EJERCICIOS – NÚMEROS COMPLEJOS

1. Realiza las siguientes operaciones con números complejos:

- **a.** $4i + (3 - 5i) - (2 + i)$
- **b.** $2 \cdot \left(3 - \frac{1}{2}i\right) - 3 \cdot (-1 + i)$
- **c.** $(4 - 2i) \cdot (5 + 3i)$
- **d.** $(3 + i) \cdot (3 - i)$
- **e.** $(2 + 3i)^2$
- **f.** $(6 - 2i)^2$
- **g.** $\frac{1+3i}{3-i}$
- **h.** $\frac{2-5i}{4+2i}$
- **i.** $\frac{5}{2i}$
- **j.** $\frac{-1-i}{i}$

2. Halla los valores de:

- **a.** i^{17}
- **b.** i^{50}
- **c.** i^{301}
- **d.** i^{723}
- **e.** $i^{1000} + i^{485}$
- **f.** $1 + 2 \cdot i^{48}$
- **g.** $i^{18} \cdot (4 + 2i)$
- **h.** $\frac{i^{13}}{i^7}$
- **i.** $\frac{i^9}{1+i}$
- **j.** $\frac{i^{402}}{i^{500} - i^{274}}$

3. Dados los siguientes números complejos en forma binómica:

$$z_1 = 2 + 2i \qquad z_2 = -3 + 4i \qquad z_3 = 1 - 2i$$

- **a.** Halla sus opuestos y sus conjugados
- **b.** Representa gráficamente los números con sus conjugados y sus opuestos.
- **c.** Expresa los números dados, z_1, z_2 y z_3, en forma polar y en forma trigonométrica.

4. Halla las soluciones expresadas en números complejos de las siguientes ecuaciones:

- **a.** $x^2 + 2x + 10 = 0$
- **b.** $x^2 + 1 = 0$
- **c.** $x^2 - 3x + 3 = 0$

Soluciones - Ejercicios - Números Complejos

①

a) $4i + (3-5i) - (2+i) = 4i + 3 - 5i - 2 - i = \boxed{1-2i}$

b) $2\cdot\left(3-\frac{1}{2}i\right) - 3(-1+i) = 6 - i + 3 - 3i = \boxed{9-4i}$

c) $(4-2i)\cdot(5+3i) = 20 + 12i - 10i - 6i^2 = 20 + 2i + 6 = \boxed{26+2i}$

$(i^2=-1)$

d) $(3+i)\cdot(3-i) = 9 - \cancel{3i} + \cancel{3i} - i^2 = 9 + 1 = \boxed{10}$

e) $(2+3i)^2 = (2+3i)\cdot(2+3i) = 4 + 6i + 6i + 9\underbrace{i^2}_{-1} = 4 + 12i - 9 = \boxed{-5+12i}$

f) $(6-2i)^2 = (6-2i)\cdot(6-2i) = 36 - 12i - 12i + 4\underbrace{i^2}_{-1} = 36 - 24i - 4 = \boxed{32-24i}$

g) $$\frac{1+3i}{3-i} = \frac{(1+3i)\cdot(3+i)}{(3-i)\cdot(3+i)} = \frac{3+i+9i+3\overset{-1}{i^2}}{9+\cancel{3i}-\cancel{3i}-\underbrace{i^2}_{-1}} = \frac{\cancel{3}+10i-\cancel{3}}{9+1} = \frac{10i}{10} = \boxed{i}$$

h) $$\frac{2-5i}{4+2i} = \frac{(2-5i)\cdot(4-2i)}{(4+2i)\cdot(4-2i)} = \frac{8-4i-20i+10i^2}{16-\cancel{8i}+\cancel{8i}-4i^2} = \frac{8-24i-10}{16+4} = \frac{-2-24i}{20} =$$

$$= \frac{-2}{20} - \frac{24}{20}i = \boxed{-\frac{1}{10} - \frac{6}{5}i}$$

i) $$\frac{5}{2i} = \frac{5i}{2i^2} = \frac{5i}{-2} = \boxed{-\frac{5}{2}i}$$

j) $$\frac{-1-i}{i} = \frac{(-1-i)\cdot i}{i^2} = \frac{-i-i^2}{-1} = \frac{-i+1}{-1} = \frac{-i}{-1} + \frac{1}{-1} = i - 1 = \boxed{-1+i}$$

