20) $\lim\limits_{x\to 0} \dfrac{x\cos x - \operatorname{sen} x}{x^3} = \dfrac{0}{0}\text{ Ind } (\text{L'Hop}) = \lim\limits_{x\to 0} \dfrac{\cancel{\cos x} - x\operatorname{sen} x - \cancel{\cos x}}{3x^2} =$

$= \dfrac{0}{0}\text{ Ind } (\text{L'Hop}) = \lim\limits_{x\to 0} \dfrac{-(\operatorname{sen} x + x\cos x)}{6x} = \lim\limits_{x\to 0} \dfrac{-\operatorname{sen} x - x\cos x}{6x} =$

$= \dfrac{0}{0}\text{ Ind } (\text{L'Hop}) = \lim\limits_{x\to 0} \dfrac{-\cos x - (\cos x - x\operatorname{sen} x)}{6} = \dfrac{-1-(1-0)}{6} = \dfrac{-2}{6} = \dfrac{-1}{3}$

21) $\lim\limits_{x\to +\infty} \left(x \ln \dfrac{x+1}{x}\right) \overset{*}{=} \lim\limits_{x\to +\infty} \left[\ln\left(\dfrac{x+1}{x}\right)^x\right] = \ln \underbrace{\lim\limits_{x\to +\infty} \left(1+\dfrac{1}{x}\right)^x}_{=e} =$

$= \ln e = 1$

* Puede resolverse por L'Hôpital.

22) $\lim\limits_{x\to 1} \dfrac{x^3-x^2-x+1}{2x^3-5x^2+4x-1} = \dfrac{0}{0}\text{ Ind} = L$

• $x^3-x^2-x+1=0$

$$\begin{array}{c|cccc} & 1 & -1 & -1 & 1 \\ 1 & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$x^2-1=0 \Rightarrow x=\pm 1.$

$x^3-x^2-x+1 = (x-1)^2(x+1)$

• $2x^3-5x^2+4x-1=0.$

$$\begin{array}{c|cccc} & 2 & -5 & 4 & -1 \\ 1 & & 2 & -3 & 1 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

$2x^2-3x+1=0 \rightarrow x = \dfrac{3\pm\sqrt{9-8}}{4} \begin{cases} x=1 \\ x=\frac{2}{4}=\frac{1}{2} \end{cases}$

$2x^3-5x^2+4x-1 = (x-1)^2(2x-1)$

$L = \lim\limits_{x\to 1} \dfrac{\cancel{(x-1)^2}(x+1)}{\cancel{(x-1)^2}(2x-1)} = \dfrac{2}{1} = 2$

23) $\lim\limits_{x\to 1} x^{\frac{x}{\operatorname{sen}(\pi x)}} = 1^{\infty}\text{ Ind} = L$

$\ln L = \ln \lim\limits_{x\to 1} x^{\frac{x}{\operatorname{sen}(\pi x)}} = \lim\limits_{x\to 1} \ln x^{\frac{x}{\operatorname{sen}(\pi x)}} = \lim\limits_{x\to 1} \left[\dfrac{x}{\operatorname{sen}(\pi x)} \cdot \ln x\right] =$

$= \infty\cdot 0\text{ Ind} = \lim\limits_{x\to 1} \dfrac{x\ln x}{\operatorname{sen}(\pi x)} = \dfrac{0}{0}\text{ Ind } (\text{L'Hop}) = \lim\limits_{x\to 1} \dfrac{\ln x + \cancel{x}\cdot\frac{1}{\cancel{x}}}{\pi\cos(\pi x)} =$

$= \dfrac{0+1}{-\pi} = \dfrac{-1}{\pi}$ $\qquad \ln L = \dfrac{-1}{\pi} \rightarrow L = e^{-1/\pi} = \dfrac{1}{e^{1/\pi}} = \dfrac{1}{\sqrt[\pi]{e}}$

24) $\lim\limits_{x\to +\infty} \Big[(4x+1)\big[\ln(x-3) - \ln(x+2)\big]\Big] = \infty - \infty\text{ Ind} =$

