text{ Ind} = \lim_{x \rightarrow 0} \frac{x}{\tan(2x)} = \frac{0}{0} \text{ Ind} = \lim_{x \rightarrow 0} \frac{x}{\frac{\sin(2x)}{\cos(2x)}} =$$

$$= \lim_{x \rightarrow 0} \frac{x \cos(2x)}{\sin(2x)} = \frac{0}{0} \text{ Ind} = \lim_{x \rightarrow 0} \frac{x \cos(2x)}{2x} = \lim_{x \rightarrow 0} \frac{\cos(2x)}{2} = \frac{1}{2}$$

* Aplicando infinitesimales equivalentes: $x \rightarrow 0$: $\sin x \simeq x$.

$$14) \lim_{x \rightarrow \pi/2} \left[\left(\frac{\pi}{2} - x \right) \cotg x \right] = 0 \cdot \infty \text{ Ind} = \lim_{x \rightarrow \pi/2} \frac{\frac{\pi}{2} - x}{\cotg x} = \frac{0}{0} \text{ Ind (L'Hop)} =$$

$$= \lim_{x \rightarrow \pi/2} \frac{-1}{-1/\sin^2 x} = \lim_{x \rightarrow \pi/2} \sin^2 x = 1$$

$$* y = \cotg x = \frac{1}{\tan x} \rightarrow y' = \frac{-1/\cos^2 x}{\tan^2 x} = \frac{-1/\cos^2 x}{\frac{\sin^2 x}{\cos^2 x}} = \frac{-1}{\sin^2 x}$$

$$15) \lim_{x \rightarrow \sqrt{2}} \frac{e^{2x^2+x-1} - e^{x^2+x+1}}{x^2-2} = \frac{0}{0} \text{ Ind (L'Hop)} =$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(4x+1)e^{2x^2+x-1} - (2x+1)e^{x^2+x+1}}{2x} = \frac{(4\sqrt{2}+1)e^{3+\sqrt{2}} - (2\sqrt{2}+1)e^{3+\sqrt{2}}}{2\sqrt{2}} =$$

$$= \frac{e^{3+\sqrt{2}}(4\sqrt{2}+1-2\sqrt{2}-1)}{2\sqrt{2}} = \frac{2\sqrt{2}e^{3+\sqrt{2}}}{2\sqrt{2}} = e^{3+\sqrt{2}}$$

0 (ind) - x

$$\begin{aligned}
 16) \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\ln(1+x)} \right) &= \infty - \infty \text{ Ind} = \lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x \ln(1+x)} = \\
 &= \frac{0}{0} \text{ Ind (L'Hôp)} = \lim_{x \rightarrow 0} \frac{\frac{1}{1+x} - 1}{\ln(1+x) + \frac{x}{1+x}} = \lim_{x \rightarrow 0} \frac{\frac{-x}{1+x}}{\frac{(1+x)\ln(1+x) + x}{1+x}} = \\
 &= \lim_{x \rightarrow 0} \frac{-x}{(1+x)\ln(1+x) + x} = \frac{0}{0} \text{ Ind (L'Hôp)} = \lim_{x \rightarrow 0} \frac{-1}{\ln(1+x) + \frac{1+x}{1+x} + 1} = \\
 &= \frac{-1}{0+1+1} = -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 17) \lim_{x \rightarrow 0} \frac{x^3 \cos x}{\sin x + \sqrt{x}} &= \frac{0}{0} \text{ Ind (L'Hôp)} = \\
 &= \lim_{x \rightarrow 0} \frac{3x^2 \cos x - x^3 \sin x}{\cos x + \frac{1}{2\sqrt{x}}} = \lim_{x \rightarrow 0} \frac{x^2(3\cos x - x \sin x)}{\frac{2\sqrt{x} \cos x + 1}{2\sqrt{x}}} = \\
 &= \lim_{x \rightarrow 0} \frac{2\sqrt{x} x^2 (3\cos x - x \sin x)}{2\sqrt{x} \cos x + 1} = \frac{0}{0+1} = 0
 \end{aligned}$$

$$\begin{aligned}
 18) \lim_{x \rightarrow +\infty} (\sqrt{x^2+2x} - \sqrt{x^2-x+3}) &= \infty - \infty \text{ Ind} = \\
 &= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2+2x} - \sqrt{x^2-x+3})(\sqrt{x^2+2x} + \sqrt{x^2-x+3})}{\sqrt{x^2+2x} + \sqrt{x^2-x+3}} = \\
 &= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2+2x})^2 - (\sqrt{x^2-x+3})^2}{\sqrt{x^2+2x} + \sqrt{x^2-x+3}} = \lim_{x \rightarrow +\infty} \frac{3x-3}{\sqrt{x^2+2x} + \sqrt{x^2-x+3}} = \\
 &= \lim_{x \rightarrow +\infty} \frac{3x}{\sqrt{x^2} + \sqrt{x^2}} = \lim_{x \rightarrow +\infty} \frac{3x}{2x} = \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 19) \lim_{x \rightarrow +\infty} \left(\frac{2x+1}{2x+4} \right)^{\frac{x^2}{x+1}} &= 1^\infty \text{ Ind} = \\
 &= \lim_{x \rightarrow +\infty} \left(1 + \frac{2x+1}{2x+4} - 1 \right)^{\frac{x^2}{x+1}} = \lim_{x \rightarrow +\infty} \left(1 + \frac{-3}{2x+4} \right)^{\frac{x^2}{x+1}} = \\
 &= \lim_{x \rightarrow +\infty} \left[\left(1 + \frac{1}{\frac{2x+4}{-3}} \right)^{\frac{2x+4}{-3}} \right]^{\frac{x^2}{x+1} \cdot \frac{-3}{2x+4}} = e^{\lim_{x \rightarrow +\infty} \frac{-3x^2}{(x+1)(2x+4)}} = \\
 &= e^{-3/2} = \frac{1}{e^{3/2}} = \frac{1}{\sqrt{e^3}} = \frac{1}{e\sqrt{e}} \cdot \frac{\sqrt{e}}{\sqrt{e}} = \frac{\sqrt{e}}{e^2}
 \end{aligned}$$