②

a) $i^{17} = i^{4\cdot4+1} = (i^4)^4\cdot i^1 = \underbrace{1^4}_{1}\cdot i = \boxed{i}$

$17 \,\underline{|\,4\,}$
$1 \quad 4$

$i^4 = 1$

b) $i^{50} = i^{12\cdot4+2} = \underbrace{(i^4)^{12}}_{1}\cdot i^2 = i^2 = \boxed{-1}$

$50 \,\underline{|\,4\,}$
$10 \quad 12$
2

c) $i^{301} = i^{4\cdot75+1} = \underbrace{(i^4)^{75}}_{1}\cdot i^1 = \boxed{i}$

$301 \,\underline{|\,4\,}$
$21 \quad 75$
1

d) $i^{723} = i^{180\cdot4+3} = \underbrace{(i^4)^{180}}_{1}\cdot i^3 = i^3 = \boxed{-i}$

$723 \,\underline{|\,4\,}$
$32 \quad 180$
03

$i^3 = i^2\cdot i = -i$

e) $i^{1000} + i^{485} = i^{250\cdot4} + i^{4\cdot121+1} = \underbrace{(i^4)^{250}}_{1} + \underbrace{(i^4)^{121}}_{1}\cdot i^1 = \boxed{1+i}$

$1000 \,\underline{|\,4\,}$
$20 \quad 250$
00
0

$485 \,\underline{|\,4\,}$
$08 \quad 121$
05
1

f) $1 + 2\cdot i^{48} = 1 + 2\cdot i^{4\cdot12} = 1 + 2\cdot\underbrace{(i^4)^{12}}_{1} = 1 + 2 = \boxed{3}$

$48 \,\underline{|\,4\,}$
$08 \quad 12$
0

$i^2=-1$ $\quad$ $i^3=i^2 i=-i$

g) $i^{18}\cdot(4+2i) = i^{4\cdot4+2}\cdot(4+2i) = (i^4)^4\cdot i^2\cdot(4+2i) = i^2(4+2i) = 4i^2+2i^3 =$

$= \boxed{-4-2i}$

$\begin{array}{l|l} 18 & 4 \\ \cline{2-2} 2 & 4 \end{array}$

h) $\frac{i^{13}}{i^7} = i^6 = i^4\cdot i^2 = i^2 = \boxed{-1}$ otra forma: $\frac{i^{13}}{i^7} = \frac{i^{4\cdot3+1}}{i^{4\cdot1+3}} = \frac{(i^4)^3\cdot i^1}{i^4\cdot i^3} = \frac{i}{i^3} = \frac{i}{-i} = \boxed{-1}$

$\begin{array}{l|l} 6 & 4 \\ \cline{2-2} 2 & 1 \end{array}$ $\quad$ $\begin{array}{l|l} 13 & 4 \\ \cline{2-2} 1 & 3 \end{array}$ $\quad$ $\begin{array}{l|l} 7 & 4 \\ \cline{2-2} 3 & 1 \end{array}$

i) $\frac{i^9}{(1+i)} = \frac{i^{4\cdot2+1}}{1+i} = \frac{(i^4)^2\cdot i^1}{1+i} = \frac{i}{1+i} = \frac{i(1-i)}{(1+i)\cdot(1-i)} = \frac{i-\overset{-1}{(i^2)}}{1-\cancel{i}+\cancel{i}-\underset{-1}{(i^2)}} = \frac{i+1}{1+1} = \frac{1+i}{2} =$