$= \lim\limits_{x\to +\infty} \left[\dfrac{\ln(x-3) - \ln(x+2)}{\frac{1}{4x+1}}\right] = \dfrac{\infty}{\infty}\text{ Ind } (\text{L'Hop}) =$

$= \lim\limits_{x\to +\infty} \dfrac{\frac{1}{x-3} - \frac{1}{x+2}}{\frac{-4}{(4x+1)^2}} = \lim\limits_{x\to +\infty} \dfrac{\frac{\cancel{x}+2-\cancel{x}+3}{(x-3)(x+2)}}{\frac{-4}{(4x+1)^2}} = \lim\limits_{x\to +\infty} \dfrac{5(4x+1)^2}{-4(x-3)(x+2)} =$

$$= \lim_{x\to+\infty} \frac{5\cdot 16x^2+\dots}{-4x^2\dots} = \frac{5\cdot 16}{4} = 5\cdot 4 = \underline{\underline{20}}$$

25) $\lim_{x\to 0^+} (\sqrt{x}\cdot \log x) = 0\cdot(-\infty)$ Ind $= \lim_{x\to 0^+} \frac{\log x}{\frac{1}{\sqrt{x}}} = \frac{\infty}{\infty}$ Ind (L'Hôp) =

$$= \lim_{x\to 0^+} \frac{\frac{1}{x\cdot \log 10}}{\frac{-1}{2x\sqrt{x}}} = \lim_{x\to 0^+} \frac{-2\cancel{x}\sqrt{x}}{\cancel{x}\log 10} = \frac{-0}{\log 10} = \underline{\underline{0}}$$

26) $\lim_{x\to+\infty}\left(\frac{4x-2}{2x^3+6x^2+1}+1\right)^{x^2-2x} = 1^{\infty} =$

$$= \lim_{x\to+\infty}\left[\left(1+\frac{1}{\frac{2x^3+6x^2+1}{4x-2}}\right)^{\frac{2x^3+6x^2+1}{4x-2}}\right]^{x^2-2x\cdot\frac{4x-2}{2x^3+6x^2+1}} = e$$

$$= e^{\lim_{x\to+\infty}\frac{(4x-2)(x^2-2x)}{2x^3+6x^2+1}} = e^{\lim_{x\to+\infty}\frac{4x^3\dots}{2x^3\dots}} = \underline{\underline{e^2}}$$

27) Calcular a para que f sea continua en $\mathbb{R}$:

$$f(x) = \begin{cases} |x-4| & \text{si } x<a \\ x^2-6x+8 & \text{si } x\geq a \end{cases}$$

$g(x) = |x-4| \Rightarrow x-4=0 \Rightarrow x=4$

Signo de $x-4$:

	$-\infty$ … 4	4 … $+\infty$
$x-4$	$-$	$+$

$$g(x) = |x-4| = \begin{cases} -x+4 & \text{si } x<4 \\ x-4 & \text{si } x>4. \end{cases}$$

Para que f sea continua $|x-4|$ tiene que cortar a x^2-6x+8.

- Corte entre $g_1(x) = -x+4$ y $s(x) = x^2-6x+8$.

$$-x+4 = x^2-6x+8 \rightarrow x^2-5x+4=0 \rightarrow x = \frac{5\pm\sqrt{25-16}}{2}\begin{cases} x=4 \\ x=2 \end{cases}$$

- Corte entre $g_2(x) = x-4$ y $s(x) = x^2-6x+8$:

$$x-4 = x^2-6x+8 \rightarrow x^2-7x+12=0 \rightarrow x = \frac{7\pm\sqrt{49-48}}{2} = \begin{cases} x=4 \\ x=3 \end{cases}$$

no es válida para $x>4$.

Por lo tanto $a = \underline{\underline{2 \text{ ó } 4}}$.