$= \boxed{\frac{1}{2}+\frac{1}{2}i}$

$\begin{array}{l|l} 9 & 4 \\ \cline{2-2} 1 & 2 \end{array}$

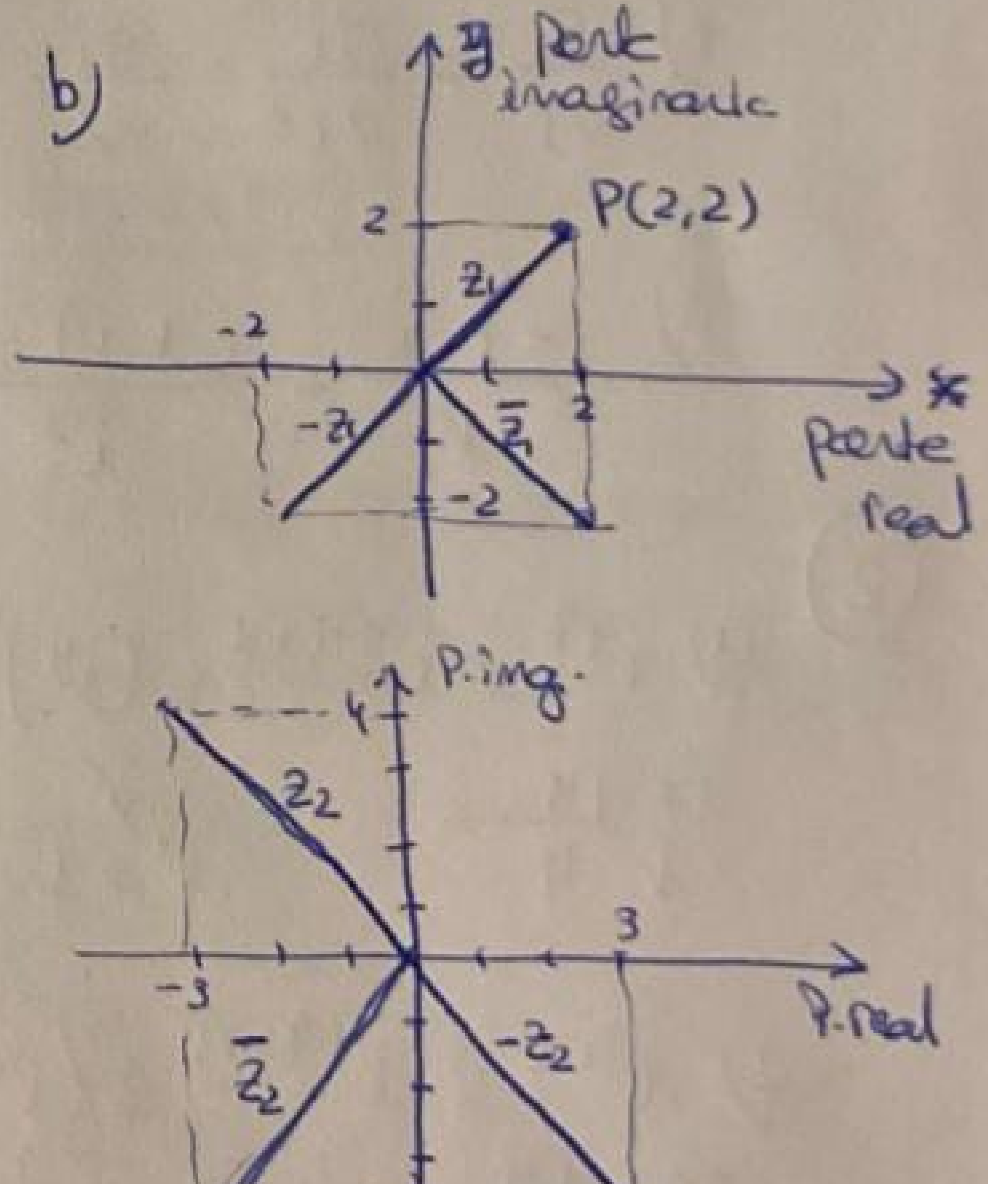
j) $\frac{i^{402}}{i^{500}-i^{274}} = \frac{i^{4\cdot100+2}}{i^{4\cdot125}-i^{4\cdot68+2}} = \frac{\overbrace{(i^4)^{100}}^{1}\cdot i^2}{\underbrace{(i^4)^{125}}_{1}-\underbrace{(i^4)^{68}}_{1}\cdot i^2} = \frac{i^2}{1-i^2} =$

$\begin{array}{l|l} 402 & 4 \\ \cline{2-2} 00 & 100 \\ 02 & \end{array}$ $\quad$ $\begin{array}{l|l} 500 & 4 \\ \cline{2-2} 10 & 125 \\ 20 & \\ 0 & \end{array}$ $\quad$ $\begin{array}{l|l} 274 & 4 \\ \cline{2-2} 34 & 68 \\ 2 & \end{array}$ $\quad$ $= \frac{-1}{1+1} = \boxed{\frac{-1}{2}}$

③ a) Opuesto de z_1 es $-z_1 \Rightarrow -z_1 = -2-2i$

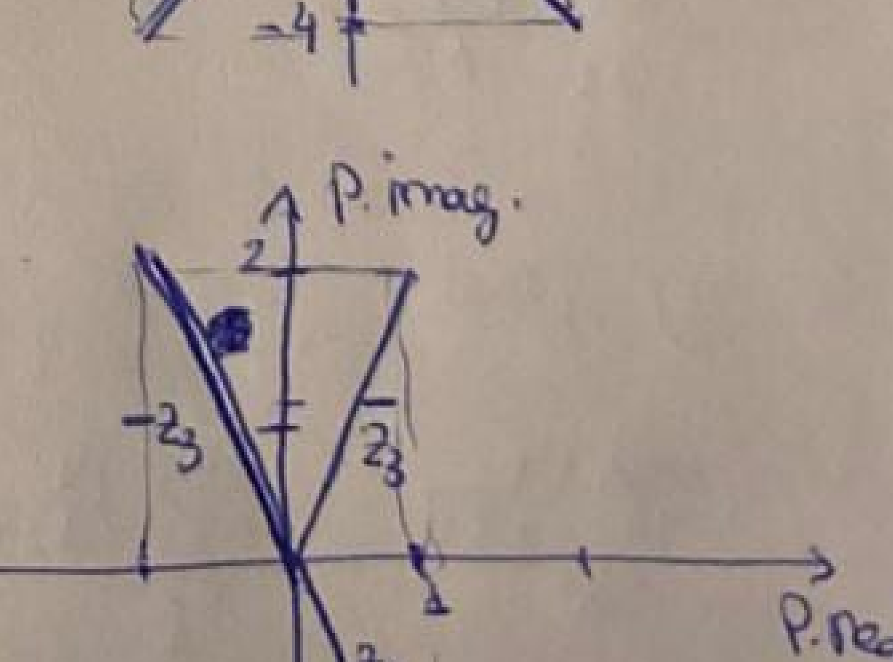
Conjugado de $z_1 \Rightarrow \overline{z_1} = 2-2i$

b)