28) $\lim_{x\to+\infty}(x+e^x)^{2/x} = \infty^0$ Ind $= L$

$$\ln L = \ln \lim_{x\to+\infty}(x+e^x)^{2/x} = \lim_{x\to+\infty}\left[\ln(x+e^x)^{2/x}\right] = \lim_{x\to+\infty}\left[\frac{2}{x}\ln(x+e^x)\right]$$

$$= 0\cdot\infty \text{ Ind} = \lim_{x\to+\infty}\frac{2\ln(x+e^x)}{x} = \frac{\infty}{\infty}\text{ Ind (L'Hôp)} = \lim_{x\to+\infty}\frac{2\,\frac{1+e^x}{x+e^x}}{1} =$$

$$= \lim_{x\to+\infty}\frac{2(1+e^x)}{x+e^x} = \lim_{x\to+\infty}\frac{\frac{2}{e^x}+\frac{2\cancel{e^x}}{\cancel{e^x}}}{\frac{x}{e^x}+\frac{\cancel{e^x}}{\cancel{e^x}}} = \frac{0+2}{0+1} = 2$$

Dividiendo cada término por e^x

$$\ln L = 2 \Rightarrow L = e^2$$

29) Determina el valor de k para que f. sea continua en x=0.

$$f(x)=\begin{cases}(1+x)^{\frac{\ln k}{x}} & \text{si } x\neq 0\\ 5 & \text{si } x=0\end{cases}$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}(1+x)^{\frac{\ln k}{x}} = 1^{\infty}\ \text{Ind} = L$$

$$\ln L = \ln \lim_{x\to 0}(1+x)^{\frac{\ln k}{x}} = \lim_{x\to 0}\ln(1+x)^{\frac{\ln k}{x}} = \lim_{x\to 0}\left[\frac{\ln k}{x}\cdot\ln(1+x)\right]=$$

$$= \infty\cdot 0\ \text{Ind} = \lim_{x\to 0}\frac{\ln k\cdot\ln(1+x)}{x} = \frac{0}{0}\ \text{Ind}\ (\text{L'Hôp}) =$$

$$= \lim_{x\to 0}\frac{\ln k\cdot\frac{1}{1+x}}{1} = \lim_{x\to 0}\frac{\ln k}{1+x} = \ln k.$$

$$\ln L = \ln k \Rightarrow L = k$$

Para que f. sea continua: $f(0) = \lim_{x\to 0} f(x)$.

$$\left.\begin{array}{l} f(0) = 5\\ \lim_{x\to 0} f(x) = k\end{array}\right\}\quad k=5$$

30) $$\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right) = \infty-\infty\ \text{Ind} = L$$

- $1-x^3=0$

$$\begin{array}{r|rrrr} & -1 & 0 & 0 & 1\\ 1 & & -1 & -1 & -1\\ \hline & -1 & -1 & -1 & \boxed{0}\end{array}$$

$$1-x^3 = (x-1)(-x^2-x-1) = (1-x)(x^2+x+1)$$

$$L = \lim_{x\to 1}\frac{x^2+x+1-3}{(1-x)(x^2+x+1)} = \frac{0}{0}\ \text{Ind} = L$$

- $x^2+x-2=0 \Rightarrow x = \frac{-1\pm\sqrt{1+8}}{2} = \begin{cases} x=1\\ x=-2\end{cases}$

$$L = \lim_{x\to 1}\frac{\cancel{(x-1)}(x+2)}{-\cancel{(x-1)}(x^2+x+1)} = \frac{-3}{4}$$

31) $$\lim_{x\to 0}\frac{1-\cos x^2}{8x^4} = \frac{0}{0}\ \text{Ind}\ (\text{L'Hop}) = \lim_{x\to 0}\frac{+\cancel{2}x\,\text{sen}\,x^2}{\overset{16}{\cancel{32}}x^3} = \frac{0}{0}\ \text{Ind}(\text{L'H}) =$$

$$= \lim_{x\to 0}\frac{\text{sen}\,x^2 + x\cdot 2x\cos x^2}{48x^2} = \frac{0}{0}\ \text{Ind}(\text{L'Hop}) = \lim_{x\to 0}\frac{2x\cos x^2 + 4x\cos x^2 - 4x^3\,\text{sen}\,x^2}{96x}$$

$$= \lim_{x\to 0}\frac{\overset{3}{\cancel{6}}\cancel{x}\cos x^2 - \overset{2}{\cancel{4}}x^{\cancel{3}\,2}\,\text{sen}\,x^2}{\overset{48}{\cancel{96}}\cancel{x}} = \frac{3-0}{48} = \frac{3}{48} = \frac{1}{16}$$