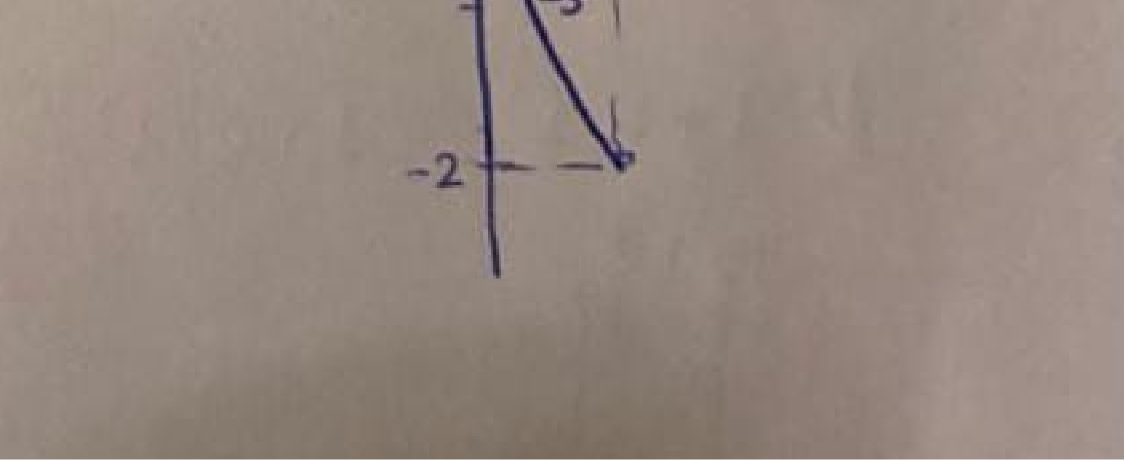
Opuesto de z_2: $-z_2 = 3-4i$

Conjugado de z_2: $\overline{z_2} = -3-4i$



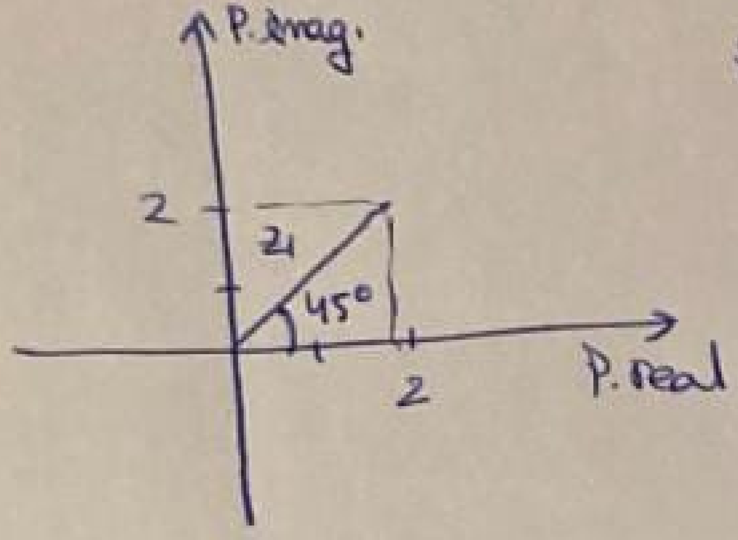
Opuesto de z_3: $-z_3 = -1+2i$

Conjugado de z_3: $\overline{z_3} = 1+2i$



c) $\boxed{z_1 = 2+2i} \Rightarrow \begin{cases} a=2 \\ b=2 \end{cases} \Rightarrow$ Forma polar: $\begin{cases} r=\sqrt{a^2+b^2} = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2} \\ \operatorname{tg}\alpha = \frac{b}{a} \Rightarrow \operatorname{tg}\alpha = \frac{2}{2} = 1 \Rightarrow \alpha = \operatorname{arctg} 1 = 45^\circ \end{cases}$

forma binómica

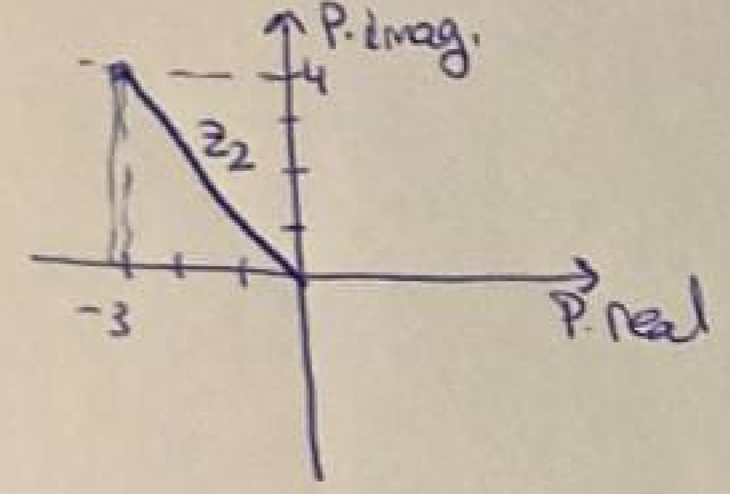


Está en el I cuadrante, entonces 45° es el ángulo.

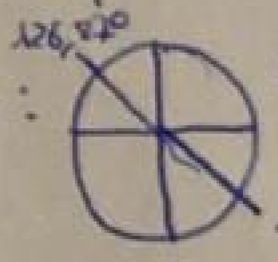
Forma polar: $z_1 = r_\alpha = \boxed{2\sqrt{2}_{45^\circ}}$

Forma trigonométrica: $\boxed{z_1 = 2\sqrt{2}\,(\cos 45^\circ + i \operatorname{sen} 45^\circ)}$

$\boxed{z_2 = -3+4i} \Rightarrow \begin{cases} a=-3 \\ b=4 \end{cases} \Rightarrow \begin{cases} r = \sqrt{9+16} = \sqrt{25} = 5 \\ \operatorname{tg}\alpha = \frac{4}{-3} \Rightarrow \alpha = \operatorname{arctg}\left(-\frac{4}{3}\right) = -53{,}13^\circ \Rightarrow 306{,}87^\circ \Rightarrow \end{cases}$



$\Rightarrow$ 306,87° está en el IV cuadrante pero el nº complejo está en el II cuadrante, luego:



$180 - 53{,}13 = \underline{= 126{,}87^\circ}$

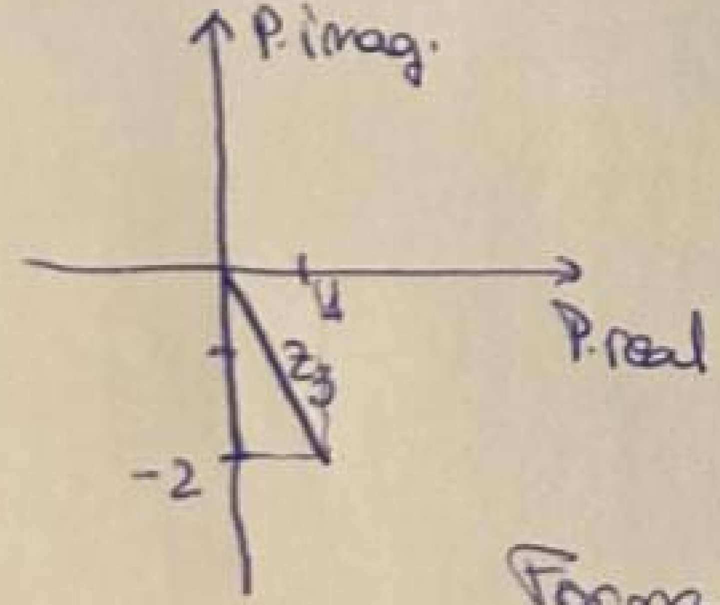
El ángulo es $\underline{126{,}87^\circ} \Rightarrow$

$\Rightarrow$ Forma polar: $\boxed{z_2 = 5_{126{,}87^\circ}}$

Forma trigonométrica: $\boxed{z_2 = 5 \cdot (\cos 126{,}87^\circ + i \operatorname{sen} 126{,}87^\circ)}$

$\boxed{z_3 = 1-2i} \Rightarrow \begin{cases} r = \sqrt{1^2+(-2)^2} = \sqrt{1+4} = \sqrt{5} \\ \operatorname{tg}\alpha = \frac{-2}{1} = -2 \Rightarrow \alpha = \operatorname{arctg}(-2) = -63{,}435^\circ \; (296{,}56^\circ) \Rightarrow \end{cases}$

$\Rightarrow$ IV cuadrante, luego el ángulo es $\underline{296{,}56^\circ}$:



Forma polar: $\boxed{z_3 = \sqrt{5}_{296{,}56^\circ}}$

Forma trigonométrica: $\boxed{z_3 = \sqrt{5} \cdot (\cos 296{,}56^\circ + i \operatorname{sen} 296{,}56^\circ)}$

④ a) $x^2+2x+10=0 \Rightarrow x = \frac{-2 \pm \sqrt{4-40}}{2} = \frac{-2 \pm \sqrt{-36}}{2} = \frac{-2 \pm \sqrt{36}\cdot\sqrt{-1}}{2} = \frac{-2 \pm 6i}{2} =$

$= \frac{-2}{2} \pm \frac{6}{2} i \Rightarrow \begin{cases} \boxed{-1+3i} \\ \boxed{-1-3i} \end{cases}$

b) $x^2+1=0 \Rightarrow x^2 = -1 \Rightarrow x = \pm\sqrt{-1} = \pm i \begin{cases} \boxed{i} \\ \boxed{-i} \end{cases}$

c) $x^2-3x+3=0 \Rightarrow x = \frac{3 \pm \sqrt{9-12}}{2} = \frac{3 \pm \sqrt{-3}}{2} = \frac{3 \pm \sqrt{3}i}{2} = \begin{cases} \boxed{\frac{3}{2} + \frac{\sqrt{3}}{2} i} \\ \boxed{\frac{3}{2} - \frac{\sqrt{3}}{2} i} \end{cases}$