32) Calcula k sabiendo que $\lim_{x\to 0}\left[\cos(kx)\right]^{\frac{1}{x^2}} = \frac{1}{e^2} = S$

$$\lim_{x\to 0}\left[\cos kx\right]^{1/x^2} = 1^{\infty}\text{Ind} = L$$

$$\ln L = \ln \lim_{x\to 0}(\cos kx)^{1/x^2} = \lim_{x\to 0} \ln(\cos kx)^{1/x^2} = \lim_{x\to 0} \frac{1}{x^2}\ln(\cos kx) =$$

$$= \infty\cdot 0\,\text{Ind} = \lim_{x\to 0}\frac{\ln(\cos kx)}{x^2} = \frac{0}{0}\text{Ind}\,(\text{L'Hôp}) = \lim_{x\to 0}\frac{-\frac{k\cdot \operatorname{sen} kx}{\cos kx}}{2x} =$$

$$= \lim_{x\to 0}\frac{-k\operatorname{sen} kx}{2x\cdot\cos kx} = \frac{0}{0}\text{Ind}\,(\text{L'Hôp}) = \lim_{x\to 0}\frac{-k^2\cos kx}{2\cos kx - 2x\cdot k\operatorname{sen} kx} =$$

$$= \frac{-k^2}{2-0} = \frac{-k^2}{2} \qquad \ln L = \frac{-k^2}{2} \Rightarrow L = e^{-k^2/2}$$

• Como la solución del límite: $S = \frac{1}{e^2} = e^{-2}$

$$\text{Como } L = S \Rightarrow e^{\frac{-k^2}{2}} = e^{-2} \Rightarrow \frac{\cancel{-}k^2}{2} = \cancel{-}2 \Rightarrow k^2 = 4 \Rightarrow k = \pm 2$$

33) $$\lim_{x\to 2}\frac{x^2-\sqrt{8x}}{\sqrt{2x}-2} = \frac{0}{0}\text{Ind} = \lim_{x\to 2}\frac{(x^2-\sqrt{8x})(x^2+\sqrt{8x})(\sqrt{2x}+2)}{(\sqrt{2x}-2)(\sqrt{2x}+2)\,(x^2+\sqrt{8x})} =$$

$$= \lim_{x\to 2}\frac{(x^2)^2-(\sqrt{8x})^2}{(\sqrt{2x})^2-2^2} = \lim_{x\to 2}\frac{x^4-8x}{2x-4} = L$$

• $x^4-8x = x(x^3-8) = 0$

$$x^2+2x+4=0 \Rightarrow x = \frac{-2\pm\sqrt{4-16}}{2} \nexists \text{ sol.}$$

	1	0	0	-8
2		2	4	8
	1	2	4	0

$$x^4-8x = x(x-2)(x^2+2x+4)$$

$$L = \lim_{x\to 2}\frac{x\cancel{(x-2)}(x^2+2x+4)}{2\cancel{(x-2)}} = \frac{\cancel{2}\cdot 12}{\cancel{2}} = 12$$

34) ¿Para qué valores de k es f continua en x=0?

$$f(x) = \begin{cases} \left(\frac{x+1}{2x+1}\right)^{1/x} & \text{si } x<0 \\ 6x+k & \text{si } x\geq 0 \end{cases}$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(6x+k) = k$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left(\frac{x+1}{2x+1}\right)^{1/x} = 1^{\infty}\text{Ind} = \lim_{x\to 0^-}\left(1+\frac{x+1}{2x+1}-1\right)^{1/x} =$$

$$= \lim_{x\to 0^+}\left[\left(1+\frac{-x}{2x+1}\right)^{\frac{2x+1}{-x}}\right]^{\frac{1}{\cancel{x}}\cdot\frac{-\cancel{x}}{2x+1}} = e^{\lim_{x\to 0^+}\frac{-1}{2x+1}} = e^{-1} = \frac{1}{e}$$

$$\text{Como } \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) \rightarrow k = \frac{1}{e}$$

Para $k=\frac{1}{e}$: $\exists \lim_{x\to 0} f(x) = 1/e = f(0)$. por lo tanto f. es continua.